Accurately calculating equilibrium quantities with any Grad-Shafranov solver

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Static MHD equilibria in toroidally axisymmetric devices are computed by solving the Grad-Shafranov equation. While the output of any numerical Grad-Shafranov solver is the poloidal flux function ψ, many physically important equilibrium quantities are functions of the derivatives of ψ instead of ψ itself. For example, the magnetic field and the safety factor are functions of the first derivatives of ψ, and some quantities that play a critical role in MHD stability and transport calculations, such as the parallel current density and the magnetic curvature, depend on the second derivatives of ψ.

The direct methods finite difference and finite element Grad-Shafranov solvers use to evaluate derivatives of ψ lead to the loss of at least one order of convergence of the numerical error per derivative calculated numerically. In contrast, we present a new, general method that allows the evaluation of any derivative of ψ with a numerical error that converges as fast as the error on ψ itself. Our method is based on two main ingredients: 1) we first analytically differentiate the Grad-Shafranov equation to obtain linear elliptic partial differential equations for the partial derivatives of ψ we want to evaluate; 2) we then use an integral equation formulation to compute with high accuracy the values of the derivatives of ψ on the plasma boundary, which are needed as boundary conditions for the linear PDE.

Our method can be applied to any direct Grad-Shafranov solver (as opposed to inverse solvers). We demonstrate its effectiveness for the particular case of bicubic Hermite finite element solvers, which are very popular in the fusion community.