# Numerical Solutions to Thermal Magnetic Reconnection Equations

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# Summary

- We are studying magnetic reconnection modes driven by **electron temperature gradients**.

- An eighth order system of equations is derived for the perturbed displacement, temperature, and magnetic field within the reconnection layer.

- These equations include the effects of **viscosity**, **resistivity**, and **heat conductivity**, in addition to a **mode inductivity**, and are derived from a planar geometry for simplicity.

# Motivation

The conventional theory of magnetic reconnection by the tearing mode in weakly collisional and collisionless plasmas involves characteristic length scales that are unrealistically small for astrophysical plasmas of interest. This fact motivates the search for modes that produce magnetic reconnection over microscopic scale distances that remain significant when large macroscopic scale distances (such as space plasmas) are considered.

# Planar Current Sheet Model

$$\mathbf{B} = B_0(x)\mathbf{e_z} + B_y(x)\mathbf{e_y}$$

$$B_y^2 << B_0^2$$

$$\hat{\mathbf{B}} = \mathbf{B}(x) exp(-i\omega t + iky + ik_z z)$$

$$\mathbf{k} \cdot \mathbf{B} = \mathbf{k} \cdot \mathbf{B}'(x - x_0), k_y^2 << k_z^2$$

$$k_y B_y + k_z B_z = 0 \text{ at } x - x_0$$

$$\hat{\mathbf{E}} = -\nabla\Phi - \frac{1}{c}\frac{\partial \hat{\mathbf{A}}}{\partial t}$$

$$\hat{\mathbf{B}} = \nabla \times \hat{\mathbf{A}}$$

$$\hat{E}_{\|} = -i\frac{\mathbf{k} \cdot \mathbf{B}}{B}\hat{\Phi} + i\frac{\omega}{c}\hat{\mathbf{A}} \cdot \frac{\mathbf{B}}{b} = -\frac{i}{B}\left[(\mathbf{k} \cdot \mathbf{B}')(x - x_0)\hat{\Phi} - \frac{\omega}{c}\hat{A}_z\right]$$

# Governing Equations

$$-(\omega - \omega_{di})\left(\omega - iD_\mu \frac{d^2}{dx^2}\right)\frac{d^2\xi_x}{dx^2} = i\frac{(\mathbf{k}\cdot\mathbf{B})}{4\pi m_i n}\frac{d^2 B_x}{dx^2}$$

$$(\omega - \omega_{*e} - \omega_{*T})B_x = i(\mathbf{k}\cdot\mathbf{B})\left[(\omega - \omega_{*e})\xi_x + \omega_{*T}\frac{T_e}{T_e'}\right] + \left[\omega\left(S_L - \frac{c^2}{\omega_{pe}^2}\frac{k_\parallel^2 T_e}{m_e\omega^2}\right) + iD_\eta\right]\frac{d^2 B_x}{dx^2}$$

$$i\omega T_e'\xi_x + D_{\perp e}\frac{d^2 T_e}{dx^2} - D_{\parallel e}k_\parallel^2 Te = -ik_\parallel D_{\parallel e}T_e'\frac{B_x}{B}$$

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$$\bar{B} = \frac{B_x}{B_{x0}} \qquad \bar{U} = \frac{T_e}{iT_e'}\left(\frac{\mathbf{k}\cdot\mathbf{B}'}{B_{x0}}\delta_T\right) \qquad \bar{W} = i\xi_x\left(\frac{\mathbf{k}\cdot\mathbf{B}'}{B_{x0}}\delta_T\right)$$

# Nondimensional Equations

$$\frac{d^2\bar{U}}{d\bar{x}^2} - \bar{x}^2\bar{U} = -\bar{x}\bar{B}$$

$$\eta_* \frac{d^2\bar{U}}{d\bar{x}^2} = \Delta\bar{\omega}(\bar{x}^2\bar{W} - \bar{x}\bar{B}) + \frac{L_0}{\epsilon_s}\bar{\omega}\bar{S}_L\bar{x}\frac{d^2\bar{B}}{d\bar{x}^2}$$

$$\epsilon_s(\bar{\omega} - \bar{\omega}_{di})\left(\bar{\omega} - i\nu_\mu \frac{d^2}{d\bar{x}^2}\right)\frac{d^2\bar{W}}{d\bar{x}^2} = \bar{x}\frac{d^2\bar{B}}{d\bar{x}^2}$$

$$\eta_* = \frac{\omega_{*T}}{\omega{*e}}$$

$$\epsilon_s = \frac{\omega_{*e}^2}{\omega_H^2}\frac{1}{k_y^2\delta_T^2}$$

$$\omega_{*e} = -k_y \frac{cT_e}{eB}\left(\frac{1}{n}\frac{dn}{dx}\right)$$

$$\bar{S}_L = 1 - \frac{\epsilon_s}{L_0}\left(\frac{1}{\beta_{p*}}\frac{\bar{x}^2}{\bar{\omega}^2} - i\frac{\epsilon_\eta}{\bar{\omega}}\right)$$

$$\omega_{*T} = -(1+\alpha_T)k_y\frac{cT_e}{eB}\left(\frac{1}{T_e}\frac{dT_e}{dx}\right)$$

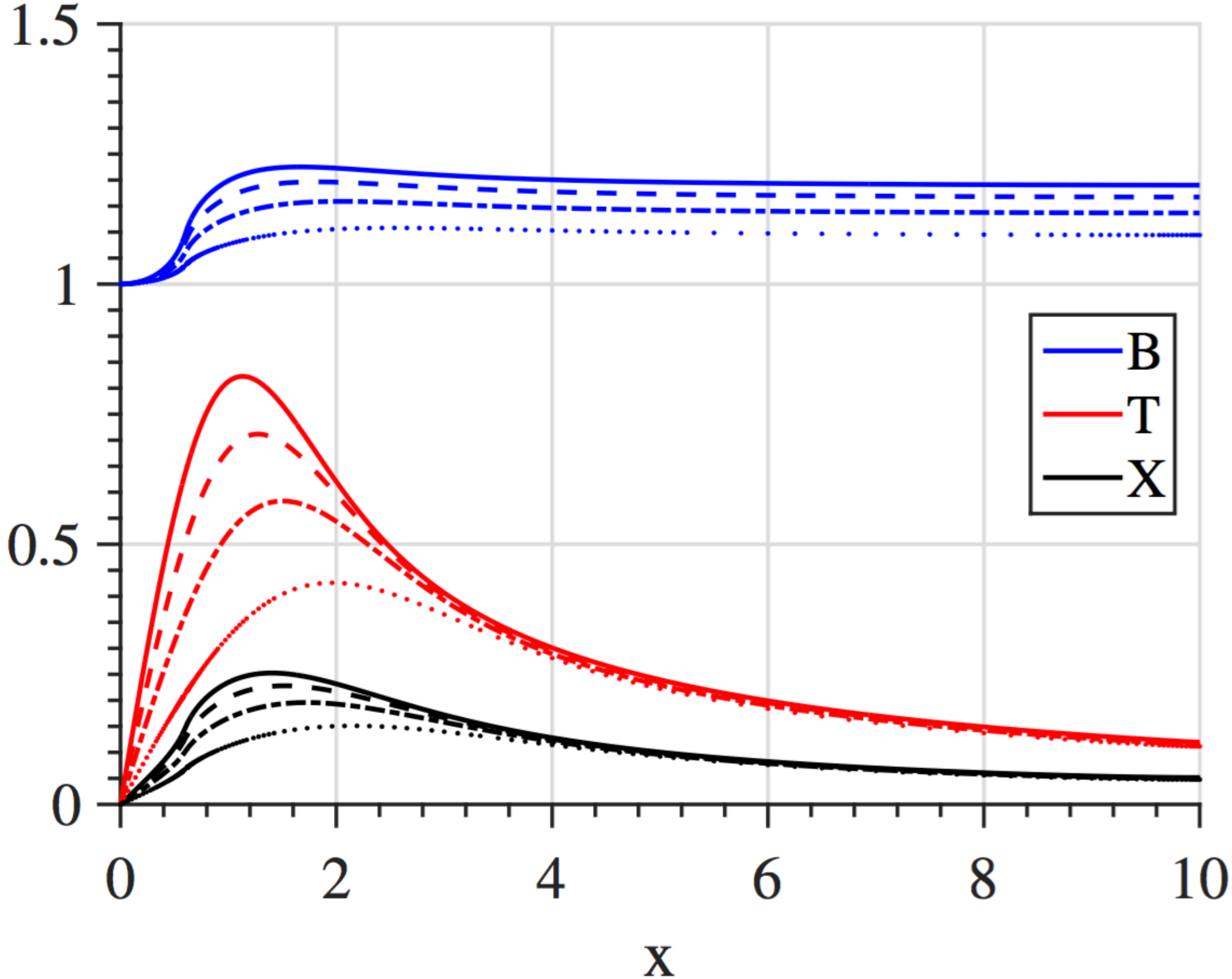
$$\omega_{di} = k_y\frac{cT_i}{eB}\left(\frac{1}{n}\frac{dn}{dx} + \frac{1}{T_i}\frac{dT_i}{dx}\right)$$

# Numerics

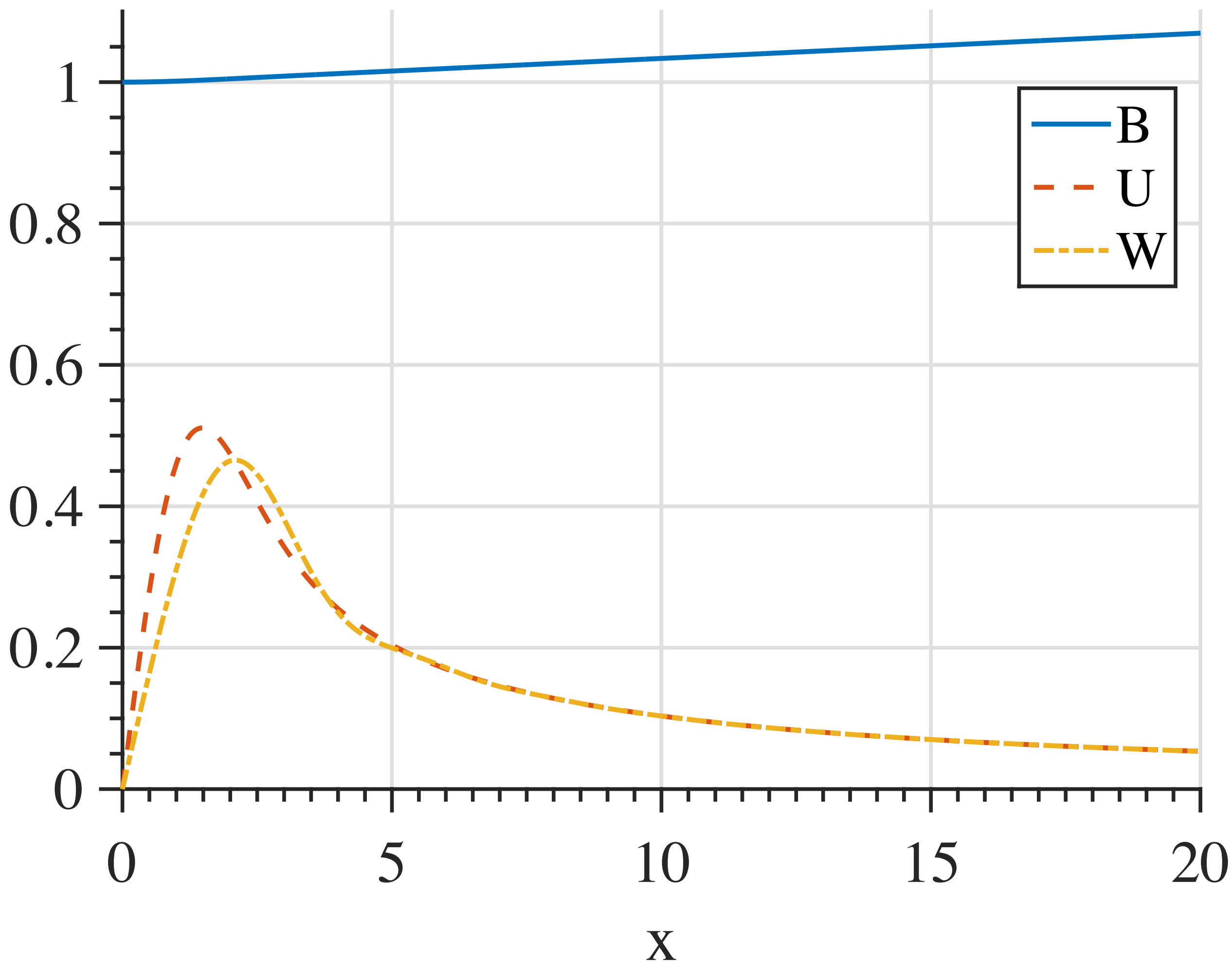
- Collocation algorithm used to solve resulting eighth order system of ODEs
- Singularites at $\bar{x}^2 = \beta_{p*}\bar{\omega}^2 \left(\frac{L_0}{\epsilon_s} + \frac{i\epsilon_\eta}{\bar{\omega}}\right)$ handled with small resistivity and complex frequency
- Boundary conditions from parity considerations and matched to asymptotic solution at $\infty$

$$
\begin{aligned}
\bar{B}(0) &= 1 & \bar{B}'(\infty) &= 0 \\
\bar{B}'(0) &= 0 & \bar{U}(\infty) &= 0 \\
\bar{U}(0) &= 0 & \bar{W}(\infty) &= 0 \\
\bar{W}(0) &= 0 & \bar{W}'(\infty) &= 0 \\
\bar{W}''(0) &= 0 & &
\end{aligned}
$$

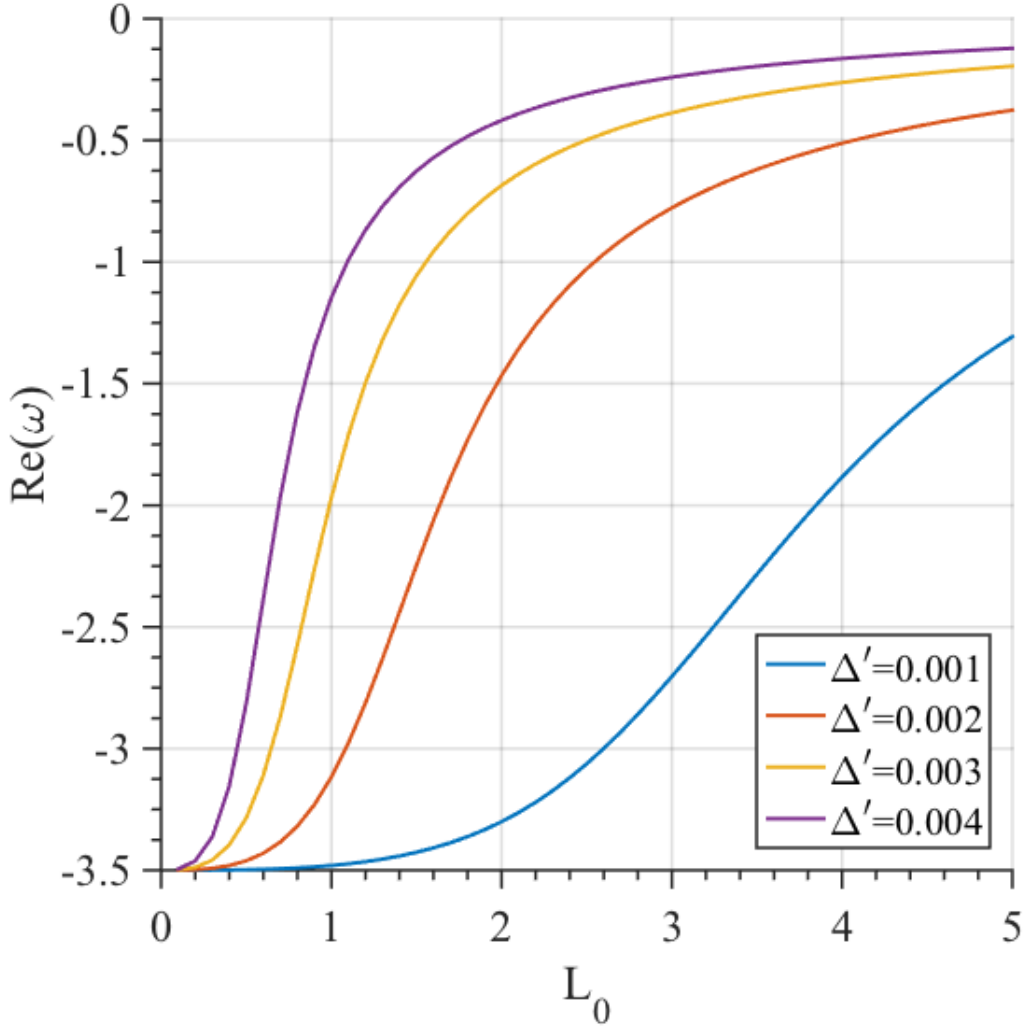
# Δ’ = 0 Solutions



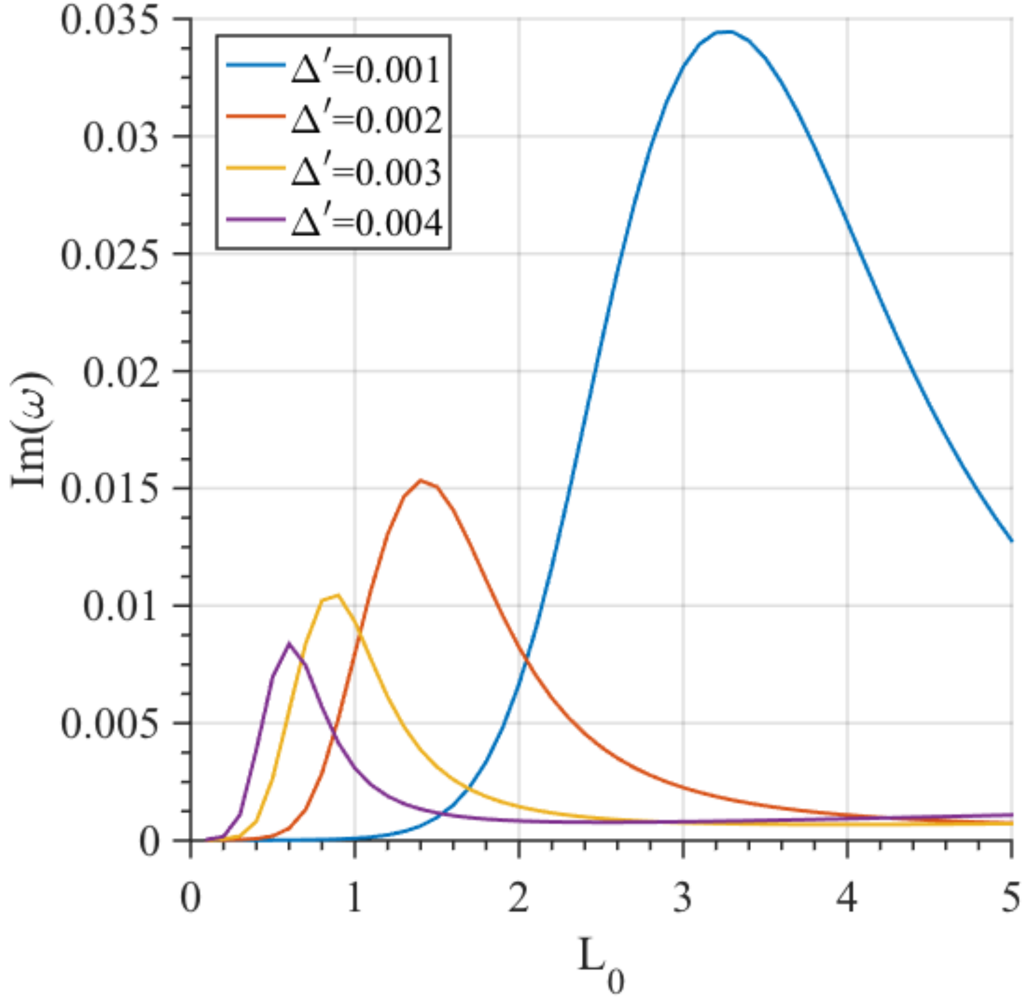
# Δ’ > 0 Solutions



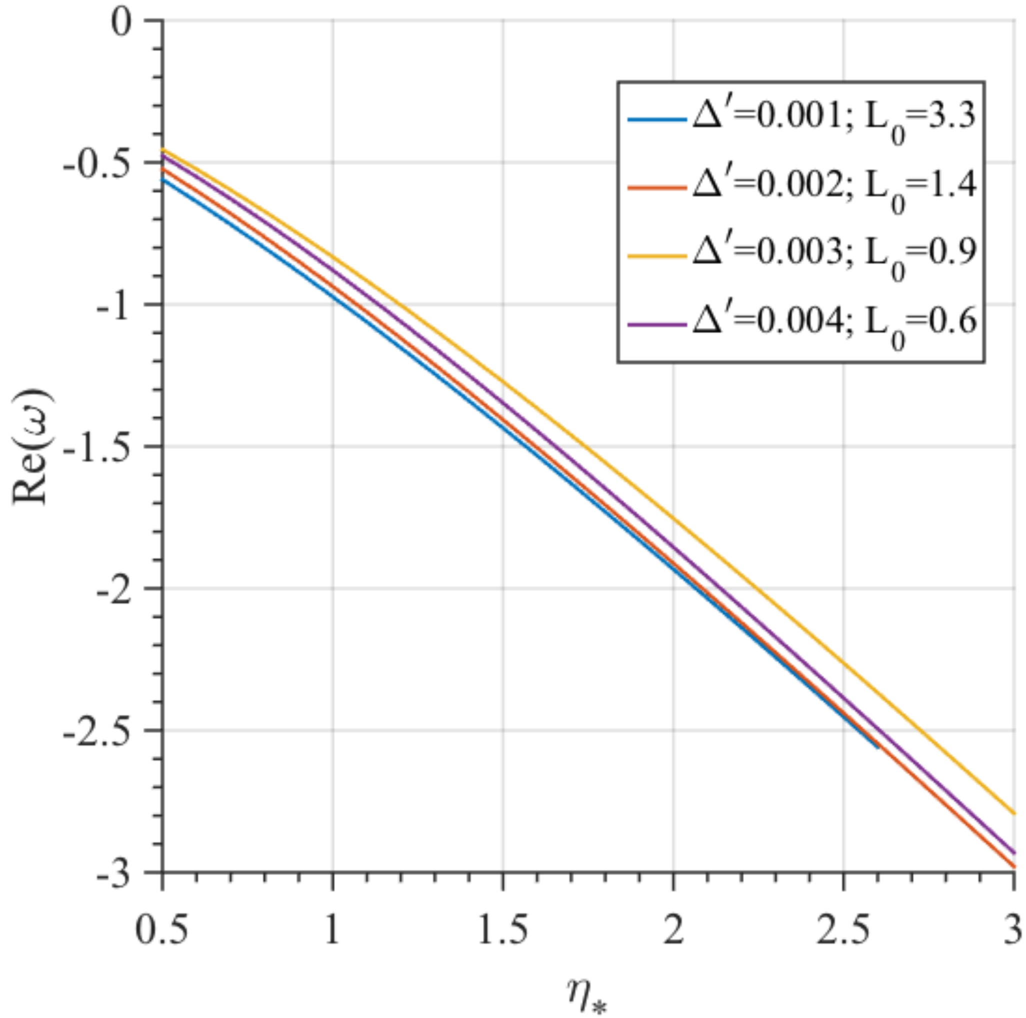
# Frequency vs. Inductivity



# Growth Rate vs. Inductivity



# Frequency vs. Gradient Ratio



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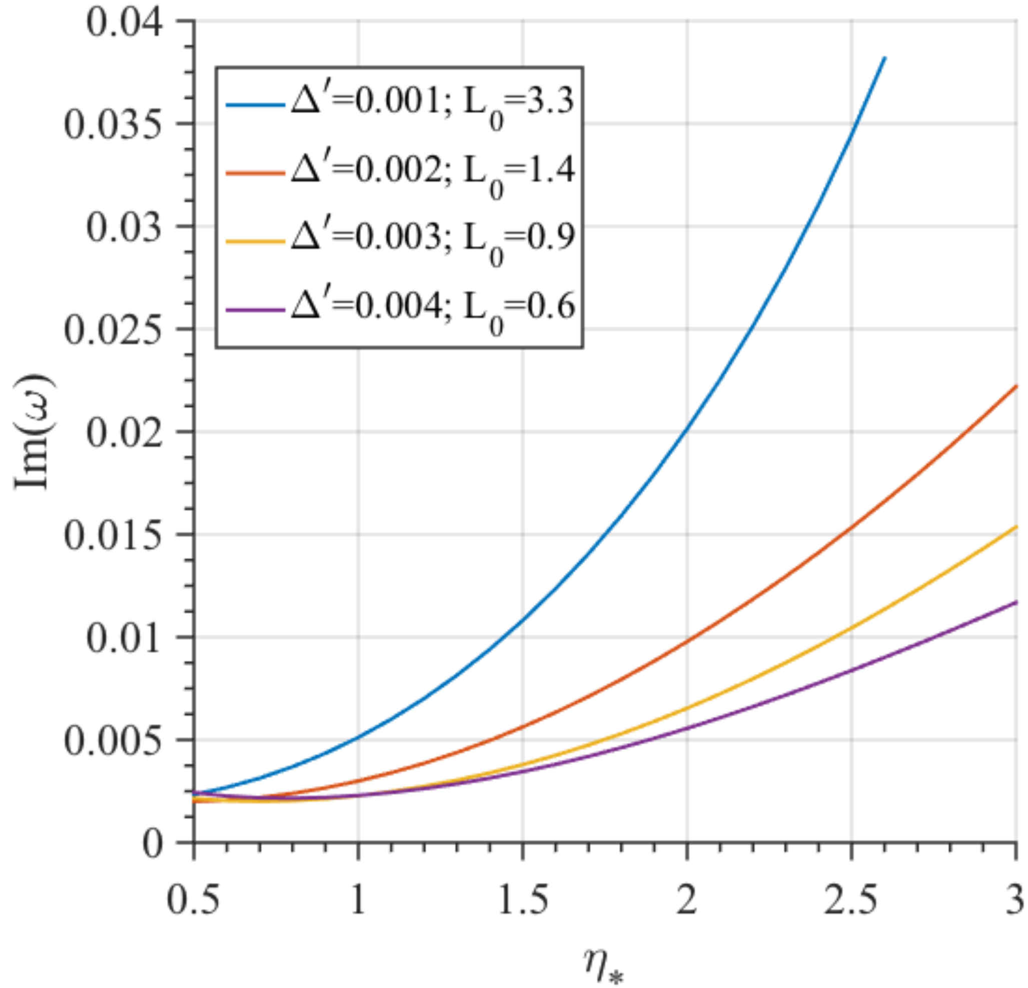

# Concluding Remarks

- The solutions exhibit the expected structure within the layer and show that the thermal layer width is the primary scale distance.
- No localized modes are found.
- The imaginary part of the frequency is small and positive, indicating a dissipative instability.

# Concluding Remarks (II)

- We observe mode phase velocities in the direction of the electron diamagnetic velocity when delta-prime is zero. The solutions with positive delta-prime yield frequencies with negative real part, indicating the mode phase velocity is in the direction of the ion diamagnetic velocity.

- **Temperature fluctuations play an important role in the development of reconnecting modes.**