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Influence of Dupree diffusivity on the occurrence scattering time in a turbulent plasma

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ABSTRACT

The effects of turbulence on the occurrence scattering time advance for the electron-ion collision are investigated in thermal turbulent plasmas. The second-order eikonal method and the effective far-field potential term associated with the plasma turbulence are employed to obtain the occurrence scattering time as a function of the diffusion coefficient, impact parameter, collision energy, thermal energy, and Debye length. The result shows that the occurrence scattering time advance decreases with an increase of the diffusion coefficient. Hence, we have found that the influence of plasma turbulence diminishes the occurrence time advance for forward electron-ion collisions in thermal turbulent plasmas. It is shown that the propensity of the occurrence time advance increases with increasing scattering angle. It is also found that the effect of turbulence on the occurrence scattering time advance decreases with an increase of the thermal energy.

Second-order eikonal analysis and effective Shukla-Spatschek potential

- The projectile electron in turbulent plasmas would be strongly affected by the random fluctuating electric fields since the response of field fluctuations plays an important role in binary encounters.

- The occurrence scattering time characterizing the quantum collision process exemplifies the occurrence time of emergence of the wave packet during the collision process.

- The occurrence scattering time advance or delay would be expected to reveal when the scattering occurs with charged particles for a given scattering angle.

- The effects of turbulence on the occurrence scattering time advance for the electron-ion collision in thermal turbulent plasmas is investigated in this presentation.

Non-relativistic Schrödinger equation

$$\left[\nabla^2 - \frac{2\mu_{ei}}{\hbar^2}V_{ei}(\mathbf{r}) + k^2\right]\varphi(\mathbf{r}) = 0,$$

where μ_{ei} is reduced mass of the electron-ion collision system, $\hbar$ is the rationalized Planck constant, $k[=(2\mu_{ei}E)^{1/2}/\hbar]$ is the wave number, and E is the collision energy.

Using the Green function of the Helmholtz operator, the scattering eikonal wave function can be written in the form

$$\varphi_E^{(+)}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}}\left[e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{2\mu_{ei}}{\hbar^2}\int d^3\mathbf{r}'\, e^{i\mathbf{k}\cdot\mathbf{r}'}\, G_0^{(+)}(\mathbf{r}-\mathbf{r}')V_{ei}(\mathbf{r}')\right].$$

Hence, the eikonal scattering amplitude is obtained as

$$\begin{aligned} f_E(\mathbf{k}_i,\mathbf{k}_f) &= -ik_i\int_0^\infty db\, b\, J_0(Qb)\left[\exp\left(i\chi_E(\mathbf{k}_i,\mathbf{b})\right) - 1\right] \\ &= \left|f_E(\mathbf{k}_i,\mathbf{k}_f)\right|\exp\left[i\xi(\mathbf{k}_i,\mathbf{k}_f)\right], \end{aligned}$$

where $\mathbf{b}$ is the impact parameter, $\hat{\mathbf{n}}$ is the unit vector perpendicular to the momentum transfer $\mathbf{Q}(\equiv \mathbf{k}_i - \mathbf{k}_f)$, $\mathbf{k}_i$ and $\mathbf{k}_f$ are the incident and final wave vectors, $Q = (k_i^2 + k_f^2 - 2k_i k_f \cos\theta)^{1/2}$, θ is the angle between the incident $\mathbf{k}_i$ and final $\mathbf{k}_f$ wave vectors, $\chi_E(\mathbf{k}_i, \mathbf{b})$ is the total eikonal scattering phase, and $\xi(\mathbf{k}_i, \mathbf{k}_f)$ is the argument of the scattering amplitude, respectively.

Here, the eikonal scattering phase can be represented by

$$\chi_E(\mathbf{k}_i, \mathbf{b}) = \frac{\chi_1(b)}{k_i} + \frac{\chi_2(b)}{k_i^3},$$

where

$$\frac{\chi_1(b)}{k_i} = -\frac{\mu}{k_i \hbar^2} \int_{-\infty}^{\infty} dz\, V_{ei}(r), \quad \frac{\chi_2(b)}{k_i^3} = \frac{1}{2k_i^3}\left(\frac{\mu}{\hbar^2}\right)^2 \int_{-\infty}^{\infty} dz\, [\nabla \alpha(b,z)] \cdot [\nabla \beta(b,z)],$$

$$\alpha(b,z) = -\int_{-\infty}^{z} dz' V_{ei}(r') \qquad \beta(b,z) = -\int_{z}^{\infty} dz' V_{ei}(r')$$

Effective Shukla-Spatschek model

Screened interaction potential between the projectile electron and the target ion with nuclear charge *Ze*

$$V_{SS}(r) = -\frac{Ze^2}{r}\exp(-r/\lambda_D) - \frac{Ze^2}{r}\frac{2\sqrt{2}}{\sqrt{\pi}}\cos\alpha\left(\frac{\lambda_D}{r}\right)^2\left(\frac{v}{v_T}\right)\left(1-\frac{9}{4}\sqrt{\pi}\frac{D}{v_T^3}r\right),$$

where $r=[b^2+(v_0t)^2]^{1/2}$, λ_D is the Debye length, v_0 is the collision velocity, $v_T(=\omega_p\lambda_D)$ is the thermal velocity, ω_p is the plasma frequency, and α is the angle between **r** and **v**.

The total eikonal scattering phase is found as

$$\chi_E(\bar{k}_i,\bar{b}) = \frac{1}{\bar{k}_i}\left[2K_0(\bar{b}/\bar{\lambda}_D) + \frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5} + \frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}} - \frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right],$$

where $\bar{k}_i \equiv k_i a_Z$, $\bar{b}(\equiv b/a_Z)$ is the scaled impact parameter, $a_Z(=a_0/Z)$ is the Bohr radius of the hydrogenic ion with nuclear charge Ze, $a_0(=\hbar^2/m_e e^2)$ is the Bohr radius of the hydrogen atom, m_e is the mass of the electron, $\bar{\lambda}_D(\equiv \lambda_D/a_Z)$ is the scaled Debye length, $\bar{E}_T \equiv E_T/Z^2 Ry$, $Ry(=m_e e^4/2\hbar^2 \approx 13.6\,\mathrm{eV})$ is the Rydberg constant, $E_T(=k_B T)$ is the thermal energy, k_B is the Boltzmann constant, T is the plasma temperature, $\bar{D}(\equiv D a_Z/v_Z^3)$ is the scaled diffusion coefficient, $v_Z(=Z\alpha c)$ is the Bohr velocity of the hydrogenic ion with nuclear charge Ze, $\alpha(=e^2/\hbar c \approx 1/137)$ is the fine structure constant, c is the speed of light in vacuum.

K_0 is the zeroth order of the MacDonald function

$$K_n(p) = [\pi^{1/2}/(n-1/2)!](p/2)^n \int_1^\infty dq\, e^{-pq}(q^2-1)^{n-1/2}$$

Occurrence scattering time for the elastic electron-ion collision

The occurrence scattering time τ_{OST} can be expressed by the first-derivative of the argument of the scattering amplitude $\xi(\mathbf{k}_i, \mathbf{k}_f)$ with respect to the initial wave number k_i :

$$\tau_{OST}(\theta, k) = \frac{\mu_{ei}}{\hbar} \left[\frac{\partial \xi(\mathbf{k}_i, \mathbf{k}_f)}{k_i \partial k_i} \right] \Bigg|_{k_i = k_f = k},$$

with the constraint for elastic collisions: $k \equiv k_i = k_f$.

Eikonal scattering amplitude would be represented by a power series in the potential strength with the scattering angle such as

$$f_E(\theta, \bar{k}_i, \bar{k}_f) = f_{\mathrm{Re}}(\theta, \bar{k}_i, \bar{k}_f) + i\, f_{\mathrm{Im}}(\theta, \bar{k}_i, \bar{k}_f),$$

where $$f_{\mathrm{Re}}(\theta,\bar{k}_i,\bar{k}_f) = 2a_Z \int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}\,J_0(\bar{Q}\bar{b})\,\bar{\chi}_E(\bar{b}),$$

$$f_{\mathrm{Im}}(\theta,\bar{k}_i,\bar{k}_f) = \frac{2a_Z}{\bar{k}_i}\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}\,J_0(\bar{Q}\bar{b})\,\bar{\chi}_E^2(\bar{b}),$$

$\bar{k}_f \equiv k_f a_Z$, $\bar{Q}(\theta,\bar{k}_i,\bar{k}_f)[\equiv Qa_Z = (\bar{k}_i^2 + \bar{k}_f^2 - 2\bar{k}_i\bar{k}_f\cos\theta)^{1/2}]$ is the scaled momentum transfer, and $\bar{\chi}_E(\bar{b})(\equiv \bar{k}_i\chi_E/2)$ is the scaled total eikonal scattering phase.

The scaled occurrence scattering time $\bar{\tau}_{OST}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D)$ for the elastic electron-ion collision in turbulent plasma is then found to be

$$\boxed{\begin{aligned}\bar{\tau}_{OST}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D) &\equiv \tau(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D)v/a_Z\\ &= \frac{1}{f_{\mathrm{Re}}^2+f_{\mathrm{Im}}^2}\left(f_{\mathrm{Re}}\frac{\partial f_{\mathrm{Im}}}{\partial\bar{k}_i} - f_{\mathrm{Im}}\frac{\partial f_{\mathrm{Re}}}{\partial\bar{k}_i}\right)\Bigg|_{\bar{k}_i=\bar{k}_f=\bar{k}},\end{aligned}}$$

where $\bar{k}(\equiv ka_Z = \bar{E}^{1/2})$ is the scaled wave number, the functions related to the scattering amplitude $f_{\mathrm{Re}}\big|_{\bar{k}_i=\bar{k}_f=\bar{k}}$, $f_{\mathrm{Im}}\big|_{\bar{k}_i=\bar{k}_f=\bar{k}}$, $\partial f_{\mathrm{Re}}/\partial\bar{k}_i\big|_{\bar{k}_i=\bar{k}_f=\bar{k}}$, and $\partial f_{\mathrm{Im}}/\partial\bar{k}_i\big|_{\bar{k}_i=\bar{k}_f=\bar{k}}$ are, respectively, given by

$$
\begin{aligned}
f_{\mathrm{Re}}\Big|_{\bar{k}_i=\bar{k}_f=\bar{k}} &= f_{\mathrm{Re}}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D) \\
&= 2a_Z\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}\,J_0\left(2\bar{E}^{1/2}\bar{b}\sin(\theta/2)\right) \\
&\quad\times\left[2K_0(\bar{b}/\bar{\lambda}_D)+\frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5}+\frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}}-\frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
f_{\mathrm{Im}}\Big|_{\bar{k}_i=\bar{k}_f=\bar{k}} &= f_{\mathrm{Im}}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D) \\
&= \frac{2a_Z}{\bar{E}}\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}\,J_0\left(2\bar{E}^{1/2}\bar{b}\sin(\theta/2)\right) \\
&\quad\times\left[2K_0(\bar{b}/\bar{\lambda}_D)+\frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5}+\frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}}-\frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right]^2,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f_{\mathrm{Re}}}{\partial\bar{k}}\Bigg|_{\bar{k}_i=\bar{k}_f=\bar{k}} &= f'_{\mathrm{Re}}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D) \\
&= -2a_Z\sin(\theta/2)\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}^2\,J_1\left(2\bar{E}^{1/2}\bar{b}\sin(\theta/2)\right) \\
&\quad\times\left[2K_0(\bar{b}/\bar{\lambda}_D)+\frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5}+\frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}}-\frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right],
\end{aligned}
$$

$$
\left.\frac{\partial f_{\mathrm{Im}}}{\partial \bar{k}}\right|_{\bar{k}_i=\bar{k}_f=\bar{k}} = f'_{\mathrm{Im}}(\theta,\bar{E},\bar{E}_T,\bar{D},\bar{\lambda}_D)
$$

$$
\begin{aligned}
&= -\frac{2a_Z}{\bar{E}^{1/2}}\sin(\theta/2)\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}^2\,J_1\left(2\bar{E}^{1/2}\bar{b}\sin(\theta/2)\right) \\
&\quad\times\left[2K_0(\bar{b}/\bar{\lambda}_D)+\frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5}+\frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}}-\frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right]^2 \\
&\quad-\frac{2a_Z}{\bar{E}}\int_{\bar{\lambda}_D}^{\infty} d\bar{b}\,\bar{b}\,J_0\left(2\bar{E}^{1/2}\bar{b}\sin(\theta/2)\right) \\
&\quad\times\left[2K_0(\bar{b}/\bar{\lambda}_D)+\frac{\bar{\lambda}_D^4}{\bar{E}_T}\left(-\frac{3}{\bar{b}^5}+\frac{48}{\sqrt{\pi}}\frac{\bar{D}}{\bar{b}^4\bar{E}_T^{3/2}}-\frac{81\pi}{4}\frac{\bar{D}^2}{\bar{b}^3\bar{E}_T^3}\right)\right]^2 .
\end{aligned}
$$

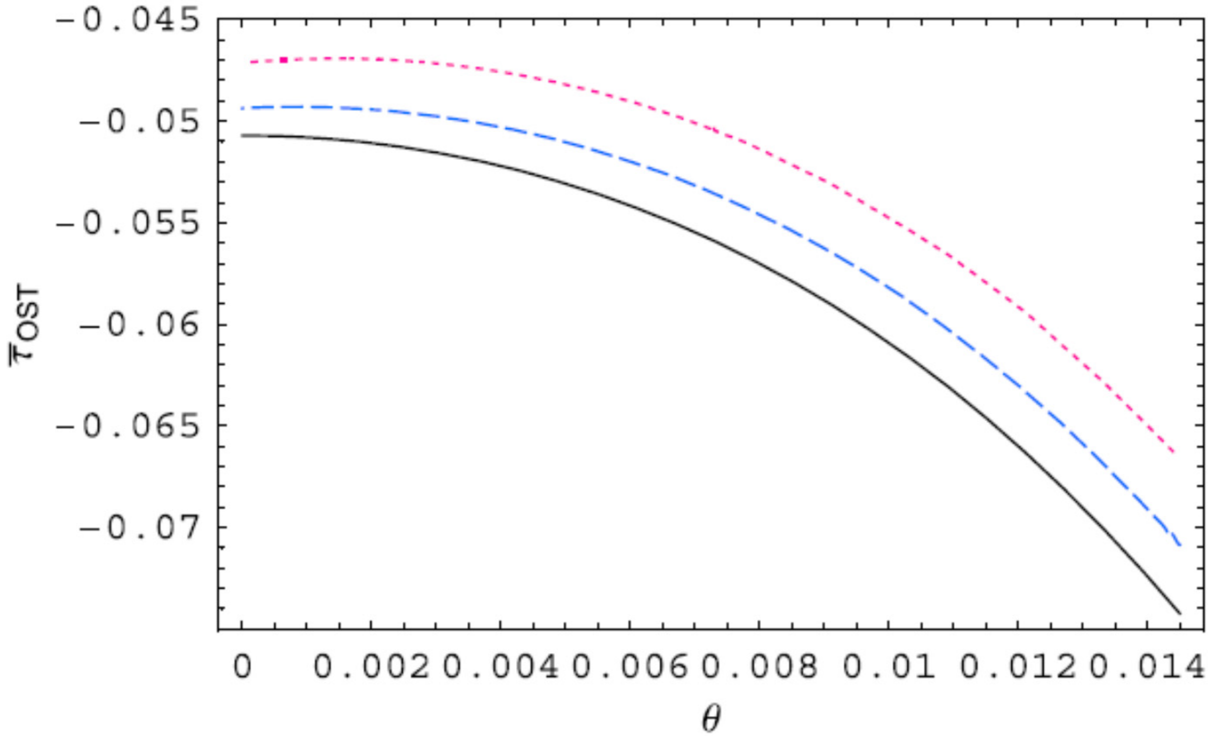

FIG. 1. The scaled occurrence scattering time advance $\bar{\tau}_{OST}$ as a function of the scattering angle θ when $\bar{\lambda}_D = 30$, $\bar{E}_T = 3$, and $\bar{E}_T = 9$. The solid line represents the case of $\bar{D} = 0$. The dashed line represents the case of $\bar{D} = 0.6$. The dotted line represents the case of $\bar{D} = 0.8$.

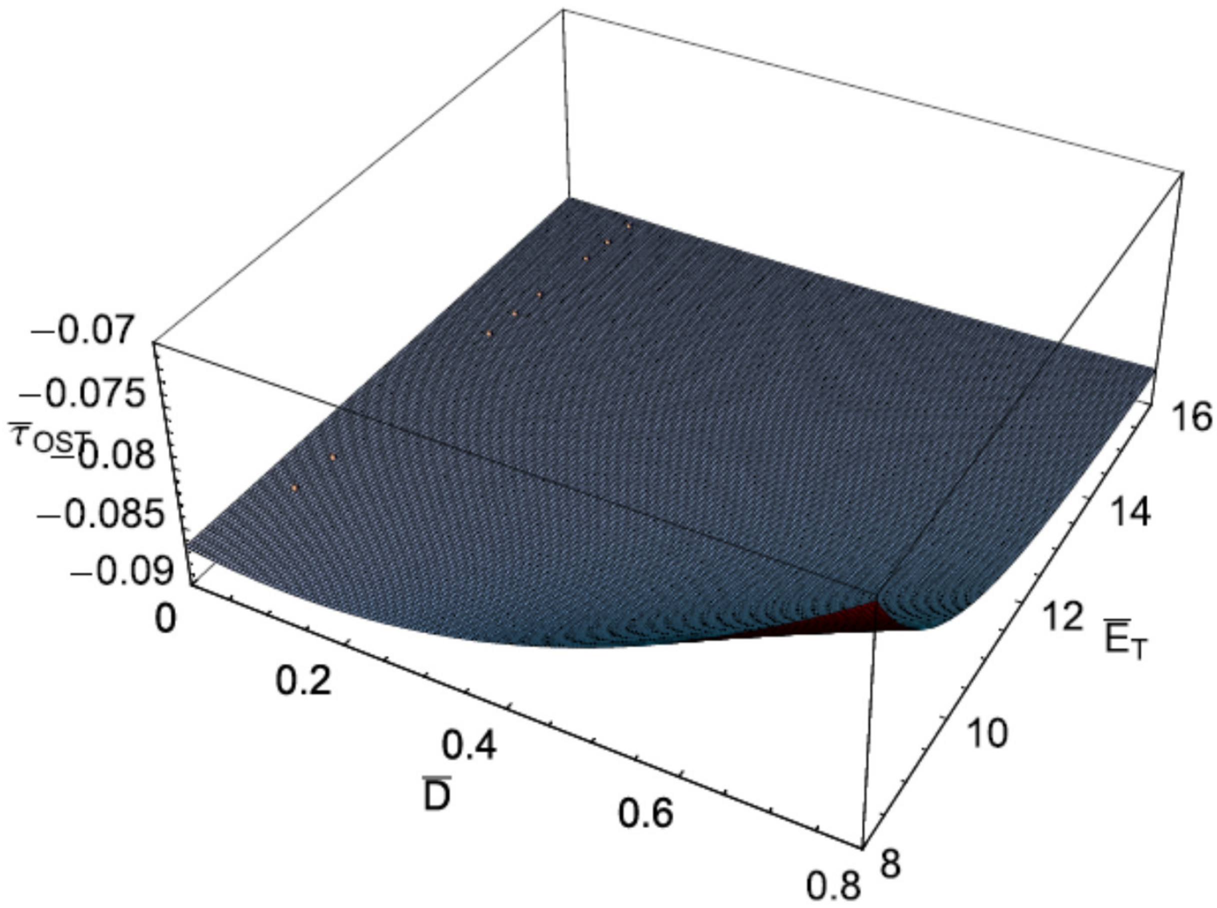

FIG. 2. The surface plot of the scaled occurrence scattering time advance $\bar{\tau}_{OST}$ as a function of the scaled diffusion coefficient $\bar{D}$ and the scaled thermal energy $\bar{E}_T$ when $\theta = 1^{\circ}$, $\bar{\lambda}_D = 30$, and $\bar{E}_T = 3$.

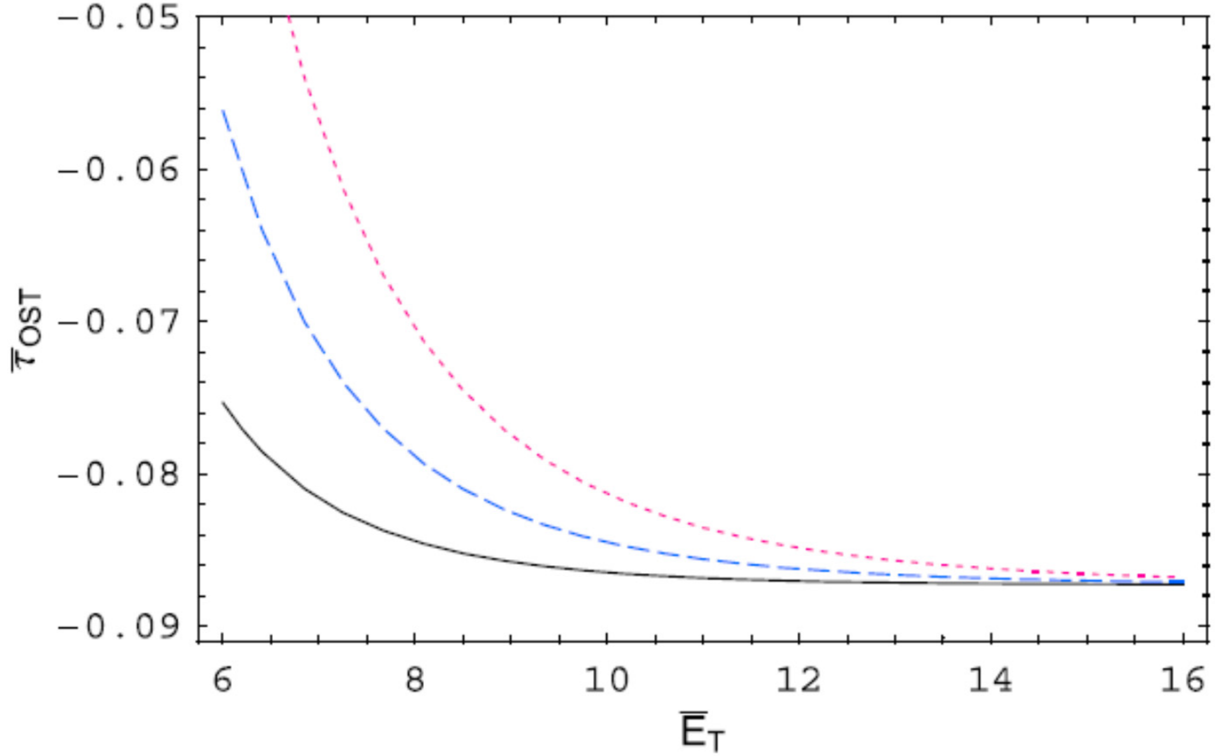

FIG. 3. The scaled occurrence scattering time advance $\bar{\tau}_{OST}$ as a function of the scaled thermal energy $\bar{E}_T$ when $\theta = 1^\circ$, $\bar{\lambda}_D = 30$, and $\bar{E}_T = 3$. The solid line represents the case of $\bar{D} = 0.4$. The dashed line represents the case of $\bar{D} = 0.6$. The dotted line represents the case of $\bar{D} = 0.8$.

Conclusions

- The propensity of the occurrence time advance increases with increasing scattering angle
- Plasma turbulence decreases the occurrence scattering time for forward electron-ion collisions in thermal turbulent plasmas.
- The occurrence time advance for the electron-ion collision in non-turbulent plasmas has more advanced character than that in turbulent plasmas.
- The effect of turbulence on the occurrence scattering time advance decreases with an increase of the thermal energy.
- We expect that the occurrence time advance for the electron-ion collision in hot turbulent plasmas has more advanced character than that in cold turbulent plasmas.

- The dependence of thermal energy on the occurrence scattering time increases with increasing scaled diffusion coefficient.

- The influence of diffusional turbulence on the occurrence scattering time advance decreases with an increase of the scaled thermal energy.

- The turbulence character of the plasma can be explored by the occurrence scattering time advance for the electron-ion collision in cold-temperature domains.

- The temperature dependence of the occurrence scattering time increases with an increase of the scaled diffusion coefficient.