Relativistic Boltzmann collision operator for runaway-avalanche studies

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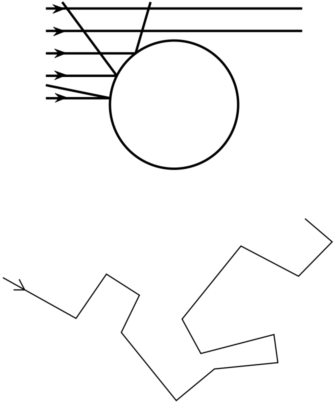
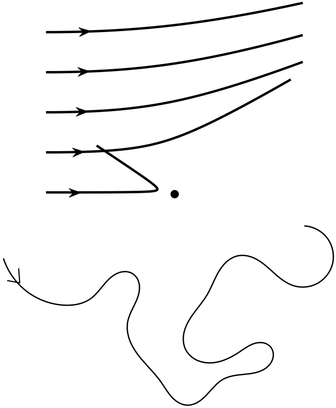
Hard-sphere collisions



Coulomb collisions



Boltzmann:

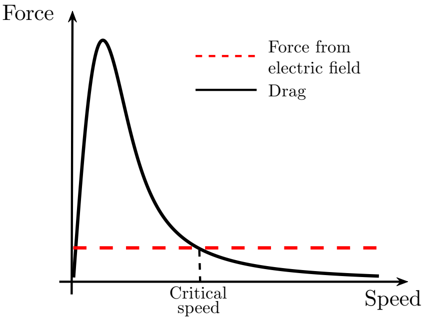
$$\frac{\mathrm{d}f_e}{\mathrm{d}t}(\mathbf{p}) = \int \mathrm{d}\mathbf{p}_1\, n_e v_1 f_e(\mathbf{p}_1)\frac{\partial\sigma}{\partial\mathbf{p}}(\mathbf{p}_1 \to \mathbf{p}) - n_e v f_e(\mathbf{p}) \int \mathrm{d}\mathbf{p}_1\, \frac{\partial\sigma}{\partial\mathbf{p}_1}(\mathbf{p} \to \mathbf{p}_1)$$

Fokker-Planck:

$$\frac{\mathrm{d}f_e}{\mathrm{d}t} = \frac{\partial}{\partial\mathbf{p}} \cdot \left[\mathbf{A}f_e + \frac{\partial}{\partial\mathbf{p}} \cdot \left(\mathsf{D}f_e\right)\right]$$

Electron runaway

Runaways are created in the presence of strong electric fields by primary (Dreicer) or secondary (avalanche, knock-on) generation



Avalanche generation

Runaway growth rates:

$$\textbf{Knock-on}: \frac{\partial n_{\mathrm{RE}}}{\partial t} \sim \mathcal{O}\left[n_{\mathrm{RE}}\left(\frac{E}{E_c} - 1\right)\right]$$

$$\textbf{Primary}: \frac{\partial n_{\mathrm{RE}}}{\partial t} \sim \mathcal{O}\left[n_e \ln\Lambda \exp\left(-\frac{E_D}{4E}\right)\right]$$

where $E_D = (m_e c^2/T_e)E_c$

$$\frac{\text{Knock-on}}{\text{Primary}} \sim \text{“}\frac{1}{\ln\Lambda}\text{“}$$

BUT: Knock-on still wins for small E/E_D or large n_{RE}/n_e!

[Connor, Hastie NF 1975; Rosenbluth, Putvinski NF 1998]

Avalanche generation

To describe knock-on collisions we add a (simplified) Boltzmann operator:

$$\frac{\mathrm{d}f_e}{\mathrm{d}t} = C_{\mathrm{FP}}\{f_e\} + C_{\mathrm{boltz}}\{f_e\}$$

Beware of double counting!

Generally we can linearize ($n_{\mathrm{RE}} \ll n_e$)

$$C_{\mathrm{boltz}}\{f_e,\ f_e\} \approx \underbrace{C_{\mathrm{boltz}}\{f_e,\ f_{e0}\}}_{\text{test-particle}} + \underbrace{C_{\mathrm{boltz}}\{f_{e0},\ f_e\}}_{\text{field-particle}}.$$

Avalanche generation

The two most established knock-on models today:

$$C_{\text{knock-on}} = C_{\text{boltz}}\{n_e\delta(\mathbf{p}),\ f_e\} \qquad \text{(only field-particle term)}$$

Rosenbluth-Putvinski: $$f_e(\mathbf{p}) = n_{\text{RE}} \lim_{p_0\to\infty} \frac{1}{p^2}\delta(p-p_0)\delta(\cos\theta - 1)$$

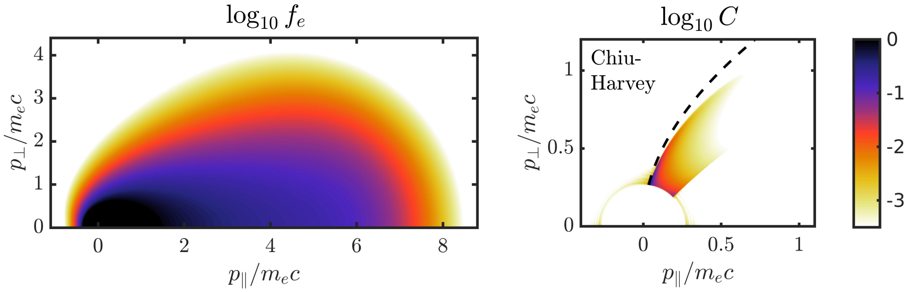
Chiu-Harvey: $$f_e(\mathbf{p}) = F(p)\delta(\cos\theta - 1)$$

$$\left(F(p) = \int_{-1}^{1} f_e(\mathbf{p})\,\mathrm{d}(\cos\theta)\right)$$

[Rosenbluth, Putvinski NF 1997; Chiu, Rosenbluth, Harvey NF 1998]

Avalanche generation

So how do these operators behave?



Avalanche generation

Both models have limitations:

- Double counting collisions
- Non-conservation of momentum and energy
  - Rosenbluth-Putvinski even creates infinite energy and momentum!
- Chiu-Harvey model ignores pitch-angle distribution
- Arbitrary cut-off affecting solutions

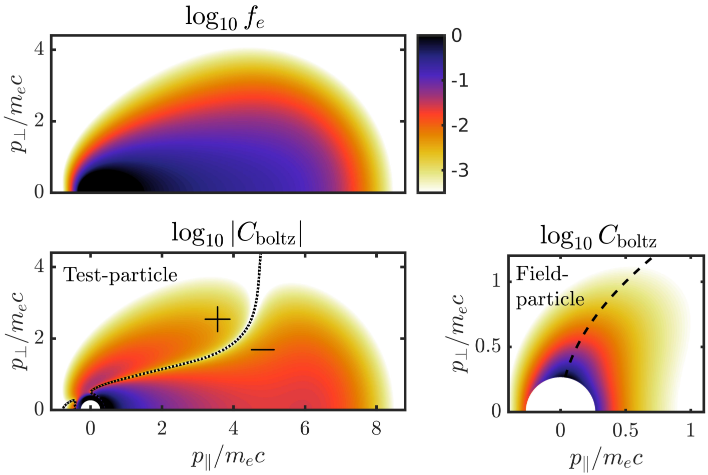
Avalanche generation

We solved this, by

- Accounting for full $f_e(\mathbf{p})$
- Including the test-particle term [restores conservation laws]
- Modify $\ln \Lambda$ in Fokker-Planck operator [avoids double counting]

[O. Embréus *et al.*, APS 2015, PP12.00107; T. Fülöp *et al.*, IAEA 2016, TH/P4-1]

Avalanche generation



CODE (COllisional Distribution of Electrons)

The various knock-on operators have been implemented in the 0D+2P kinetic-equation solver CODE.
The tool contains all momentum-space physics needed to describe runaway generation and decay:

- Primary, secondary and hot-tail generation from first principles
- Synchrotron and bremsstrahlung radiation losses and interactions with partially ionized ions

[M. Landreman, A. Stahl, T. Fülöp, CPC **185**, 847 (2014)]

[A. Stahl, O. Embréus, G. Papp, M. Landreman, T. Fülöp, NF **56**, 112009 (2016)]

[A. Stahl, E. Hirvijoki, J. Decker, O. Embréus, T. Fülöp, PRL **114**, 115002 (2015)]

Avalanche generation

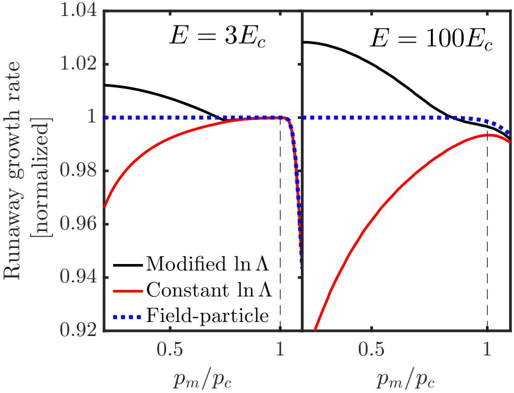
Cut-off momentum p_m:

$$\ln\Lambda \mapsto \ln\Lambda - \ln\sqrt{\frac{\gamma-\gamma_m}{\gamma_m-1}}.$$

$\Rightarrow$ Energy-loss rate independent of p_m.

The correction follows from

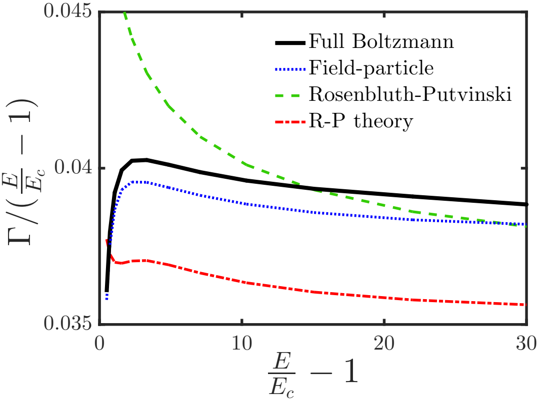
$$C = \underbrace{C(\theta<\theta_m)}_{\text{FP approx.}} + \underbrace{C(\theta>\theta_m)}_{\text{Boltzmann}}$$



Avalanche generation

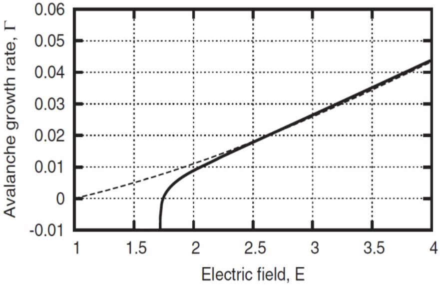
We can now revisit a classical calculation [R-P, NF 1998]:
The steady state avalanche growth rate

$$\Gamma = \frac{1}{n_{\mathrm{RE}}}\frac{\mathrm{d}n_{\mathrm{RE}}}{\mathrm{d}t}$$



Avalanche generation in a near-threshold electric field

An interesting situation occurs when $E \sim E_c$, as radiation losses become important.



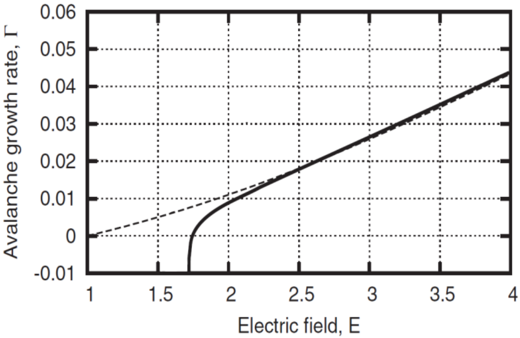
[P. Aleynikov and B. N. Breizman, PRL **114**, 155001 (2015)]

Near-threshold electric field

Approximate Γ calculated from the avalanche cross-section

$$\Gamma(E) \approx v \int_{\gamma_{min}}^{\gamma_{max}} \frac{\partial \sigma}{\partial \gamma} \mathrm{d}\gamma.$$

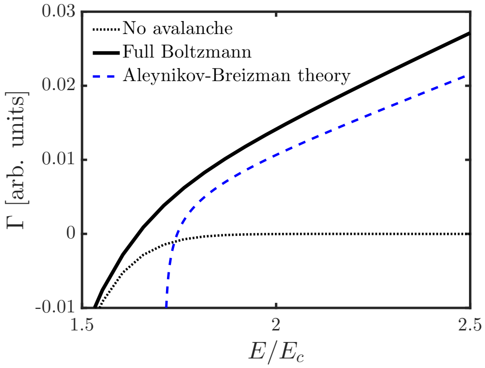
Negative growth for small E: Reverse knock-ons predicted!



[P. Aleynikov and B. N. Breizman, PRL **114**, 155001 (2015)]

Near-threshold electric field

- Significant reverse knock-on however ***not*** observed in kinetic simulations
- Runaway decay is described mainly by Fokker-Planck dynamics when $\Gamma \lesssim 0$.

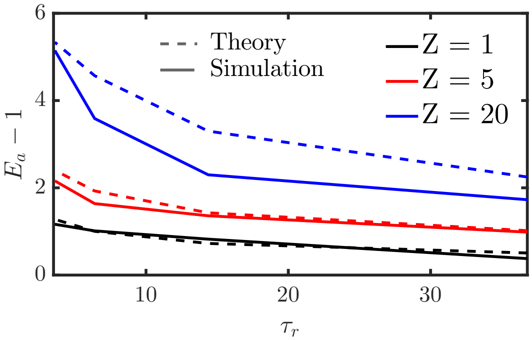


Near-threshold electric field

A very important parameter is the effective critical field $E_a : \Gamma(E_a) = 0$, as this quantity sets the current-quench time when $I_{\mathrm{RE}} \gg 100\,\mathrm{kA}$.

[B. N. Breizman, NF **54**, 072002 (2014)]

$\tau_r \propto n_e/B^2$, relative strength of synchrotron energy loss

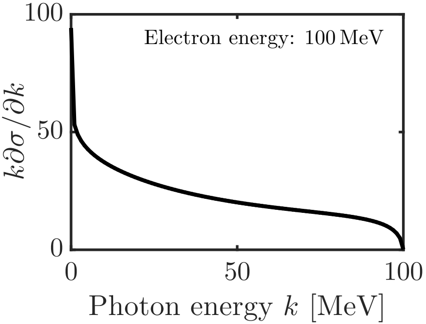


Bremsstrahlung

Previous work (fusion, lightning, astrophysics) in plasma physics considered only the *stopping-power force*

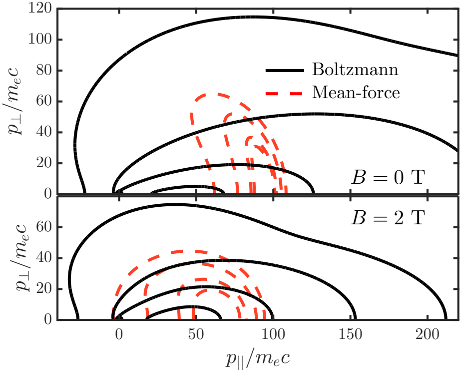
$$F_{\text{brems}}(p) \sim \frac{m_e c}{\tau_{\text{brems}}}\gamma \ln 2\gamma$$
$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

However, the average photon energy is large!



Bremsstrahlung

A Boltzmann model for radiation losses give significantly different runaway dynamics:
Maximum runaway energy significantly underestimated by previous calculations.



[O. Embréus, A. Stahl, T. Fülöp, New Journal of Physics **18**, 093023 (2016)]

Summary

The Boltzmann operator has two important applications for runaways:

- **Avalanche runaway generation**
  - Conservative knock-on operator
  - Formally eliminating double counting collisions
  - Describing reverse knock on
- **Bremsstrahlung energy loss**
  - Capturing the finite momenta of emitted photons
  - Accurately describing the energy limit due to radiation losses

Selection of related publications: [Embréus NJP 2016; Stahl PRL 2015; Stahl CPC 2017; Stahl JPCS 2017; Stahl NF 2016; Decker PPCF 2016; Hirvijoki JPP 2015]