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PRINCETON PLASMA PHYSICS LABORATORY

THE DIMITS SHIFT IN A ONE-FIELD FLUID MODEL

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Turbulence, Zonal Flows, and the Dimits Shift

- Ion temperature gradients can cause turbulence to develop in toroidal plasmas, driving heat diffusion.
- The critical temperature gradient needed for turbulence to develop can be calculated using linear theory.

The Dimits shift is the nonlinear upshift of the critical temperature gradient that signals the onset of turbulence.

- This is caused by a shearing away of turbulent streamers by poloidal zonal flows (ZF) generated through a secondary instability.
- This has been witnessed in both gyrofluid and gyrokinetic simulations.
- If the linear drive is sufficiently large, the system undergoes a tertiary instability.

Questions and Motivation

Even though the existence of the shift has been known for two decades, two fundamental questions still go unanswered:

- **Why should the Dimits shift exist?**
  - Some energy transfer mechanism must dominate over the usual forward cascade.
- **What is the true nature of the tertiary instability?**
  - Is it actually related to the Kelvin-Helmholtz instability? Do zonal flows really go "unstable"?

In order to answer these questions, a simple model is needed that captures the Dimits shift with a minimal amount of physics.

The Modified Terry-Horton Equation

The Terry-Horton equation is modified to include proper adiabatic electron response and is made to capture Rosenbluth-Hinton states:

$$\frac{\partial \zeta}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\zeta = \beta\frac{\partial \varphi}{\partial y} - \widehat{\alpha} D\zeta,$$

where $\boldsymbol{v} = (-\partial_y\varphi, \partial_x\varphi)$, D is a damping operator and

$$\widehat{\zeta} = -(k_\perp^2 - \mathrm{i}\widehat{\delta}_{\boldsymbol{k}} + \widehat{\alpha}_{\boldsymbol{k}})\widehat{\varphi}$$

in Fourier space.

The modification is done by the introduction of the operator

$$\widehat{\alpha}\varphi \doteq \varphi - \langle\varphi\rangle \doteq \varphi - \frac{1}{L_y}\int_0^{L_y} \mathrm{d}y\,\varphi,$$

which is zero when acting on zonal modes and unity otherwise.

The modified Terry Horton equation has the linear growth rates $\lambda_{\boldsymbol{k}} = \gamma_{\boldsymbol{k}} - \mathrm{i}\omega_{\boldsymbol{k}}$ given by

$$\gamma_{\boldsymbol{k}} = -D_{\boldsymbol{k}} + \frac{\beta k_y \widehat{\delta}_{\boldsymbol{k}}}{(1+k_\perp^2)^2 + \widehat{\delta}_{\boldsymbol{k}}^2}, \qquad \omega_{\boldsymbol{k}} = \frac{\beta k_y(1+k_\perp^2)}{(1+k_\perp^2)^2 + \widehat{\delta}_{\boldsymbol{k}}^2}.$$

This system contains destabilizing electron effects and can be made to model ion Landau damping as well.

Simplified Models

Two simplifications are considered which exhibit the Dimits shift:

Quasilinear System:

- Fields are decomposed into zonal-averaged and fluctuating components ($\varphi = \langle\varphi\rangle + \varphi'$).
- Eddy-eddy self-interactions are neglected.
- Model isolates the interaction between zonal flows and drift-waves.

Four-Mode Truncation:

- Four Fourier modes plus c. c. are kept with the following wavenumbers:

$$\text{radial drift wave:} \quad \boldsymbol{p} = (0, p_y),$$
$$\text{pure zonal flow:} \quad \boldsymbol{q} = (q_x, 0),$$
$$\text{sidebands:} \quad \boldsymbol{r}_\pm = (\pm q_x, p_y).$$

- Allows one to investigate the behaviour of specific triad interactions.

Direct Numerical Simulation

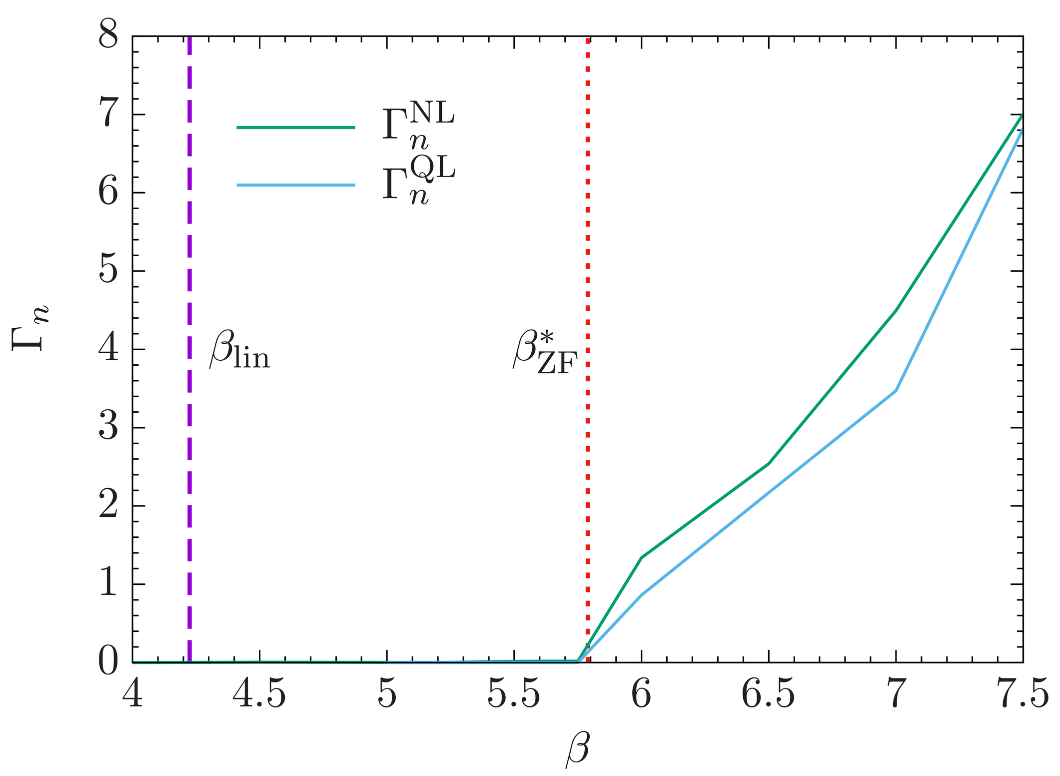

Figure 1: Particle flux for the nonlinear and quasilinear systems with $D = 1 - 0.01\nabla_\perp^2$ and $\widehat{\delta}_{\boldsymbol{k}} = 1.5\mathrm{i}k_y$ as a function of β. The dashed-dotted line denotes the linear threshold for instability $\beta_{\rm lin}$, while the dashed line denotes the calculated upshifted critical gradient, $\beta^*_{\rm ZF}$.

Zonal Stability Analysis of the 4MT

Stability analysis of the 4MT with a background zonal mode φ_0 results in the unstable eigenvalue

$$\mathrm{Re}(\lambda_+) = \frac{1}{2}\left[\gamma_+ + \sqrt{\frac{1}{2}}\left(\Omega + \sqrt{\Omega^2+\Theta^2}\right)^{1/2}\right],$$

where $\gamma_\pm = \gamma_{\boldsymbol{p}} \pm \gamma_{\boldsymbol{r}}$, $\omega_\pm = \omega_{\boldsymbol{p}} \pm \omega_{\boldsymbol{r}}$,

$$\Omega \doteq \gamma_-^2 - \omega_-^2 - 8|\varphi_0|^2(M_{\boldsymbol{p}}^{\rm Re}M_{\boldsymbol{r}}^{\rm Re} - M_{\boldsymbol{p}}^{\rm Im}M_{\boldsymbol{r}}^{\rm Im}),$$
$$\Theta \doteq 2\omega_-\gamma_- + 8|\varphi_0|^2(M_{\boldsymbol{p}}^{\rm Re}M_{\boldsymbol{r}}^{\rm Im} + M_{\boldsymbol{p}}^{\rm Im}M_{\boldsymbol{r}}^{\rm Re}).$$

In general, a valid end-state of the Dimits shift scenario should be stable to drift-wave perturbations, though this can only constitute an upper bound.

Maximally-Coupled Modes

The drift-wave and sideband modes become maximally coupled when the discriminant vanishes, i.e.,

$$\Omega = 0, \qquad \Theta = 0.$$

A Maximally-Coupled Mode occur when two individual modes a and b with eigenvalues λ_a and λ_b are nonlinearly coupled to form two new modes $a \pm \gamma b$ with eigenvalue $(\lambda_a + \lambda_b)/2$ for some constant γ.

In order for these modes to be formed, the zonal-flow interaction must be stabilizing, rendering $\Omega \leq 0$.

When dealing with many modes $\varphi_{\boldsymbol{k}}$, one can consider the behavior of the phase-space.

Necessary condition for stability: Phase-space of $\varphi_{\boldsymbol{k}}$'s must be contracting ($\sum_{\boldsymbol{k}}\gamma_{\boldsymbol{k}} < 0$, see Terry & Horton 1982).

Nonlocality and $\widehat{\alpha}_{\boldsymbol{k}}$

For zonal flows to be stabilizing, the condition $M_{\boldsymbol{p}}^{\rm Re}M_{\boldsymbol{r}}^{\rm Re} > M_{\boldsymbol{p}}^{\rm Im}M_{\boldsymbol{r}}^{\rm Im}$ must be satisfied. Consider Hasegawa-Mima ($\widehat{\delta}_{\boldsymbol{k}} = 0$):

Incorrect Electron Response

$$q_x^2 \lesssim p_y^2 \quad \Rightarrow \quad \textbf{LOCAL}$$

Correct Electron Response

$$q_x^2 \lesssim 1 + p_y^2 \quad \Rightarrow \quad \textbf{NONLOCAL}$$

Thus, the $\widehat{\alpha}$ operator enables nonlocal zonal interactions in k space.

CENTRAL IDEA

The tertiary instability *is not* a Kelvin-Helmholtz instability.

Rather, the Dimits shift ends when the collection of modes within the interaction range of the primary drift-wave mode becomes unstable.

Estimation of the Dimits Shift

The Dimits shift can be estimated by calculating the range of the interaction (q_x^*) and the stability condition for the collection of nonlocally coupled modes (phase-space expansion):

$$\left.\frac{\partial\gamma(0,p_y)}{\partial p_y}\right|_{p_y=p_y^*} = 0, \qquad \textbf{Fastest Growing Mode},$$

$$\int_0^{q_x^*}\mathrm{d}k_x\,\gamma(k_x, p_y^*) = 0, \qquad \textbf{Phase-Space Expansion/ Stability},$$

$$\begin{Bmatrix}\Theta = 0\\ \Omega = 0\end{Bmatrix}, \qquad \textbf{Interaction Range in } k \textbf{ space}.$$

These equations are solved for $\beta^*_{\rm ZF}$, the value of the upshifted critical gradient.

Theoretical Results

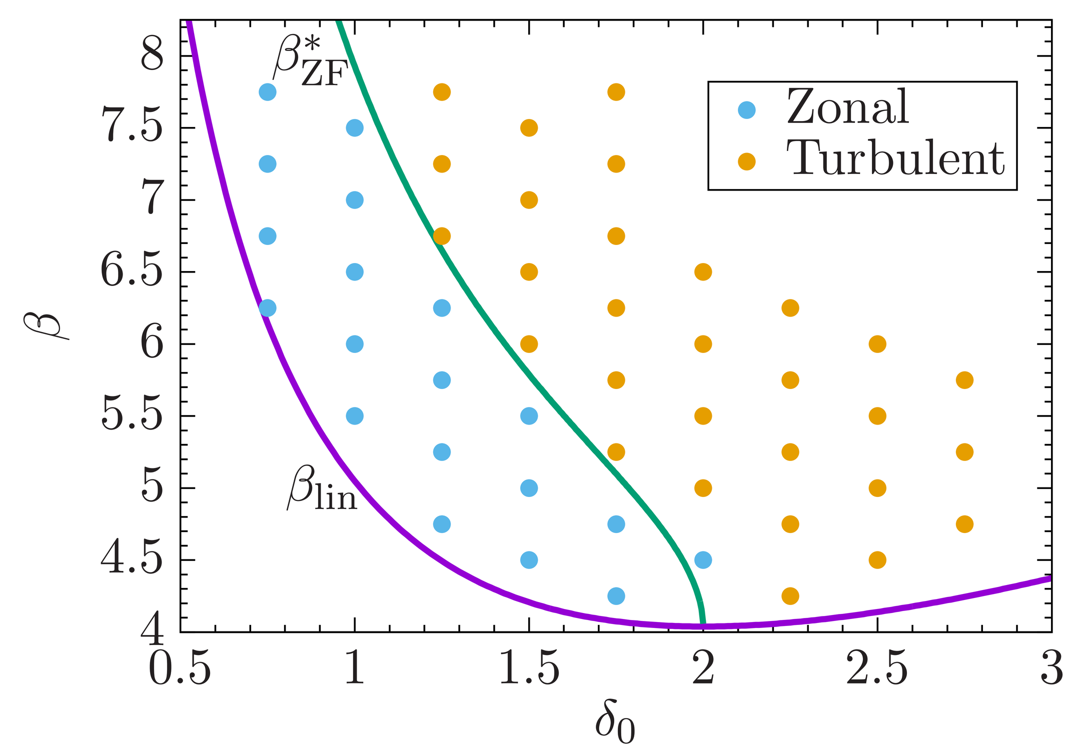

Figure 2: Parameter scan of the nonlinear system with $\widehat{\delta}_{\boldsymbol{k}} = \mathrm{i}\delta_0 k_y$ and $D_{\boldsymbol{k}} = 1 + 0.01k_\perp^2$. The purple line marks the linear stability threshold, while the green line denotes the predicted end of the Dimits shift β^* rom the analytic model.

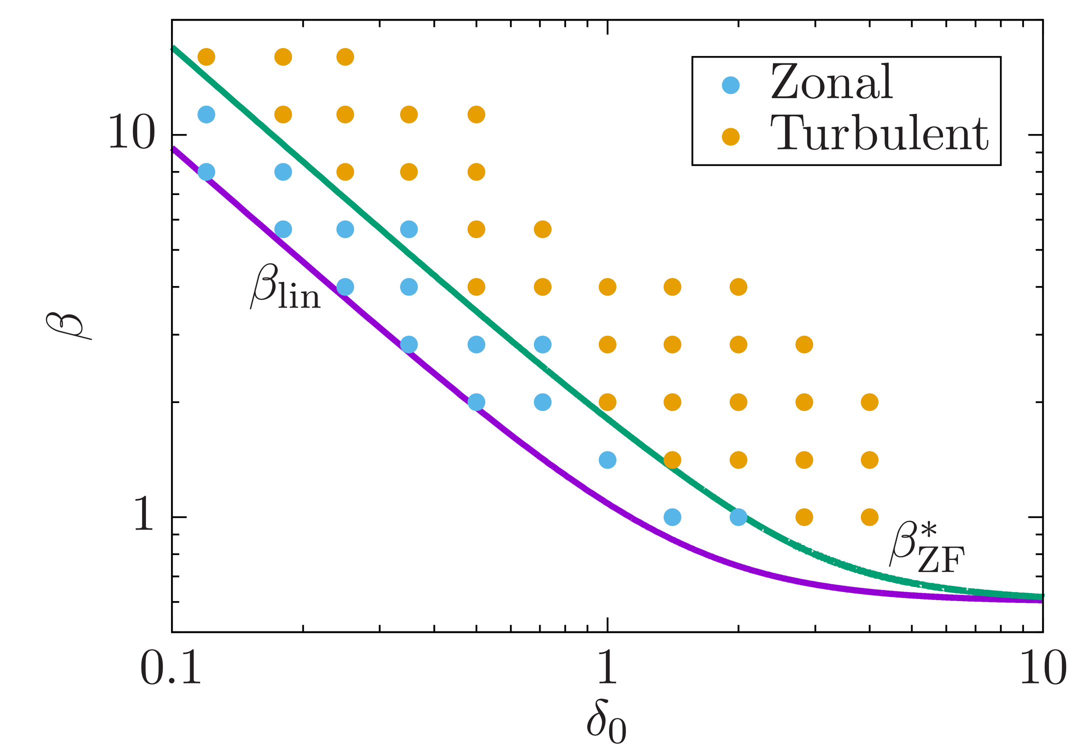

Figure 3: Parameter scan of the nonlinear system with $\widehat{\delta}_{\boldsymbol{k}} = \mathrm{i}\delta_0 k_y$ and $D_{\boldsymbol{k}} = 0.3|k_y|$. The purple line marks the linear stability threshold, while the green line denotes the predicted end of the Dimits shift β^* from the analytic model.

Physical Picture

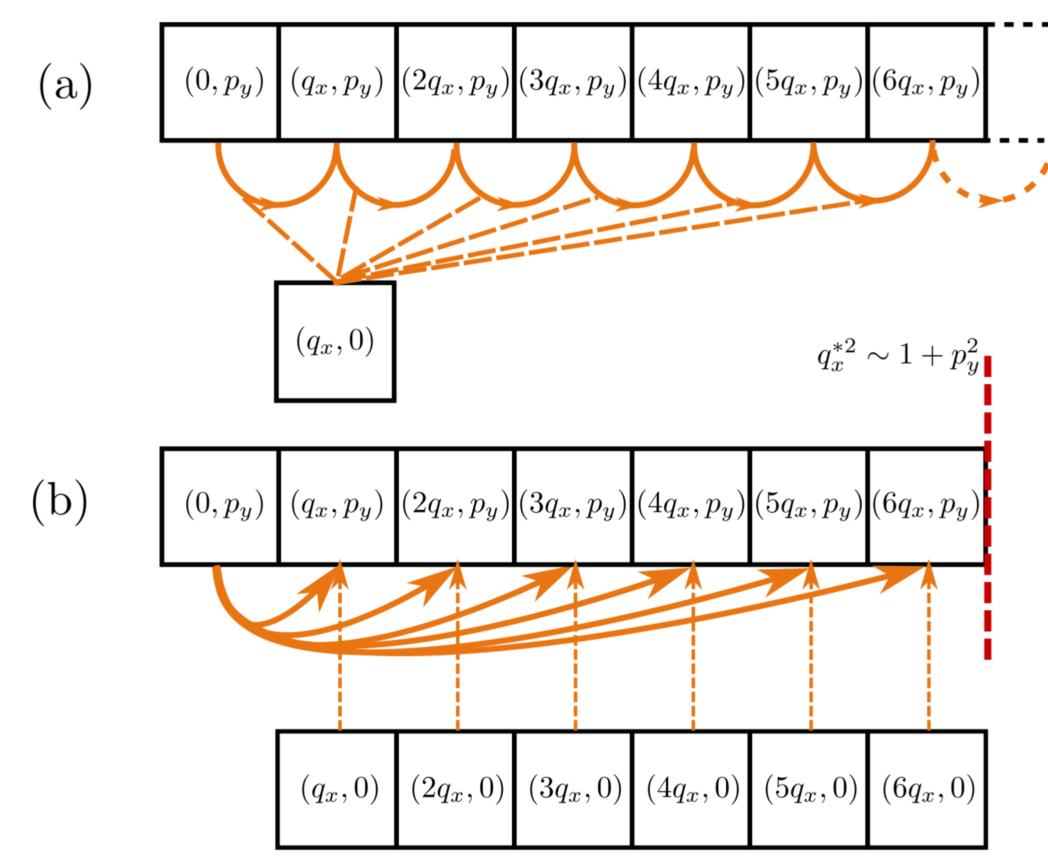

Figure 4: Visualization of (a) forward zonal cascade usually found in turbulence and (b) nonlocal zonal shearing that is crucial in the Dimits shift regime. Typically, $q_x \ll p_y$. Red dashed line denotes the interaction range in k space.

The Dimits Shift scenario is summarized as follows:

1. Small-amplitude drift-wave perturbations grow exponentially in the initial linear regime.
2. These drift waves cause a Kelvin-Helmholtz-type secondary instability, causing zonal flows to grow.
3. A spectrum of zonal flows is established, and flows begin to shear drift waves.
4. This shearing is fast and nonlocal in k-space, transferring energy quickly from unstable to stable drift waves.
5. The Dimits shift ends when the cluster of modes within the interaction range of the primary drift-wave mode is no longer stable.

CONCLUSIONS

- The $\widehat{\alpha}$ operator renders the zonal interaction nonlocal in k space.
- The tertiary instability *is not* a Kelvin-Helmholtz instability.
- The Dimits shift roughly ends when the collection of nonlocally coupled modes goes unstable.
- This calculation results in a shift that agrees well with direct numerical simulation.
- The quantitative size of the shift encompasses all aspects of the underlying model, both linear and nonlinear.

Acknowledgements

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