# Hot Particle Equilibrium code (HPE) with plasma anisotropy and toroidal rotation

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The development of HPE code addresses the needs for accurate evaluation of the effects of plasma toroidal rotation and anisotropic pressure generated by NBI on tokamak equilibrium. These effects are becoming increasingly important after the upgrades of DIII-D and NSTX machines in the US where the equilibrium is expected to change significantly. One of important novel elements of the presented formulation is the finite width of the fast ions drift orbits. HPE adopts the following equations

$$\begin{aligned} &\mathbf{K} \equiv (\nabla \times \sigma \mathbf{B}), \quad \nabla \bar{p}_{pl} + \nabla \bar{p}_{\|} - \bar{\rho}\Omega^2 r \mathbf{e}_r = B\tau\nabla B + (\mathbf{K}\times\mathbf{B}), \quad \sigma \equiv 1-\tau, \\ &\mathbf{B} = \frac{1}{r}(\nabla\bar{\Psi}\times\mathbf{e}_\phi) + \frac{1}{r}\bar{F}\mathbf{e}_\phi, \quad \bar{p}_{pl} \equiv \mu_0 p_{pl}, \quad \bar{p}_{\|} \equiv \mu_0 p_{\|}, \quad \bar{\rho} \equiv \mu_0 \rho, \end{aligned} \tag{1.1}$$

where $r$ is radius in cylindrical coordinates, $p_{pl}, p_{\|}$ are the isotropic and parallel pressures, $\rho$ is the plasma density, $\Omega = \Omega(\bar{\Psi})$ is toroidal rotation frequency. The function $\tau$, which is determined by Eq. (1.1), is related the plasma anisotropy

$$\tau \equiv \frac{\bar{p}_{\|} - \bar{p}_{\perp}}{B^2}, \quad B \equiv |\mathbf{B}| \tag{1.2}$$

and allows for determination of perpendicular pressure $p_{\perp}$.

With an additional assumption in HPE on plasma temperature $T_{e,i} = T_{e,i}(\bar{\Psi})$ and parallel pressure given as $\bar{p}_{\|} = \bar{p}_{\|}(\bar{\Psi}, B)$ the set of equilibrium equations in HPE is reduced to

$$\Delta^{*\sigma}\bar{\Psi} \equiv r\nabla\cdot\left(\frac{\sigma\nabla\bar{\Psi}}{r}\right) = -r^2\left(\widehat{p}(\bar{\Psi})e^{\mathcal{M}^2\frac{r^2-R^2}{2R^2}} + \tilde{p}_{\|}\right)'_{\bar{\Psi}} - \frac{1}{\sigma}T, \quad B\tau = \frac{\partial \tilde{p}_{\|}}{\partial B}, \quad T \equiv \sigma\bar{F}\frac{d\sigma\bar{F}}{d\bar{\Psi}}. \tag{1.3}$$

Here the first term $\propto \widehat{p}$ represents the total isotropic pressure, $\tilde{p}_{\|}(\bar{\Psi}, B)$ is the oscillatory part of the parallel pressure. The plasma density is consumed by the Mach number with $R$ as the major radius of magnetic axis

$$\mathcal{M}^2(\Psi) \equiv \frac{m_i\Omega^2R^2}{T_e(\Psi)+T_i(\Psi)} \tag{1.4}$$

The input profiles in HPE are $\widehat{p}(a)$, $\bar{p}_{\|}(\bar{\Psi}, B)$, $\mathcal{M}^2(a)$ and $q(a)$ where $a$ is the normalized minor radius used in the code, $\bar{\Psi} = \bar{\Psi}(a)$. Note, that the "$q$-solver" regime of HPE, taken from the ESC code, provides the same sub-second convergence of 3-5 iterations as for the standard right-hand-side of the Grad-Shafranov equation.

At present, the mono-energetic hot particle model of the parallel pressure is implemented in HPE in the novel form determined by two input profiles $s(\lambda)$ and $\mathcal{F}(\bar{\Psi})$ determining pitch angle and radial distribution of fast ions

$$\bar{p}_{\|} = \int_{-1}^{1} \frac{B}{B_{min}}\sqrt{\frac{1-\lambda\frac{B}{B_{min}}}{1-\lambda}}\left[\mathcal{F}(\bar{\Psi}) \pm \frac{2}{3}\rho_L\bar{F}\mathcal{F}'_{\bar{\Psi}}\left(1-\frac{B}{B_{min}}\right)\right]s(\lambda)d\lambda, \tag{1.5}$$

where $\rho_L$ is the ion Larmor radius related to the width of the particle orbits. At the same time, the equilibrium solver of HPE is designed for arbitrary specification of plasma aisotropy.

Examples of the equilibrium of NSTX-U like plasmas will be shown emphasizing the effects of anisotropy and rotation.