

Abstracts of Oral Presentations

Drift-Kinetic Simulations of Neoclassical Transport*

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A new δf Eulerian kinetic code NEO-GK has been developed for numerical studies of neoclassical transport. NEO-GK serves a dual role: in addition to its practical value as a tool for high-accuracy neoclassical calculations, NEO-GK also functions as a stepping-stone (together with our nonlinear GK code GYRO) toward a full-F gyrokinetic code which integrates neoclassical transport and microturbulence. NEO-GK is based on a hierarchy of equations derived by expanding the fundamental drift-kinetic equation in powers of ρ_* , the ratio of the ion gyroradius to the system size. Thus, unlike NCLASS [1], NEO-GK represents a first-principles calculation of the neoclassical transport coefficients (particle flux, heat flux, bootstrap current, poloidal rotation, etc.) directly from the particle distribution function. NEO-GK extends previous numerical studies by including the self-consistent coupling of electrons and multiple ion species and the calculation of the first-order electrostatic potential via coupling with the Poisson equation. Fully general geometry is included, using either a full numerical equilibrium or the Miller local parameterized equilibrium model [2].

For benchmarking, comparisons of the second-order transport coefficients with various analytical theories are presented. Three model collision operators are compared in NEO-GK: the full Hirshman-Sigmar operator [3], which includes pitch-angle scattering dynamics and energy diffusion, and the reduced Hirshman-Sigmar operator [4] and the Connor model [5], both of which consist of just the sum of pitch angle scattering and momentum-restoring terms. For all three models, the ambipolar relation $\sum_s Z_s \Gamma_s = 0$, which can only be maintained with complete cross-species collisional coupling, is confirmed. With the full Hirshman-Sigmar collision operator, we confirm that the widely used Chang-Hinton analytical model [6] overestimates the ion energy flux in the intermediate aspect ratio regime. Agreement with the more accurate Taguchi banana regime model [7] is shown for validation. The reduced Hirshman-Sigmar operator and the Connor model are found to consistently underestimate the ion energy flux in all collisionality regimes. Through comparisons with simulations using an adiabatic electron model, the effects of kinetic electron dynamics on the ion transport coefficients are specifically identified. The effects of heavy impurity ions are also explored and limitations of multi-species collisionally-interpolated analytical theories are discussed. Furthermore, parameterized studies of the effects of shaping are performed using the Miller model. Results using DIII-D experimental profiles are also presented.

Finally, finite orbit width effects are studied via solution of the third and fourth-order drift-kinetic equations. Neoclassical transport near the magnetic axis is explored and the implications of non-local transport on the validity of the δf formulation are discussed.

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Nonlinear Consequences of Weakly Driven Energetic Particle Instabilities

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Abstract

The build-up of the energetic particle population in fusion plasmas is typically slow compared to the growth times of energetic-particle-driven instabilities. This feature draws special attention to nonlinear studies of unstable waves in the near-threshold regimes. The goal is to characterize the long-time behavior of the weakly dissipative waves and resonant particles in the presence of particle sources and sinks. This system exhibits an intricate nonlinear dynamics ranging from benign saturation of unstable modes to explosive growth of nonlinear phase space structures and avalanche-type events. The list of intriguing nonlinear effects also includes frequency-chirping phenomena.

This talk presents a first-principle theoretical approach to the nonlinear description of near-threshold instabilities, aimed at understanding a variety of experimental data from JET, MAST, DIII-D, C-Mod, NSTX and TFTR. The theory interprets the pitchfork splitting effect, observations of Alfvén Cascades, rapid frequency chirping in Alfvén modes and fishbones, anomalous losses of fast ions. The most recent progress refers to the role of transient perturbations (quasimodes) and geodesic acoustic effects in the observed spectrum of Alfvén Cascades and to the MHD-mechanism of frequency downshift during the decay of the fishbone pulse.

The talk blends recent results into a broader discussion of how the present theory responds to the experimental challenges and what kind of theoretical and computational advances could potentially resolve some of the critical outstanding issues.

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Modeling of Resistive Wall Mode with Full Kinetic Damping *

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Magnetohydrodynamic (MHD) stability of the plasma depends critically on the frequency and wave length of the perturbation. Future tokamaks are expected to operate in regimes where the external macro-scale perturbations have much lower frequencies than the intrinsic dynamical time scales of the particles [1]. This situation calls for a detailed re-examination of the assumptions on previous models of the response of the plasma to MHD perturbations [2]. We have developed a full drift kinetic version of MARS-F based on the kinetic formulation of MHD response [3]. The kinetic integrals are evaluated in a general toroidal geometry with flow, and self-consistently incorporated into the MHD formulation. In particular, the energy and momentum flux across the plasma surface is expressed in terms of the MHD perturbations. The new code has been tested on a Soloviev analytic equilibrium. It is observed that most of the kinetic damping comes from the particle precession drift resonances, from particles with nearly vanishing drift frequency. The RWM eigenmode structure is modified by the new kinetic terms. These kinetic terms may provide strong stabilization for high-pressure plasmas, such as those from DIII-D. Implication on the stability and plasma response [4] relevant for the resistive wall mode, with its time scale dramatically slowly by the external resistive wall, is discussed.

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Explosive Instability in Plasmas

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Some of the most dramatic events in nature are large scale disruptions of plasmas such as tokamak disruptions, solar flares, magnetospheric substorms and Edge Localized Modes. For all these examples the speed of disruption suggests that an explosive mechanism is responsible. I will discuss the fundamental nature of explosive growth and the issues for fusion plasmas. As an example I will outline a particular mechanism for the explosive release of energy in a plasma [1] via filamentary eruptions. I will also show experimental evidence [2] for its role in Edge Localized Modes. The size and connectivity of the filament and the mechanisms for releasing stored plasma energy will be discussed. At the heart of these mechanisms lie near singularity formation. These may include the formation of contact discontinuities and current sheets [3]

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Scalable Parallel Computation for Extended MHD Modeling of Fusion Plasmas

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A principal contribution of John Greene to plasma physics and magnetic fusion energy research is an understanding of the computational implications of the huge range of length and time scales in ideal and resistive MHD. His achievements include the efficient computation of the ideal MHD spectrum in the pioneering PEST code [1], as well as deep theoretical understanding of the role and behavior of the singular layer in linear resistive instabilities [2].

Today's version of this challenge is the need for scalable parallel computation for the nonlinear extended MHD modeling of fusion plasmas. Macroscopic modeling codes such as NIMROD and M3D have become major contributors to our understanding of Tokamaks and Innovative Confinement Concepts [3, 4]. The demands of multiple length scales lead to the need for high-order spatial representation and adaptive grids [5]. The demands of multiple time scales lead to the need for implicit time steps and the resulting requirement for efficient parallel solution of large, sparse linear systems.

Parallel solution of a linear system is called scalable if simultaneously doubling the number of dependent variables and the number of processors results in little or no increase in the computation time to solution. This property is essential for the efficient use of current and future generations of parallel supercomputers, with $10^4 - 10^5$ processors and petaflop speeds. Two approaches have been found to have this property for parabolic systems: multigrid methods [6] and domain decomposition methods [7].

Since extended MHD is primarily a hyperbolic rather than a parabolic system, dominated by the effects of ideal or two-fluid MHD waves, additional steps must be taken to parabolize the linear system to be solved by such a method. Such physics-based preconditioning methods have been pioneered by Luis Chacón, using finite volumes for spatial discretization, multigrid methods for solution of the preconditioning equations, and matrix-free Newton-Krylov methods for the accurate solution of the full nonlinear preconditioned equations [8].

The work described here is an extension of these methods to high-order spectral element methods for spatial discretization. Multigrid methods, appropriate for low-order spatial discretization, are replaced by the FETI-DP method of domain decomposition for high-order spectral elements. Application of physics-based preconditioning to a flux-source representation of the physics equations is discussed. The full set of preconditioned equations is solved by matrix-free Newton-Krylov iteration. The resulting scalability will be demonstrated for ideal and Hall MHD waves and for 2D magnetic reconnection.

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Low frequency eigenmodes due to Alfvén acoustic coupling in toroidal fusion plasma*

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Investigating the interaction of energetic ions with MHD modes is important not only for planning future self-sustained burning plasmas but it is an area where ideal MHD and kinetic theories are put to the test with great accuracy. We report on a theory and observations of a new class of global MHD solutions resulting from coupling of two Alfvén and acoustic fundamental MHD oscillations due to geodesic curvature. These modes predicted theoretically and numerically and called Beta-induced Alfvén-Acoustic Eigenmodes (BAAEs) have been recently observed in both low beta JET and high beta NSTX plasmas. They are capable of inducing strong radial transport of beam ions in NSTX especially in the presence together with multiple TAE instabilities.

Acoustic branch coupling also upshift the reversed shear Alfvén eigenmodes (RSAEs) eigenfrequency due to the finite pressure. We present a theory which explains the upshift of RSAE frequency due to finite pressure gradient. It is applied to NSTX observations at medium to high plasma beta. Experimental results supported by the ideal MHD code NOVA simulations clearly separate the effects of the plasma pressure and pressure gradient. We observe that the upshift of the RSAE frequency depends on mode number and is sensitive to the q -profile, which is in agreement with theoretical and numerical results. The upshift in frequency helps to understand observable “suppression” of RSAEs at high beta. Sweeping frequency RSAEs are seen to inflict the enhanced beam ion losses.

By understanding the range of BAAE and RSAE frequency excitation we are able to extend the use of so-called MHD spectroscopy to high beta plasma and by using frequency observations in new regimes to determine q_{min} . We have found that MSE measurements of q -profile in NSTX agree with the q_{min} values inferred from the BAAE and RSAE theories. Such observations would be a very important tool for diagnosing q -profile and other plasma parameters in ITER, CTF and other burning plasma experiments.

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On John M. Greene's MHD Equilibrium and Stability Work
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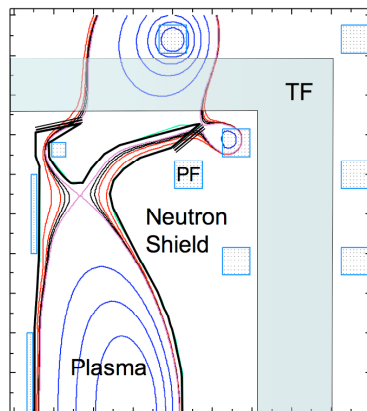
Katherine Weimer and I had the privilege of collaborating with John Greene on most of his MHD work. Here, I want to illustrate the breadth of his efforts by commenting on our incorporation of curvature effects into stellarator equilibrium models, Hamada Coordinates, the Mercier Criterion, resistive instabilities, and the PEST code. An illustration of how John kept our interactions exciting was the morning he walked into my office to say: "I know the answer; can you help me formulate the question?"

High Power Density Experiment (HPDX) - A Next Step Device in the Age of ITER

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A next generation high (reactor-level) power density machine (HPDX), capable of demonstrating Q_{XT} ($= Q_{\text{Exothermic}} = \text{fusion power} / \text{total electrical input}$) > 1 , is explored. (JET and NIF achieve $Q_{XT} \sim 1/10$). We give a solution for the extreme challenge of simultaneously maintaining $\beta \sim 6$ times ITER and power density ~ 10 times ITER and $Q_{XT} > 1$. The device will run in the advanced tokamak mode, $R=2.2\text{m}$, $A=2.5$, $\kappa=2.7$. Density peaking and enhanced elongation will be exploited to maximize beta for a given beta normal. Because HPDX is conceived as a prelude to a fusion reactor, its primary function is to demonstrate that appropriate conditions needed for the simultaneous occurrence of high beta, good confinement, stability (including thermal stability for a self-heated plasma), proper heat exhaust etc. can be created using only those methods that are likely to be pertinent under actual reactor.

A critical challenge of the beta optimized HPDX is the enormous heat load that will be incident on the divertor; the problem is further accentuated by density peaking. A radically new magnetic geometry that tends to isolate the divertor from the main plasma, the SuperX divertor (SXD), has been invented to solve this problem: its working is demonstrated by both 1D and 2D edge codes.



Example of a Super X Divertor at large R and with line length ~ 5 - 10 x standard divertor.

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Zonal Flow and Zonal Density Saturation Mechanisms for Trapped Electron Mode Turbulence

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Abstract

Mode coupling theory and gyrokinetic turbulence simulation are used to study the nonlinear saturation mechanisms of collisionless trapped electron mode (CTEM) turbulence [1, 2]. Turbulence simulations show that the importance of zonal flow is parameter sensitive, but is well characterized by the ExB shearing rate formula. The importance of zonal flow is found to be sensitive to temperature ratio, magnetic shear and electron temperature gradient. For parameter regimes where zonal flow is unimportant, zonal density (a purely radial density perturbation) is generated and is found to be the dominant saturation mechanism. In fact, CTEM turbulence saturates at physically reasonable levels with or without zonal flow. This is in stark contrast to ion temperature gradient driven turbulence where the zonal flow has an order of magnitude effect on the saturation level. A toroidal mode coupling theory is developed that agrees well with simulation in the initial nonlinear saturation phase (before fully developed turbulence ensues). The theory predicts nonlinear generation of the zonal density and then the feedback and nonlinear saturation of the unstable mode. Inverse energy cascade is also observed in CTEM turbulence and reported here. Further exploration on the magnetic fluctuation effect will be discussed. Finally, we have utilized GEM to investigate the Toroidal Alfvén Eigenmode (TAE) in tokamak plasmas. Here, we present numerical results showing the gap and continuum frequencies and compare directly with the eigenfrequency obtained from an eigenmode calculation.

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Modern Measurement Capabilities and Analysis Techniques for Validation of Turbulence Simulations

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Fluctuation diagnostics deployed at magnetic fusion experiments have advanced significantly in their breadth and depth and now measure turbulence characteristics in multiple plasma fields over a wide range of wavenumbers. The corresponding development of comprehensive nonlinear simulations of turbulence and transport is now allowing for quantitative comparisons of various turbulence properties between measurement and simulation, a crucial task required to validate simulations and ultimately to develop a predictive capability for turbulent transport. Specialized diagnostics have been developed to probe fluctuations in density, temperature, electrostatic potential, velocity and magnetic field, as well as to examine a wide range of wavenumbers from the ion gyroradius scale to electron scales. Quantities that can be compared include fluctuation amplitudes, wavenumber and frequency spectra, and spatial and temporal correlations. The corresponding development of advanced analysis techniques applied to multipoint, spatially resolved fluctuation measurements allows for extraction of critical characteristics such as zonal flow fields as well as the nonlinear dynamics of turbulence, including internal energy transfer. Detailed comparisons between measurements and simulation require "synthetic diagnostics" that model diagnostic measurement physics and performance to facilitate these direct, quantitative tests. Validation exercises also require the execution of focused experiments that systematically vary critical parameters to insure proper scaling behavior. Initial efforts to perform validation exercises and thereby test and challenge simulations will be presented.

“Oh, that.”

P. J. Morrison

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John Greene produced important research in a many areas. His work was characteristically original, of great depth and clarity, concise, mathematically clean, and broadly applicable. In this talk I will highlight some of his research, including: BGK modes, exact nonlinear solutions of the Vlasov-Poisson system; the inverse scattering transform, a method of solution for the KdV equation (and others) via a remarkable transformation; Greene’s residue criterion, which describes e.g. how magnetic surfaces break; and other work on Hamiltonian systems.

Tokamak Plasma Response to External Magnetic Perturbation

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Tokamak plasmas are sensitive to external magnetic perturbations as small as $|\delta B|/B \approx 10^{-4}$ can be important. An asymmetric external magnetic perturbation changes the plasma equilibrium, and the asymmetry of the equilibrium current contributes to the perturbed magnetic field $\delta\vec{B}$. The linear Ideal Perturbed Equilibrium Code (IPEC) finds the perturbed non-axisymmetric tokamak equilibrium with the same p and q profiles. Often the magnetic field strength is changed little at fixed points in space, but the wobble of the magnetic surfaces causes a large variation in the field strength that perturbs the action $J = \oint v_{\parallel} d\ell$ and, therefore, the particle drift motion.

The non-axisymmetry of the equilibrium currents tends to give:

(1) Strong poloidal coupling - The magnetic perturbation tends to be locally close to resonant with the magnetic field lines because that gives the largest distortion in the equilibrium plasma currents.

(2) Amplification of the external perturbation - The perturbed magnetic field can either be amplified or shielded by the perturbed plasma current, but the most important perturbations are those that are amplified by the non-axisymmetric distortions of the equilibrium plasma currents. Nevertheless, the plasma must shield the perturbation if the toroidal torque between an external magnetic perturbation and the plasma, $\int \vec{x} \times (\vec{j} \times \vec{B}) d^3x$ is sufficiently strong. Maxwell's equations imply this torque is given by an expression, which is approximately $(n/\mu_0) \oint (\delta\vec{B}^x \cdot \hat{n})(\delta\vec{B}^p \cdot \hat{n}) da \sin(n\varphi_p)$, where n is the toroidal mode number, $\delta\vec{B}^x \cdot \hat{n}$ is the perturbation due to external coil currents, $\delta\vec{B}^p \cdot \hat{n}$ the perturbation due to the plasma response, and φ_p is the phase difference between them.

The IPEC code has (1) resolved paradoxes in error field correction on NSTX and DIII-D (Phys. Rev. Lett. **99** 195003), (2) shown that ELM control coils in ITER could be designed to greatly reduce asymmetries in the central plasma while producing a strong perturbation at the plasma edge, (3) found that NSTX experiments on rotating error fields indicate strong plasma shielding due to the Maxwell limit on the torque.

Limitations of gyrokinetics on transport time scales

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ORAL PRESENTATION

Gyrokinetic models are widely used in simulations of tokamak drift wave turbulence. However, these models are only valid on time scales of the order of the saturation time of the turbulence and are unable to predict and evolve the equilibrium profiles of density, temperature and electric field. Calculating these profiles requires the extension of gyrokinetics to transport time scales. We are developing a self-consistent electrostatic model to calculate the distribution functions of both ions and electrons and the electrostatic potential for wavelengths that go from the size of the tokamak to smaller than the ion Larmor radius. A set of gyrokinetic variables is defined so that the gyrophase dependent part of the distribution is absorbed into the gyrokinetic variables by extending the linear treatment of Lee, Myra and Catto [1] to retain the usual nonlinear gyrokinetic modifications [2]. Using this procedure we find a nonlinear full f gyrokinetic equation correct to first order in a gyroradius over global scale length expansion. The electrostatic potential must be found to insure quasineutrality. In \mathcal{O} models, the gyrokinetic quasineutrality equation is normally used for this purpose. However, intrinsic ambipolarity requires that the ion distribution function be known at least to second order in gyroradius over characteristic length to calculate the long wavelength, axisymmetric components of the electrostatic potential self-consistently. Using the example of a steady-state θ -pinch, we prove that the quasineutrality equation fails to provide the axisymmetric piece of the potential even with a distribution function correct to second order. We also show that second order accuracy is enough if a moment description is used instead of the quasineutrality equation. These results demonstrate that the gyrokinetic quasineutrality equation is not the most effective procedure to find the electrostatic potential if the long wavelength components are to be retained in the analysis.

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Variational Symplectic Integrator for the Guiding Center Motion of Charged Particles for Long Time Simulations in General Magnetic Fields

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A variational symplectic integrator for the guiding center motion of charged particles in general magnetic fields is developed for long time simulation studies of magnetized plasmas [1]. Instead of discretizing the differential equations of the guiding center motion, the action of the guiding center motion is discretized and minimized to obtain the iteration rules for advancing the dynamics. Standard integrators only guarantee the error to be small in each time-step. The errors at different time-steps often accumulate coherently, and result in a large error over a large number of time-steps. The variational symplectic integrator conserves exactly a discrete Lagrangian symplectic structure, and has better numerical properties over long integration time, compared with standard integrators, such as the standard and variable time-step 4th order Runge-Kutta methods. The symplectic integrator conserves the symplectic structure exactly, and guarantees that the energy error is bounded by a small number for all the time-steps. To construct the symplectic algorithm, it is necessary to adopt the variational approach because standard symplectic integrators are only valid for canonical Hamiltonian systems, and the guiding-center dynamics in general magnetic field does not possess a (global) canonical symplectic structure. Numerical examples with more than 25 million time-steps are given to demonstrate the superiority of the variational symplectic integrator. This significant improvement in long term simulation capability of gyrokinetics is a direct, otherwise-impossible result of the geometric formulation of the gyrokinetic theory using the modern language of differential geometry [2, 3].

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A comprehensive analytical model for 2D magnetic reconnection in resistive, Hall, and electron magnetohydrodynamics*

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Magnetic reconnection – the topological rearrangement of magnetic fields immersed in highly electrically conductive plasma and the accompanying magnetic energy release – usually happens on very fast time scales and is of fundamental importance for both laboratory and naturally occurring plasmas. While computational evidence exists [1] of fast (e.g. dissipation independent) reconnection rates, there is no fundamental comprehensive analytical model that is able to explain them. In this work, we propose such a quantitative model (à la Sweet-Parker). The model describes the magnetic field dissipation region for 2D steady-state (e.g. at or around the time of maximum reconnection rate) magnetic reconnection without a guide field. It takes into account plasma resistivity, electron viscosity (hyper-resistivity), and electron inertia. It recovers the Sweet-Parker results for small ion inertial length scales [2], the electron MHD results in the limit of ion inertial length scales larger than other relevant scales [3], and is valid everywhere in between [4]. The model gives predictions for the dissipation region aspect ratio, the incoming and outgoing plasma flows magnitudes, the ratio of created to dissipated magnetic fields, and the reconnection rate as a function of dissipation and inertial parameters. It has been benchmarked (and is in excellent agreement) with more than thirty-five non-linear simulations of magnetic coalescence problem with resistivity, hyper-resistivity, and ion inertial length scale varying over many orders of magnitude each. It confirms a number of long-standing empirical results and resolves several outstanding controversies (e.g. whether only the open X-point or also the elongated dissipation regions are allowed in Hall MHD reconnection). The model can be straightforwardly expanded to include effects of ion viscosity, guide field, and so on.

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The Ideal Magnetohydrodynamic Peeling Mode Instability

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The rapid deposition of energy by Edge Localised Modes (ELMs) onto plasma facing components poses a threat to the performance of large Tokamaks such as ITER and DEMO. The trigger for ELMs is believed to be the ideal Magnetohydrodynamic Peeling-Ballooning instability, but recent numerical calculations have suggested that a plasma equilibrium with an X-point - as is found in all modern Tokamaks - is stable to the Peeling mode. This is surprising, because previous analytical calculations (G. Laval, R. Pellat, J. S. Soule, *Phys Fluids*, **17**, 835, (1974)), found the Peeling mode to be unstable in cylindrical plasmas with arbitrary cross-sectional shape. However the analytical calculation only applies to a Tokamak plasma in a cylindrical approximation. To avoid these shortcomings Webster & Gimblett[1], re-examined the assumptions made in cylindrical geometry calculations, and generalised the calculation to a Tokamak at marginal stability. The resulting equations solely describe the Peeling mode, and are not complicated by coupling to the ballooning mode, for example.

It was found that: (i) at marginal stability a radial plasma displacement induces a skin current that is parallel and proportional to both the equilibrium edge current and the amplitude of the radial plasma displacement. (ii) the boundary condition relating the plasma displacement to the vacuum field's perturbation is identical to requiring equality of the normal components of the perturbed plasma and vacuum magnetic field, evaluated at the equilibrium plasma position. (iii) marginal stability of the Peeling mode at high toroidal mode number is identical to solving $\delta W_S + \delta W_V = 0$, where δW_S , δW_V , and δW_F are the surface, vacuum, and plasma contribution to the energy principle's $\delta W = \delta W_F + \delta W_S + \delta W_V$. (iv) suggested defining the Peeling mode as one for which δW_F may be neglected, with (in)stability determined by the sign of $\delta W_S + \delta W_V$. (v) for the trial function used by Laval et al, Peeling mode (in)stability is determined by a single parameter Δ' that involves the poloidal average of the normalised jump in the radial derivative of the perturbed magnetic field's normal component. The calculation of Δ' in such a way as to capture the effect of the X-point, without the need for a discretisation of space as required by most numerical methods, is the subject of the remainder of this contribution.

For potentials satisfying Laplace's equation in systems that are approximately 2-dimensional, conformal transformations are often used to calculate fields in complicated geometries. However the usual requirement for the field's normal component to be zero on the boundary is unnecessarily restrictive, as may be seen by calculating how the boundary conditions transform. Similarly it is possible to calculate how other quantities transform as we map between the two systems, to obtain analytic expressions for Δ' and the vacuum energy δW_V , in terms of a sum of Fourier coefficients. The Fourier coefficients are given in terms of an integral involving the straight field line angle. The equilibrium vacuum field (for a shaped cross section) may be calculated also, and used to obtain an analytic expression for the straight field line angle at the plasma-vacuum boundary. Subsequently the Fourier coefficients and their sum may be calculated, and it is found that at high toroidal mode number, a perturbation from a single Fourier mode in straight field line coordinates (Laval et al's trial function), has the vacuum energy δW_V and Δ' the same as for a circular cross section, with $\delta W_V \simeq 2\pi^2 \Delta^2 \frac{|\xi_m|^2}{R} m$ and $\Delta' = -2m$. Further applications, and consequences for the stability of the Peeling mode are discussed.

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TEMPEST simulations of the neoclassical radial electric field*

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We present gyrokinetic neoclassical simulations of tokamak plasmas with self-consistent two-dimensional electric field using a fully nonlinear (full-f) continuum code TEMPEST. A set of gyrokinetic equations are discretized on a five dimensional computational grid in phase space. The present implementation is a Method of Lines approach where the phase-space derivatives are discretized with finite differences and implicit backwards differencing formulas are used to advance the system in time. The fully nonlinear Boltzmann model is used for electrons. The neoclassical electric field is obtained for the first time by solving gyrokinetic Poisson equation with self-consistent poloidal variation. Alternatively the neoclassical electric field can be evaluated according to the radial Ampere's law averaged over a closed-flux surface $4\pi\langle\mathbf{J}\cdot\nabla\psi\rangle+\partial\langle\mathbf{E}\cdot\nabla\psi\rangle\partial t=0$ where ψ is the poloidal magnetic flux, $\langle\cdots\rangle$ represents the flux surface average, and \mathbf{J} is the sum of all the current in the plasma, including the classical polarization current, gyroviscosity current, and the ion guiding-center current (the electron current is typically neglected in tokamak geometry, because it is smaller than the ion current by a factor of a mass ratio m_e/m_i) [1-3]. The steady-state neoclassical radial electric field E_ψ on a magnetic surface is obtained from the condition $\langle j_\psi\rangle=0$. However, this method is incomplete in the sense that the poloidal electric field cannot be solved simultaneously in a consistent way. This is an unsatisfactory situation since the potential varies significantly in the edge plasma around the X-point and in the divertor leg region due to contact with divertor plates. The gyrokinetic Poisson equation is seldom used because the small coefficient in front of Poisson operator associated with the gyroradius makes the equation singular when $\rho_i\ll L_P\ll L_B$. Here, $L_P=|\nabla(\ln P)|^{-1}$ is the characteristic gradient scale length for the plasma profile, $L_B=|\nabla(\ln B)|^{-1}$ the characteristic length for the magnetic field, and $\rho_i=v_{Thi}/\Omega_{ci}$ the ion gyroradius. For this reason, no single code exists to simulate both neoclassical transport and turbulence. However, there are efforts being undertaken to try to solve this dilemma [4,5]. In this work, we develop a method to efficiently solve the gyrokinetic Poisson equation to remove the singularity and to correctly yield the neoclassical radial electric field. We prove here the mathematical equivalence of the two approaches for solving neoclassical electric field in the large-aspect-ratio limit. With our TEMPEST code we compute radial particle and heat flux, the dynamics of relaxation of poloidal rotation, including Geodesic-Acoustic Mode (GAM), its radial propagation, collisional decay and the development of neoclassical electric field, which we compare with neoclassical theory with a Lorentz collision model. The present work provides a numerical scheme and a new capability for self-consistently studying important aspects of neoclassical transport, rotations and turbulence in toroidal magnetic fusion devices.

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