Ignition and Burning Plasmas: a Multi Fluid Analysis

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Motivation

- The next generation of magnetic confinement nuclear fusion experiments aims to achieve burning plasma conditions.
- A clear understanding of performance requirements needed to obtain burning or ignition conditions is desirable.
- Our knowledge to that purpose has not advanced much since Lawson's original work[1].
- We include additional physics in a zero- and one-dimensional analysis of the plasma to improve our estimate of plasma properties relevant to ignition and burning plasma conditions.

[1] J. D. Lawson, Proc. Phys. Soc. London Sect. B 70, 6 (1957)

Outline

- Modified ignition criterion:
  - Include two-fluid and α-particle effects.
- Compute and compare $\dot{T}$ vs. T curves for various models.
  - Determine the relevance two-fluid and α-particle effects on the minimum heating needed for ignition.
- Consider one-dimensional, two-parameter density and temperature profiles and evaluate their effect on ignition physics.
- Use the complete model to investigate physics of burning plasmas.

Time-Independent Analysis: In Previous Episodes . . .

The Lawson criterion is derived starting from the single-fluid zero-dimensional energy balance:

$$\frac{E_\alpha}{16} p^2 \frac{<\sigma v>}{T^2} + S_h = \frac{C_B}{4} \frac{p^2}{T^{3/2}} + \frac{3}{2} \frac{p}{\tau_E} \left[+ \frac{3}{2} \frac{dp}{dt} \right]. \quad (1)$$

A straightforward manipulation gives the ignition criterion (with $S_h = 0$)

$$p\tau_E = 2nT\tau_E \geq \frac{T^2}{\frac{E_\alpha}{24} < \sigma v > - \frac{C_B}{6} T^{1/2}}. \quad (2)$$

We now want to see how this is modified by multi-fluid effects.

The Starting Equations Are the Time-Dependent Three-Fluid Energy Conservation Equations.

The starting point is the system of zero-dimensional conservation equations for the three species, ions, electrons and αs:

$$\frac{3}{2}n\frac{\partial T_i}{\partial t} = S_{hi} - \frac{3}{2}\frac{p_i}{\tau_{Ei}} + \frac{3}{2}\frac{n(T_e - T_i)}{\tau_{eq}} \tag{3}$$

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} = S_{he} - \frac{3}{2}\frac{p_e}{\tau_{Ee}} + \frac{n_\alpha}{\tau_\alpha}E_\alpha - C_B\frac{p_e^2}{T_e^{3/2}} + \frac{3}{2}\frac{n(T_i - T_e)}{\tau_{eq}} \tag{4}$$

$$\frac{\partial n_\alpha}{\partial t} = \frac{n^2}{4} < \sigma v > - \frac{n_\alpha}{\tau_\alpha} - \frac{n_\alpha}{\tau_{E\alpha}}. \tag{5}$$

Heating terms (S_{hi}, S_{he}) are important in:

1. Transients;
2. Burning-Plasma analysis.

The System Simplifies for Steady-State.

We first write a 0D ignition criterion, focusing on the time-independent energy balance:

$$S_{hi} - \frac{3}{2}\frac{p_i}{\tau_{Ei}} + \frac{3}{2}\frac{n(T_e - T_i)}{\tau_{eq}} = 0 \quad (6)$$

$$S_{he} - \frac{3}{2}\frac{p_e}{\tau_{Ee}} + \frac{n_\alpha}{\tau_\alpha}E_\alpha - C_B\frac{p_e^2}{T_e^{3/2}} + \frac{3}{2}\frac{n(T_i - T_e)}{\tau_{eq}} = 0 \quad (7)$$

$$\frac{n^2}{4} < \sigma v > - \frac{n_\alpha}{\tau_\alpha} - \frac{n_\alpha}{\tau_{E\alpha}} = 0. \quad (8)$$

Note that for $\tau_{E\alpha} \to \infty$ (perfectly confined αs) alpha particles drop out of the system.

An Ideal Multi-Fluid Ignition Criterion Is Written.

- For convenience we assume $\tau_{Ei} = k_1 \hat{\tau}_E$, $\tau_{Ee} = k_2 \hat{\tau}_E$.
- We have used the definition: $\hat{\tau}_E = 2 \frac{\tau_{Ei}\tau_{Ee}}{\tau_{Ei}+\tau_{Ee}}$
- If Bremsstrahlung is neglected, a simple expression is obtained:

$$nT_i\hat{\tau}_E \geq \frac{6T_i^2}{E_\alpha < \sigma v >} \frac{\tau_{E\alpha}+\tau_\alpha}{\tau_{E\alpha}} \left[1 + \sqrt{1 + \frac{1}{6} \frac{n < \sigma v > E_\alpha}{T_i} \frac{\tau_{E\alpha}\tau_{eq}}{k_1 k_2 (\tau_{E\alpha}+\tau_\alpha)}} \right]. \quad (9)$$

- The right-hand side weakly depends on n through τ_{eq} and τ_α.
- As for the single-fluid case, this expression will have a minimum, which depends on the choice of parameters (τ_{Ej}).

The Multi-Fluid Ignition Criterion Is Explored.

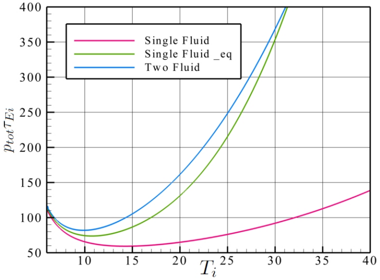
The SF Lawson criterion is recovered assuming

1. $\tau_{eq} \to 0$ and
2. $\tau_{E\alpha} \to \infty$ (in this case τ_α drops out, but will still matter for heating).

The green curve is obtained using $\tau_{E\alpha} \to \infty$, but finite τ_{eq}.
Defining $c_2 \equiv \tau_{Ee}/\tau_{Ei}$, $c_4 \equiv \tau_{E\alpha}/\tau_{Ei}$, the minimum of $p_{tot}\tau_{Ei}$ is approximated by:

$$p_{tot}\tau_{Ei} \simeq a_0\left(1+\frac{a_1}{c_2}\right)\left(1+\frac{a_2}{c_4}\right)+\frac{a_3}{c_2 c_4}+a_4\log(c_4). \quad (10)$$

Coefficients a_j are found numerically.



$p_{tot}\tau_{Ei}$ for SF and MF, $c_2 = c_4 = 1$.

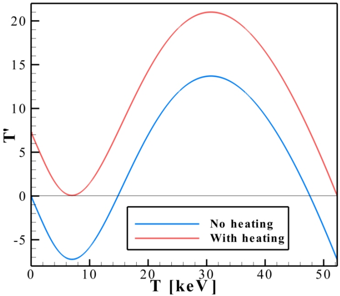
Coefficient	Value
a_0	50.0744
a_1	0.616194
a_2	0.0558493
a_3	-1.18581
a_4	-2.92248

Time-Dependent Analysis: In Previous Episodes . . .

- The standard approach considers a single fluid and the α power is immediately and entirely delivered to the plasma.
- Linear analysis is used to determine the stability of T:
  - ✓ Positive $\dot{T} \equiv dT/dt$ corresponds to an unstable temperature (temperature will grow if perturbed);
  - ✓ Negative $\dot{T}$ corresponds to a stable temperature (temperature will **not** grow if perturbed).
- $\dot{T}$ is negative for small $T \rightarrow$ heating power is needed.

In Previous Episodes . . . [Continued].

- With no heating, $\dot{T} = 0$ corresponds to ignition points (α power = losses).
- To reach an ignition point from a cold plasma, heating power is needed.
- One may also want some heating power at high temperature for burn control.
- Turning power on and off only shifts the curve up and down.
- $\dot{T} = 0$ points move farther apart with heating on.



$\dot{T}$ vs. T with and without heating, single fluid model

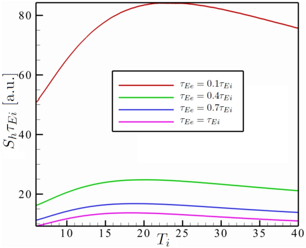
The Complete Multi-Fluid Model Is Used.

- The minimum heating power needed to take the plasma to ignition is found numerically.
- The ratios τ_{Ee}/τ_{Ei} and τ_α/τ_{Ei} are varied.
- Note that in most of our results we use:

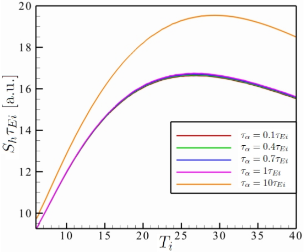
$$\tau_{eq} = \frac{1}{78\times 10^{-20} n}\left(\frac{T_e+T_i}{2}\right)^{3/2}, \qquad (11)$$

$$\tau_\alpha = 1.17\times 10^{18}\frac{T_e^{3/2}}{n}. \qquad (12)$$

- Physical values of τ_{eq}, τ_α are $\simeq 1$s when $T \simeq 20$keV, $n \simeq 10^{20}\mathrm{m}^{-3}$.



τ_{Ee}/τ_{Ei} effect on S_h for ignition



τ_α effect on S_h for ignition ($\tau_{E\alpha} \to \infty$)

One-Dimensional Parameters Are Introduced

- We introduce the density and temperature profiles:

$$n(r,t) = n_0(t)\left(1 - r^{\theta}\right)^{\eta} \qquad T_{i,e}(r,t) = T_{0;i,e}(t)\left(1 - r^{\nu}\right)^{\mu},$$

$$\text{with } \ 0.1 \leq (\mu;\eta) \leq 2 \qquad \text{and } \ 1.1 \leq (\nu;\theta) \leq 4.$$

- Spatial profiles are fixed in time even during time-dependent simulations: We assume that profile equilibration is faster than transients (i.e., time evolution of n_0 etc.).
- Ion and electron temperature profiles are kept identical, but could in principle be different. Note that $T_{0;i} \neq T_{0;e}$!

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One-Dimensional Problem Setup

- For n_α the "equilibrium" spatial profile is used, obtained from

$$\frac{\partial n_\alpha(r,t)}{\partial t} = \frac{n(r,t)^2}{4} < \sigma v > (r,t) - \frac{n_\alpha(r,t)}{\tau_\alpha(r,t)} - \frac{n_\alpha(r,t)}{\tau_{E\alpha}} \quad (13)$$

  and normalized to 1 at $r = 0$.
- Keep in mind that $< \sigma v >=< \sigma v > (T_i(r,t,))$ and $\tau_\alpha = \tau_\alpha\,(n(r,t), T_e(r,t,))$.
- The ion-electron equilibration time τ_{eq} also depends on profiles, but energy confinement times τ_{Ei}, τ_{Ee}, $\tau_{E\alpha}$ are entered as constant values for each case.

One-Dimensional Problem Setup [2]

- The full set of equations:

$$\frac{3}{2}n(r,t)\frac{\partial T_i(r,t)}{\partial t} = S_{hi} - \frac{3}{2}\frac{n(r,t)T_i(r,t)}{\tau_{Ei}} + \frac{3}{2}\frac{n(r,t)\left(T_e(r,t)-T_i(r,t)\right)}{\tau_{eq}}, \qquad (14)$$

$$\frac{3}{2}n(r,t)\frac{\partial T_e(r,t)}{\partial t} = S_{he} - \frac{3}{2}\frac{n(r,t)T_e(r,t)}{\tau_{Ee}} + \frac{n_\alpha(r,t)}{\tau_\alpha}E_\alpha - C_B\frac{\left(n(r,t)T_e^2(r,t)\right)}{T_e^{3/2}(r,t)} + \frac{3}{2}\frac{n(r,t)\left(T_i(r,t)-T_e(r,t)\right)}{\tau_{eq}}, \qquad (15)$$

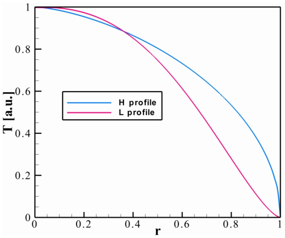
$$\frac{\partial n_\alpha(r,t)}{\partial t} = \frac{n^2(r,t)}{4} < \sigma v > - \frac{n_\alpha(r,t)}{\tau_\alpha} - \frac{n_\alpha(r,t)}{\tau_{E\alpha}} \qquad (16)$$

  is first integrated (i.e., averaged) in space at each time step, then advanced in time.

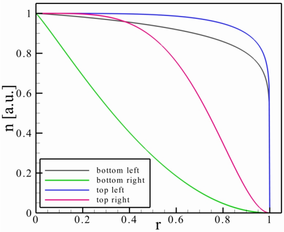
- Note that $n'_\alpha(t) \neq 0$ since only the shape (and not the numerical value) of n_α is determined from Eq. (13).

The importance of Profiles Is Studied

- The 1D profile definitions allow in principle for a 4D (η, θ, μ, ν) space to be explored for profile optimization (6D if one allows for different profiles for T_i and T_e).
- In practice, temperature profiles are determined by transport and are less amenable to external control than density profile.
- In most cases, we assign either $T_{i,e}(r) \equiv T_L(r)$ or $T_{i,e}(r) \equiv T_H(r)$ (L- or H-mode-like profiles).

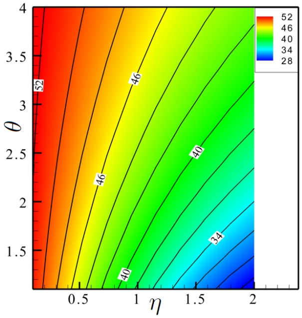


L- $(\mu = 1.5,\ \nu = 2.5)$ and H- $(\mu = 0.5,\ \nu = 1.5)$ mode temperature profiles

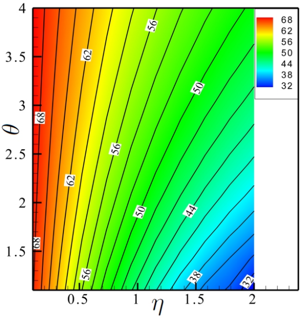


Limiting profiles for parametric scan

Minimum $p_{tot}\tau_{Ei}$ for Ignition Depends on Profiles.



Minimum $p_{tot}\tau_{Ei}$ for ignition, L-mode temperature profiles



Minimum $p_{tot}\tau_{Ei}$ for ignition, H-mode temperature profiles

- Density profiles are varied keeping temperature profiles fixed.
- Average n is fixed for all runs.
- The energy confinement time needed for ignition depends on the density and temperature profiles.
- For reference, the SF and MF 0D values are $\simeq 59$ and 82 $[10^{20}\text{m}^{-3}$ keV s].

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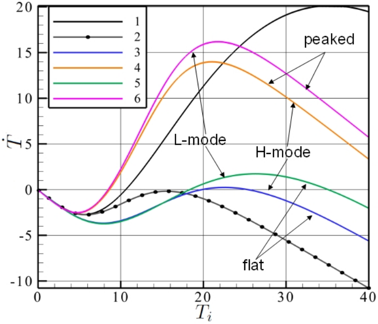
$\dot{T}$ Curves Are Obtained Also for the 1D Case.

- $\dot{T}$ curves strongly depend on parameters.
- Curves 1 and 2 are 0D (SF and MF).
- Curves 3 and 4 have H-mode T profile.
- Curves 5 and 6 have L-mode T profile.
- Curves 3 and 5 have $\theta = 2$, $\eta = 0.1$.
- Curves 4 and 6 have $\theta = 2$, $\eta = 2$.
- All curves are “at steady state”: different curves would be found in a transient.
- All $\tau_{Ej} = 3$s.



$\dot{T}_i$ vs. T_i for different models

Analysis Is Extended to Burning Plasmas

- For future experiments, the burning plasma ($P_\alpha \geq S_h$, $Q \geq 5$) state is more relevant than ignition.
- Formally, the only modification needed to extend our analysis is to have heating power on at all time.
- Experimental profiles (for e.g. ITER scenarios or DIII-D shots) are introduced as arbitrary expressions, fits or interpolations.
- Different spatial profiles are used for ion and electron temperatures.

Experimental Performance Can Be Evaluated In Terms of $p_{no-\alpha}$.

- To fix ideas, start from he single fluid, 0D case:

$$\frac{3}{2}n\frac{\partial T}{\partial t}=\frac{E_\alpha}{16}p^2\frac{<\sigma v>}{T^2}+S_h-\frac{C_B}{4}\frac{p^2}{T^{3/2}}-\frac{3}{2}\frac{p}{\tau_E}. \qquad (17)$$

- Solve

$$\dot{T}=0 \qquad \text{and} \qquad \frac{\partial \dot{T}}{\partial T}=0 \qquad (18)$$

for T and *one* of S_h, n and τ_E.

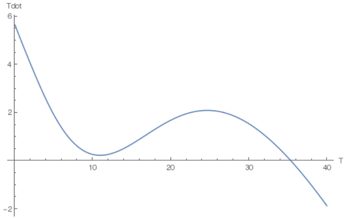
- Given the values, turn off the α heating and calculate p at steady state. This is $p_{no-\alpha}$.
- A similar procedure can be performed numerically for the full MF, 1D case.

Non-Constant Energy Confinement Time Is Considered.

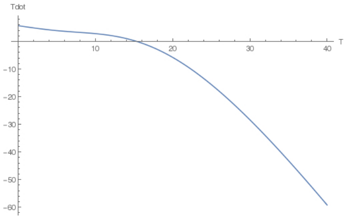
- It is more realistic to consider

$$\tau_E = \tau_{E0} \frac{S_h}{S_h + P_\alpha}. \qquad (19)$$

- This may result in a $\dot{T}$ curve without minimum.
- On the right, ITER cases with $\tau_{Ej} = 2.7$s (top), $\tau_{E0j} = 5.4$s (bottom).
- For the "standard" case, $p_{no-\alpha} \simeq 6.4 \times 10^{20}[\text{m}^{-3}\ \text{keV}]$, $\simeq 65\% p|_{min(\dot{T})}$
- In both cases, $S_h = 25$MW for both ions and electrons.



$\dot{T}$ vs. T with fixed $\tau_{Ei} = \tau_{Ee} = \tau_{E\alpha} = 2.7$s



$\dot{T}$ vs. T with fixed $\tau_{E0i} = \tau_{E0e} = \tau_{E0\alpha} = 5.4$s

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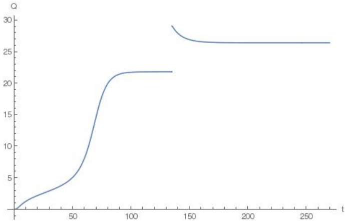
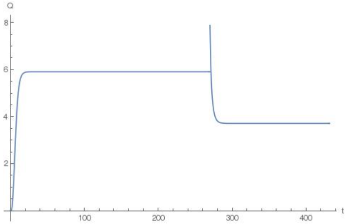
Output Power Strongly Depends on τ_E Model.

- The gain factor $Q(t)$ is calculated numerically for the fixed and variable τ_E models.
- It can be expected that a larger Q will be obtained if the heating power is reduced once the burning-plasma state is reached.
- This is verified for the fixed τ_E case only.
- Heating power is reduced by 25% halfway through the simulation.



Q vs. t with fixed $\tau_{Ei} = \tau_{Ee} = \tau_{E\alpha} = 2.7$s



Q vs. t with fixed $\tau_{E0i} = \tau_{E0e} = \tau_{E0\alpha} = 5.4$s

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Conclusions

- Two-fluid and α effects on ignition have been analyzed.
- The energy confinement time needed for ignition depends on multi-fluid physics.
- The heating power needed for ignition also depends on multi-fluid physics.
- One-dimensional temperature and density profiles influence the ignition criterion.
- For burning plasmas, a $p_{no-\alpha}$ figure of merit is introduced and the effect of the heating power on τ_E is considered.

List of Symbols

n	plasma density	n_α	α particle density
T_i	ion temperature	T_e	electron temperature
C_B	Bremsstrahlung coefficient	$< \sigma v >$	Fusion cross section
$\hat{\tau}_E$	"equivalent" energy confinement time	$\tau_{E\alpha}$	α energy confinement time
$k_1 \hat{\tau}_E$	ion energy confinement time	$k_2 \hat{\tau}_E$	electron energy confinement time
$\tau_{eq} = \tau_{eq}(T_i, T_e)$	T_i/T_e equilibration time	$\tau_\alpha = \tau_\alpha(T_e)$	α/T_e equilibration time

References

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3. http://www.auburn.edu/cosam/faculty/physics/guazzotto/research/TF_Lawson_main.html (google "Guazzotto Lawson")
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