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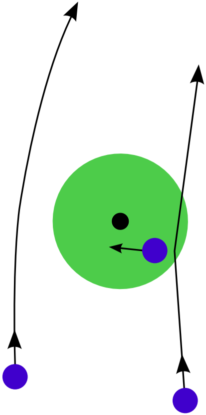
Kinetic effects of partially screened impurities in runaway-electron mitigation scenarios



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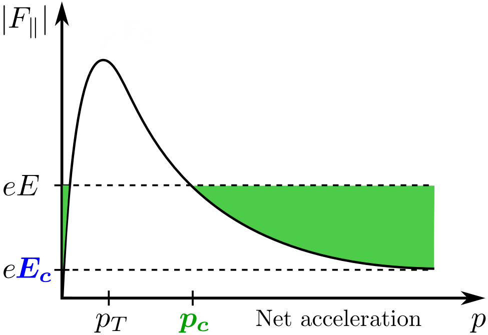
Runaway electrons

- Electrons with $p > \boldsymbol{p_c}$ accelerated if $E > \boldsymbol{E_c}$
  $p = \gamma \frac{v}{c}$
- In disruptions: runaways accelerated to $E \gtrsim 10$ MeV $\leftrightarrow p \gtrsim 20$
- Runaways may damage fusion power plants



Runaway mitigation

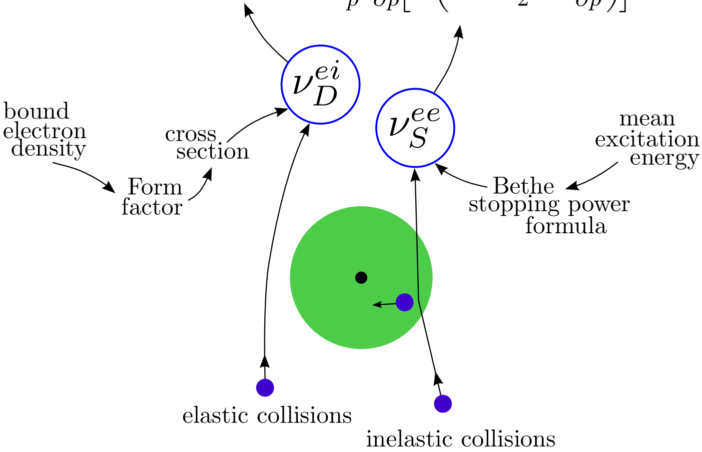
- Dissipate runaways: inject heavy ions (e.g. massive gas injection), typically $n_\mathrm{Z} > n_\mathrm{D}$
- Cold plasma: weakly ionized impurities
- Experiments: more effective than predicted[1]
- Previous work: no kinetic simulations, use simplified models[2−4]

Hollmann et al., PoP (2015)

[1]Hollmann et al., PoP (2015)
[2]Kirillov et al., Fizika Plasmy (1975)
[3]Zhogolev and Konovalov, VANT (2014)
[4]Martin-Solis et al., PoP (2015)

Generalized Fokker–Planck operator

$$C^e_{\text{test}} = \boldsymbol{\nu_D}\mathscr{L}(f_e) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\left(\boldsymbol{\nu_S} f_e + \frac{1}{2}\nu_\parallel p \frac{\partial f_e}{\partial p}\right)\right]$$



partial screening

Elastic collisions ν_D^{ei}

Cross section[5] in Born approximation, valid for $v/c \gg Z\alpha$

$$\frac{d\sigma_{ej}}{d\Omega} = \left(\frac{r_0^2}{4p^4}\right)\left(\frac{\cos^2(\theta/2)p^2 - 1}{\sin^4(\theta/2)}\right)|Z_j - F_j(q)|^2$$

$$\text{Form factor: } F_j(q) = \int \rho_{e,j}(r)\, e^{-i\mathbf{q}\cdot\mathbf{r}/a_0}\, d\mathbf{r}$$

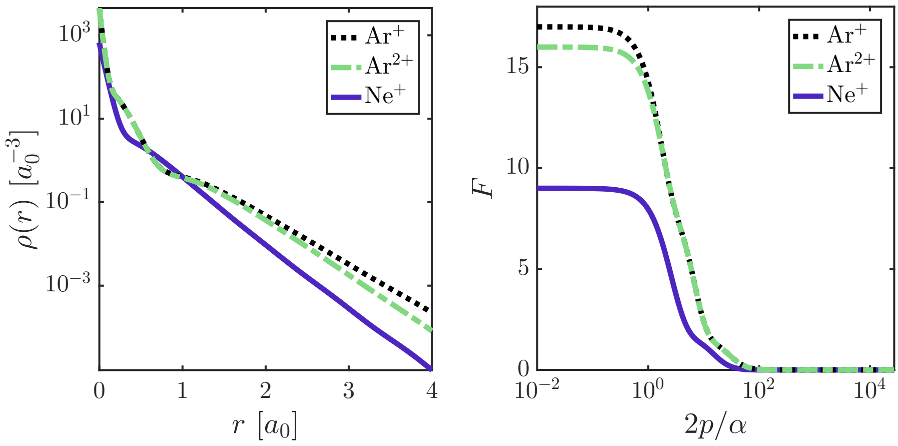
$q = \frac{2p}{\alpha}\sin(\theta/2)$, Z: atomic number, Z_0: net charge

Limits:
Low energy $|Z - F| \to Z_0$: **complete screening** (usual case)
High energy $|Z - F| \to Z$: **no screening** (interaction with nucleus)

[5] Landau and Lifshitz, *Quantum mechanics: non-relativistic theory* (2013)

Elastic collisions: density and form factor

From density functional theory (DFT)



Elastic collisions ν_D^{ei}

$$\nu_D^{ei} = \nu_{D,\mathrm{cs}}^{ei} \left(1 + \frac{1}{\sum_j n_j Z_{0,j}^2} \sum_j n_j Z_{0,j}^2 \frac{g_j(p)}{\ln \Lambda} \right)$$

completely screened collision frequency

DFT simulations

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completely screened collision frequency

Full formula

$$g_j(p) = \int_0^1 \left(\frac{[Z_j - F_j(q)]^2}{Z_{0,j}^2} - 1\right)\frac{dx}{x}$$

DFT simulations

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TF-DFT model

$$g_j(p) = \frac{2}{3}(X_j^2 - 1)\ln(y_j^{3/2}+1) - \frac{2}{3}\frac{(X_j-1)^2 y_j^{3/2}}{y_j^{3/2}+1}$$

$X_j = Z_j/Z_{0,j}$

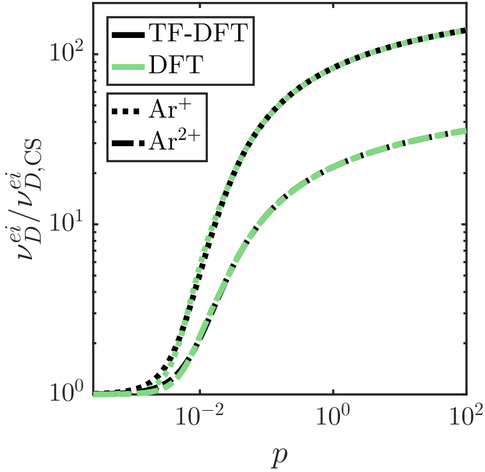
effective length a_j

$y_j = 2a_j p/\alpha$

DFT simulations

Enhancement of deflection frequency

- Compare to completely screened $\nu^{ei}_{D,\mathrm{CS}}$
- Excellent agreement with full DFT
- $p \gg 1$: $\nu^{ei}_D/\nu^{ei}_{D,\mathrm{CS}} \sim (Z/Z_0)^2 \sim 10^2$
- Significant effect already at $p \sim p_c \sim 0.1$



Inelastic collisions ν_S^{ee}

- Commonly used Rosenbluth–Putvinski rule of thumb[6]: $\nu^{ee}_{S,\text{RP}} \approx \nu^{ee}_{S,\text{CS}}\left[1 + \frac{1}{2}\sum_j n_j N_{e,j}/n_e\right]$
- Bethe stopping power formula[7] (valid for $p \gtrsim 0.03$; matched with low energy asymptote)

$$\nu_S^{ee} = \nu^{ee}_{S,\text{CS}}\left\{1 + \sum_j \frac{n_j N_{e,j}}{n_e \ln\Lambda}\left[\frac{1}{k}\ln\left(1 + h_j^k\right) - \beta^2\right]\right\}.$$

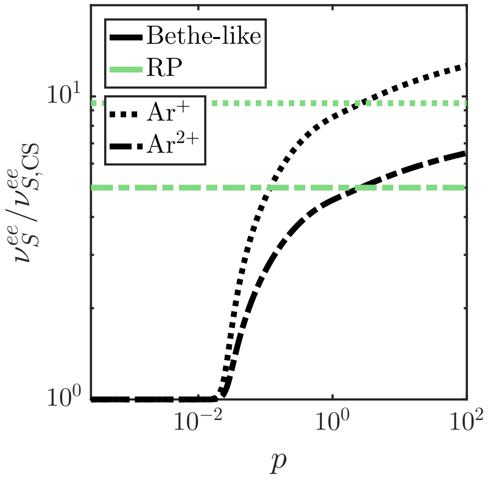
$h_j = p\sqrt{\gamma - 1}/I_j$, I_j mean excitation energy, $k = 5$

[6]Rosenbluth and Putvinski, NF (1997)

[7]Bethe, Annalen der Physik (1930)

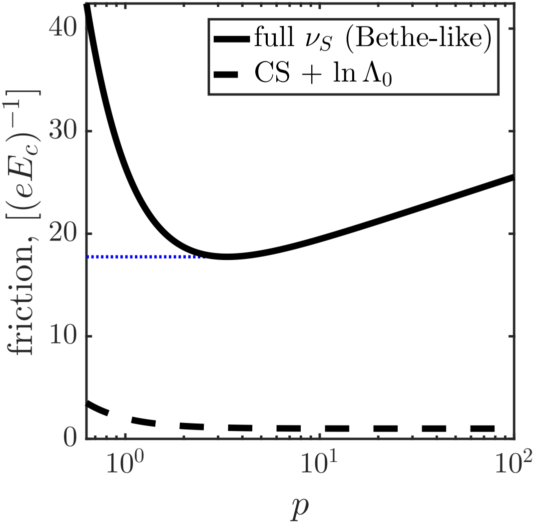
Enhancement of slowing-down frequency

- Transition around $p = 0.02$
- RP model inaccurate for $p \lesssim 0.1 \sim p_c$



Inelastic: enhanced critical electric field

- Effective critical field from force balance: $e\boldsymbol{E}_{\boldsymbol{c},\mathbf{eff}} = \min[p\nu_S(p)]$
- Constant $\ln\Lambda$ and no screening effects: $E_c = \frac{n_e e^3 \ln\Lambda_0}{4\pi\epsilon_0^2 m_e c^2}$



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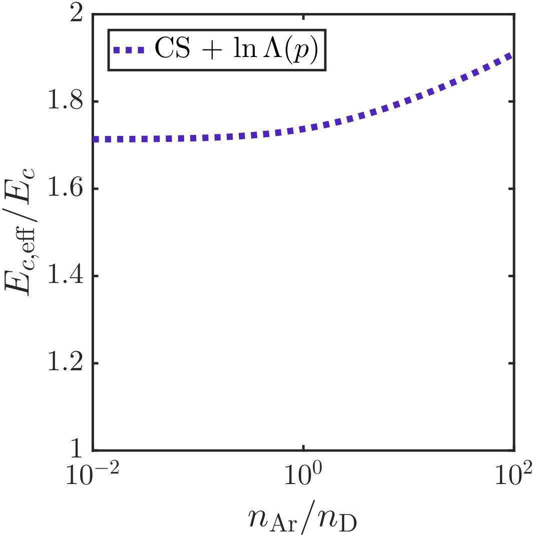
- Significant effect from energy-dependent $\ln\Lambda = \ln\Lambda_0 + \ln(p/p_T)$



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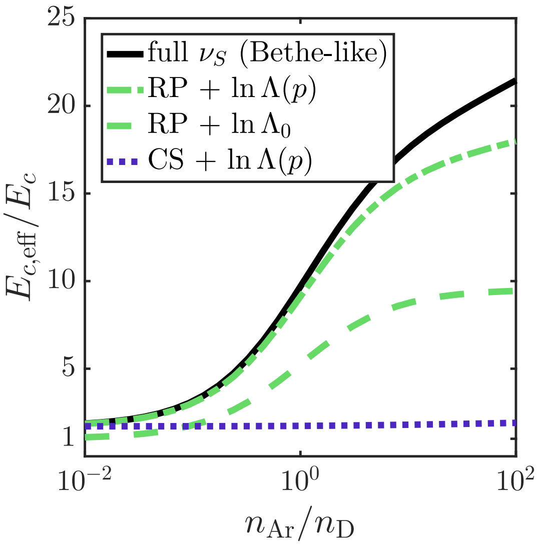
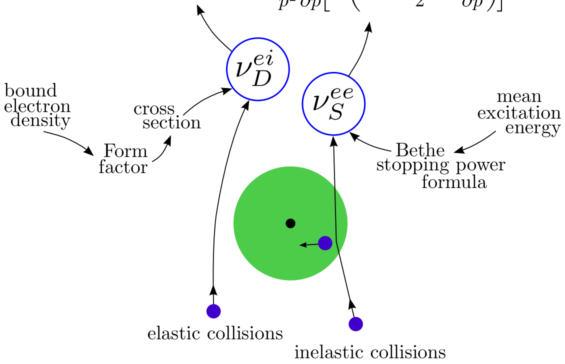
- Significant effect from energy-dependent
  $\ln\Lambda = \ln\Lambda_0 + \ln(p/p_T)$
- Large enhancement due to partial screening



Simulate dissipation of runaway beam

- Implemented in Fokker–Planck solver CODE[8]
- Test case:
  - Initial distribution using constant, high E-field
  - Compare different models

$$C^{e}_{\text{test}} = \boldsymbol{\nu_D} \mathscr{L}(f_e) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\left(\boldsymbol{\nu_S} f_e + \frac{1}{2}\nu_{\|} p \frac{\partial f_e}{\partial p}\right)\right]$$

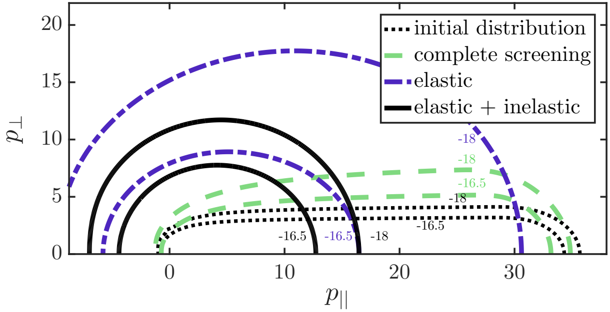


partial screening

[8]Landreman et al., CPC (2014); Stahl et al., NF (2016)

Effect on distribution function

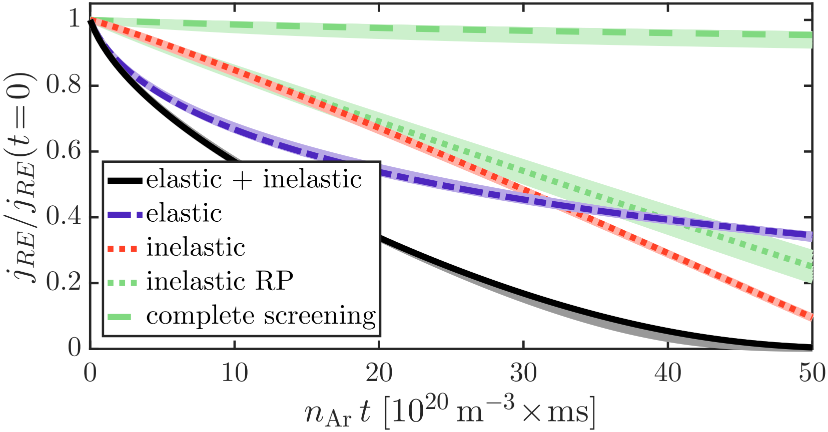
Collisional deceleration of initial beam-like distribution.
Contours of $\log_{10}(F)$, $F = (2\pi m_e T)^{3/2} f_e / n_e$



Parameters: 25 ms collisional deceleration $T = 10\,\mathrm{eV}$, Ar^+ with density $n_{\mathrm{Ar}} = n_{\mathrm{D}} = 10^{20}$ m^{-3}, $E = 2E_c$, $B = 4$ T

Current decay

- Same initial distribution as previous figure
- Bands represent $n_{\mathrm{Ar}} \in [0.5\, n_{\mathrm{D}}, 100\, n_{\mathrm{D}}]$



Self-consistent E-field [1/2]

- 0-D inductive electric field[9]

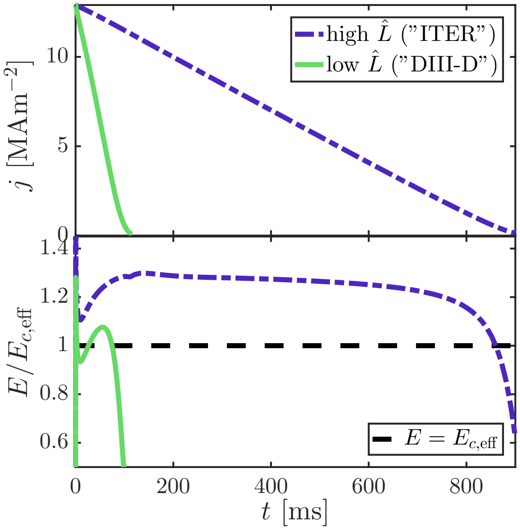
$$E = -\hat{\boldsymbol{L}}\frac{\partial j}{\partial t}, \quad \hat{\boldsymbol{L}} = \frac{AL}{2\pi R} \sim \frac{\mu_0 A}{2\pi}$$

- Forward-beamed initial distribution obtained by simulation with large E-field, Average runaway energy: 17.2 MeV
- Inductance parameter $\hat{L}$
  - Higher, ∼ITER size, $I_0 = 6.6$ MA
  - Lower, ∼DIII-D size, $I_0 = 620$ kA

[9] G. Wilkie et al., in preparation

Self-consistent E-field [2/2]

- Linear current decay: compare with Breizman NF (2014): $-\hat{L}\frac{\partial j}{\partial t} = E \overset{?}{\approx} E_{c,\mathrm{eff}}$
- Scaling with impurity density from $E_{c,\mathrm{eff}} \gg E_c$



Parameters: $T = 10\,\mathrm{eV}$, Ar^{+} with density $n_{\mathrm{Ar}} = 4n_{\mathrm{D}}$, $n_{\mathrm{D}} = 10^{20}$ m^{-3}

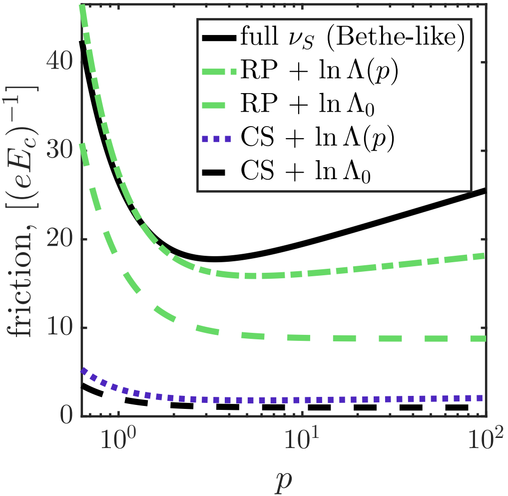
Conclusions

- **Large effect of reduced screening**
  - enhanced collision frequencies
  - more isotropic distribution function
  - faster runaway current
- Derived generalized collision operator from first principles
- Future work: disruption simulations $\Rightarrow$ improved **runaway mitigation** schemes?

Extra slides

Friction

- Friction force $F = p\nu_S(p)$
- For $n_{\mathrm{Ar}} = 11 n_{\mathrm{D}}$,
  $n_{\mathrm{D}} = 10^{20}\,\mathrm{m}^{-3}$, $T = 10\,\mathrm{eV}$



Inductance

Full inductance equation: assume a radial distribution of current

$$\nabla^2 E = \mu_0 \frac{\partial J}{\partial t} \Rightarrow E = E_a - \frac{LA}{2\pi R}\frac{\partial J}{\partial t}, \quad E_a : \text{external field}$$

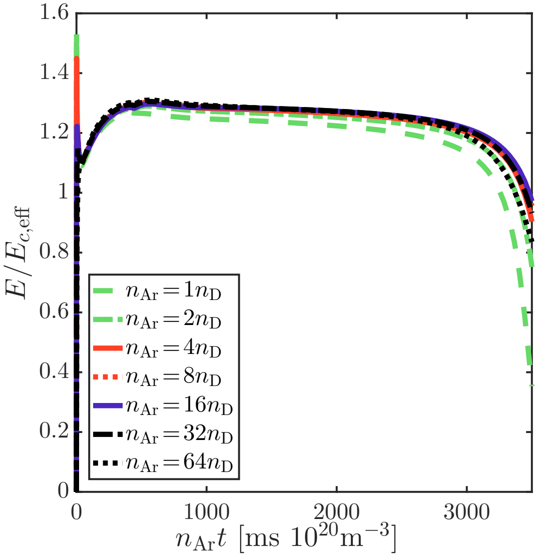
Uniform current distribution[11]: $L = \mu_0 R[\ln(8R/a) - 7/4]$

machine	$AL/\mu_0 R$	a_{RE}	a_{RE}/a
ITER	1.57	0.40	0.20
	0.14	0.10	0.05
ASDEX-U	0.14	0.12	0.25
DIII-D	1.57	0.61	0.91
	0.14	0.12	0.18
TCV	0.14	0.15	0.61

[11] Jackson, *Classical electrodynamics* (1999)

Scaling with impurity density

- 0D inductive model: $E(t) \Rightarrow j(t)$
- Scaling with impurity density well described by scaling of $n_{\mathrm{Ar}}t$ and $E_{c,\mathrm{eff}}$



Sensitivity on initial distribution

- Initial energy 2.5 MeV vs 17.2 MeV
- Both rather equilibrated in pitch-angle