# Loss of Relativistic Electrons when Magnetic Surfaces Are Broken

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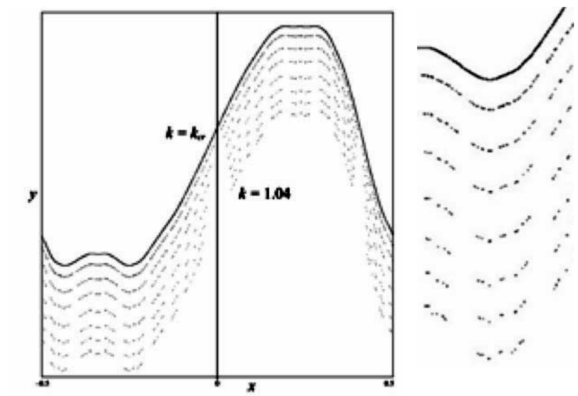
Runaway relativistic electrons in ITER may fill a large region of stochastic magnetic field lines separated from walls by a narrow annulus of confining magnetic surfaces, Izzo et al, NF 51 063032 (2011).

When the confining surfaces are lost by (1) magnetic evolution or (2) plasma drifts into the walls, the relativistic electrons are lost quickly ~0.3ms and in a narrow flux tube ~400 cm$^2$.

The damage would be severe but predictable using the turnstile concept of Hamiltonian mechanics, Meiss, Chaos 25, 097602 (2015).

# Turnstiles in Hamiltonian Mechanics

The last confining magnetic surfaces are broken by developing splits, which have neighboring regions of exiting and entering magnetic field lines called turnstiles.




For an evolving Hamiltonian the width of the split increases as a power law.

A narrow split defines a narrow flux tube of escaping magnetic field lines.

The magnetic field line Hamiltonian is $\psi_p(\psi_t,\theta,\varphi,t)$. Relativistic electrons follow the field lines in ITER with one toroidal transit taking $\tau_t = 2\pi R / c \approx 0.1\mu\mathrm{s}$.

# Two Fundamental Power Laws

$$f(t) = c_f \left( \frac{t - t_o}{\tau_{ev}} \right)^{\alpha}$$

$f$ is the fraction of the toroidal flux in the stochastic region that exits through the turnstile at time $t$, where $\tau_{ev}$ is the time scale of the system evolution and $t_o$ is the time at which the turnstile opens. $f$ is essentially the dimensionless area of the flux tube.

$$\tau_s = c_s \frac{\tau_t}{f^{\beta}}$$

$\tau_s$ is the time between independent passes at the turnstile; the number of remaining relativistic electrons $N(t)$ obeys

$$\frac{dN}{dt} = -\frac{f}{\tau_s} N$$

# Derivation of $N(t)$ and Characteristic $f$

The fundamental scaling laws imply

$$N(t) = N_0 \exp\left(-\left(\frac{t-t_o}{\tau_\ell}\right)^{\mu}\right) \quad \text{and} \quad \bar{f} \equiv \frac{1}{N_0}\int_{t_o}^{\infty} f(t)\frac{dN}{dt}dt = \bar{c}_f\left(\frac{\tau_t}{\tau_\ell}\right)^{\sigma}$$

The loss time $\tau_\ell = c_\ell \tau_t \left(\frac{\tau_{ev}}{\tau_t}\right)^{\lambda}$. The coefficient $\bar{c}_f = c_f c_\ell^{\mu\sigma}$.

The exponent $\mu = 1+(1+\beta)\alpha$ while $\lambda=1-1/\mu$ and $\sigma=1/(1+\beta)$.

**Simulations give the loss time $\tau_\ell$ and the dimensionless area in which the footpoints strike the surrounding walls $\bar{f}$.**

# Simulation Method

Simulations assume $\psi_p = \bar{\psi}_p(\psi_t) + \tilde{\psi}_p(\theta,\varphi,t)$, which can be solved by an explicit symplectic integrator:

$$\varphi^{(j+1)} = \varphi^{(j)} + k, \quad \text{used } k = 2\pi/360;$$

$$\psi_t^{(j+1)} = \psi_t^{(j+1)} - k\frac{\partial\tilde{\psi}_p(\theta^{(j)},\varphi^{(j)},t)}{\partial\theta^{(j)}};$$

$$\theta^{(j+1)} = \theta^{(j)} + k\frac{\partial\bar{\psi}_p(\psi_t^{(j+1)})}{\partial\psi_t^{(j+1)}}.$$

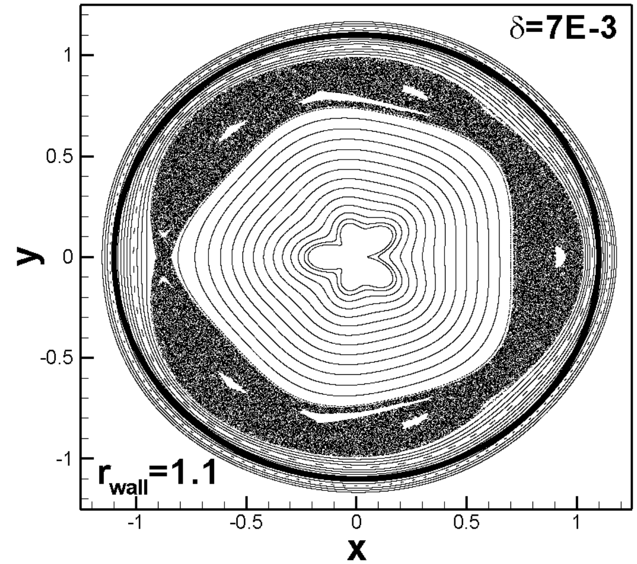
Evolutionary time $t$ is held fixed during each iteration but changed between iteration steps.

$$\bar{\iota}(\psi_t) \equiv d\bar{\psi}_p / d\psi_t = 1 - 0.8\psi_t/\psi_a$$

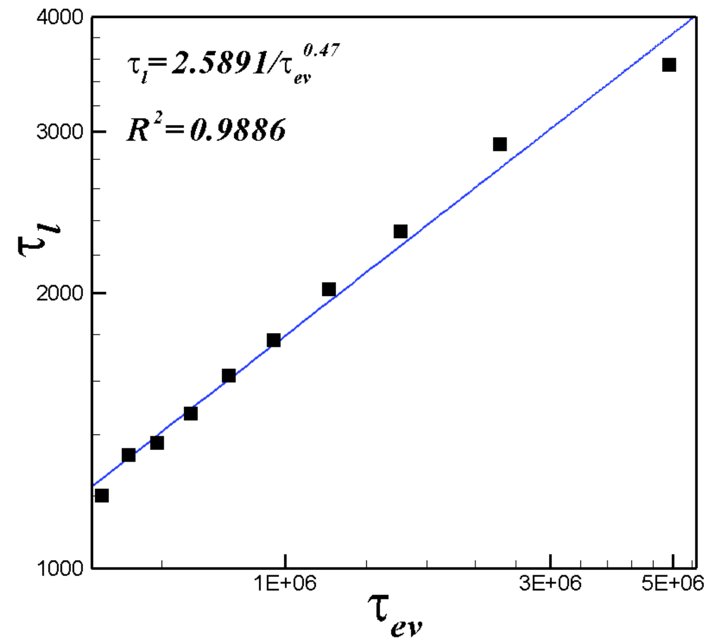
$$\tilde{\psi}_p(\theta,\varphi,t) = \delta_a(t)\big(\cos(3\theta-\varphi) + \cos(5\theta-2\varphi)\big)\psi_a$$

# Evolving $\delta_a$ Simulations

The unperturbed plasma is circular with radius $a$ and with a wall at 1.1$a$. The last confining surface is broken when $\delta_a$=11x10$^{-3}$.

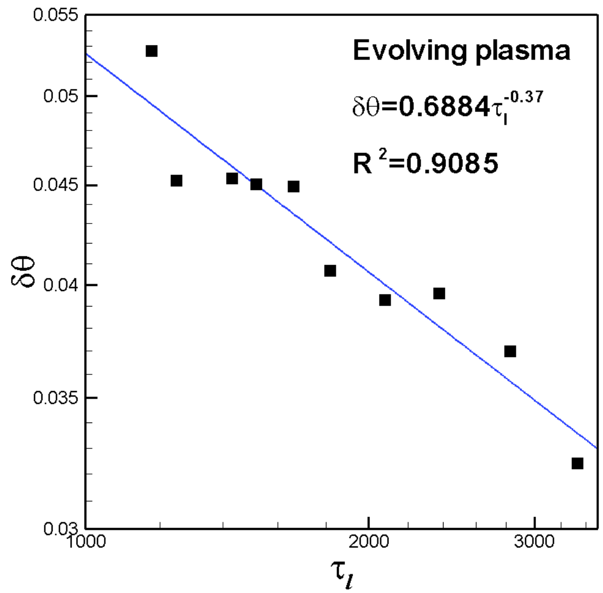


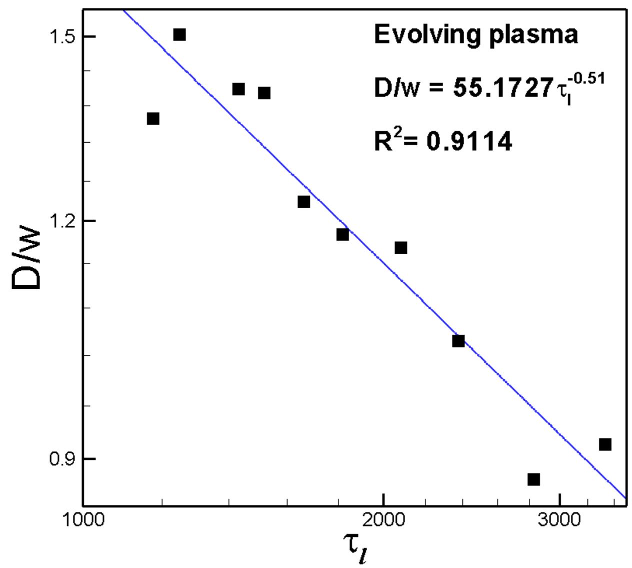

Loss time proportional to square root of evolutionary time.

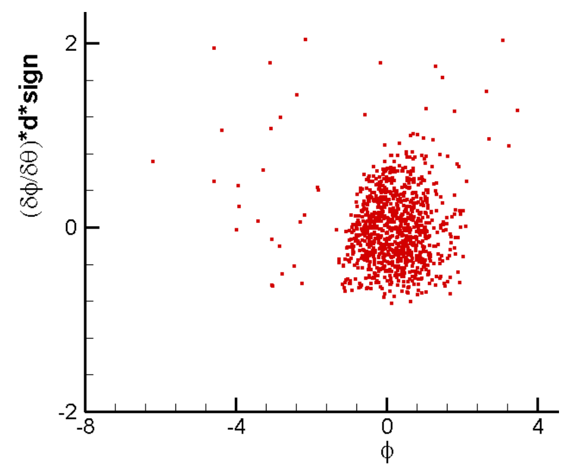


# Evolving $\delta_a$ Simulations (continued)

The characteristic dimensionless area $\bar{f} = \frac{1}{f_{sc}} \frac{D}{w} \left\langle \delta\theta^2 \right\rangle \approx 6 \frac{\tau_t}{\tau_\ell}$

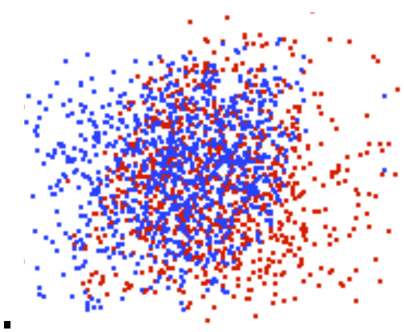


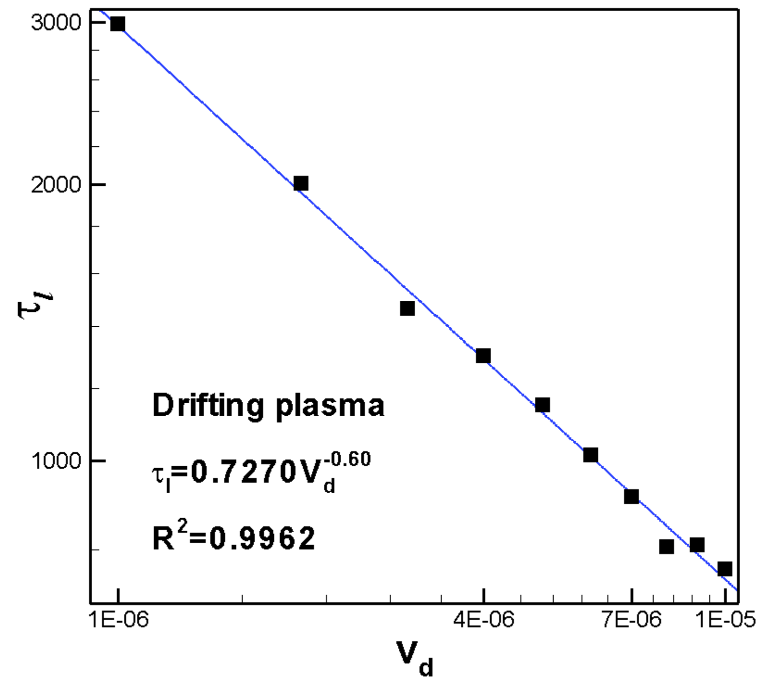




*D* is width of tube in plane perpendicular to wall.
*w* is width of flux tube in plane tangential to wall.

Footpoints are compact; have a tangled structure, red integrating forward and blue integrating backward.



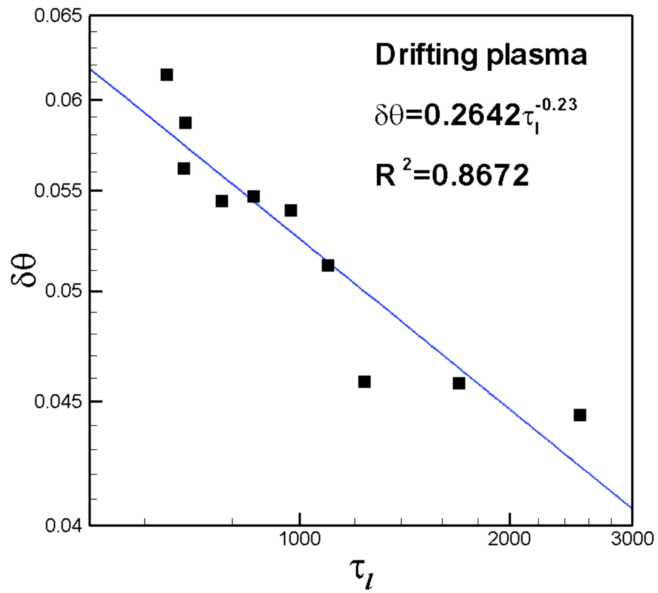
# Drifting Plasma Simulations

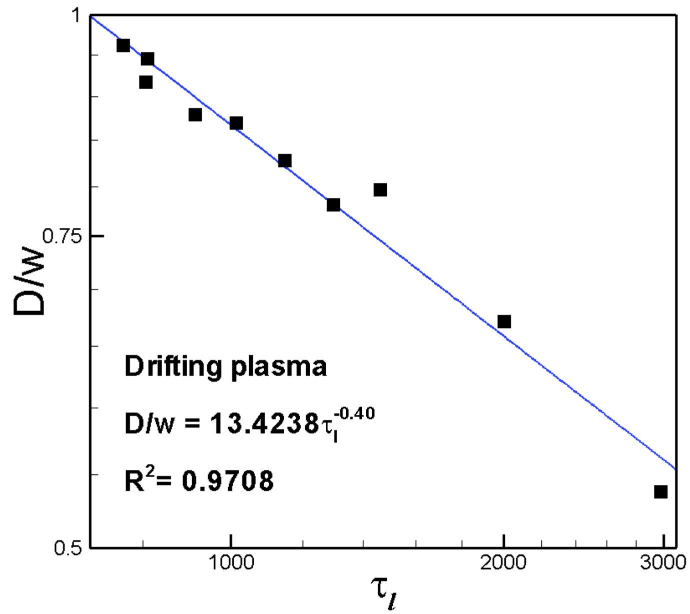


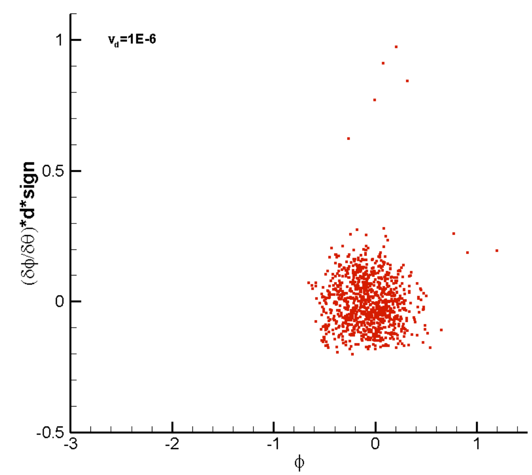

Loss time proportional to 3/5 power of evolutionary time.

Characteristic dimensionless area

$$\bar{f} = \frac{1}{f_{sc}} \frac{D}{w} \left\langle \delta\theta^2 \right\rangle \approx 4 \frac{\tau_t}{\tau_\ell}$$



# Summary

Turnstile concept allows a calculation of the flux in tubes along which relativistic electrons are lost with similar results whether the magnetic field is evolving or the plasma is drifting into a wall.

During an ITER disruption $\tau_{ev}/\tau_t \approx 100\text{ms}/0.1\mu\text{s}=10^6$, the flux tube has an area (ITER cross section) $\bar{f}$~(22 m$^2$)(2x10$^{-3}$)~400 cm$^2$.

The power per unit area flowing down the flux tube is intense

$$P \approx 0.4 \frac{(\bar{\gamma}-1)mc^2}{(\psi_{st}/B)} \frac{N(t)}{\tau_t},$$

where $(\bar{\gamma}-1)mc^2$ is average kinetic energy of relativistic electrons, $\psi_{st}/B$ is the cross-sectional area of stochastic region.

The time for $N(t)$ to decrease by an e-fold is $\tau_\ell \sim 0.3\text{ms}$.