A FLOW-STRESS TENSOR FORMULATION OF NEOCLASSICAL TOROIDAL VISCOSITY

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OVERVIEW

- There is substantial experimental and theoretical evidence that 3D non-axisymmetric B-fields affect the viscous damping of toroidal angular momentum in tokamaks [1-18].
- The Braginskii flow-rate-of-strain viscosity formalism [19] has previously been generalized to toroidal flux surface geometry and poloidal asymmetries[20,21], low collisionality[22], elongation[23,24] and more recently to allow 3D non-axisymmetric B-fields[25].
- This extended 3D Braginskii viscosity tensor is now used to indicate the development of a systematic Neoclassical Toroidal Viscosity formalism and rotation theory for 3D non-axisymmetric tokamaks.

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COORDINATE SYSTEM

- We define a general right hand orthogonal (ψ, p, ϕ) flux surface coordinate system for an axisymmetric tokamak with differential length elements $\left(dl_\psi = h_\psi d\psi, dl_p = h_p dp, dl_\phi = h_\phi d\phi\right)$ and unit vectors $\left(\hat{n}_\psi = \nabla\psi / |\nabla\psi|, \hat{n}_\varphi = R\nabla\varphi, \hat{n}_p = \hat{n}_\varphi \times \hat{n}_\psi\right)$ where ψ is a radial-like flux surface variable, p is a poloidal-like angular variable, and φ is the toroidal angle. The h_α are metric scale factors particular to the specific geometry. The 3D magnetic field structure and the non-axisymmetry will be represented within this geometry by allowing radial components of the magnetic field and a toroidal(φ) dependence of variables.

ANGULAR MOMENTUM EQUATIONS

- Toroidal angular momentum

$$nmR\frac{\partial V_{\varphi}}{\partial t}+nmR\left[\left(\mathbf{V}\bullet\nabla\right)\mathbf{V}\right]_{\varphi}+R\left(\nabla p\right)_{\varphi}+R\left(\nabla\bullet\Pi\right)_{\varphi}=$$

$$enR\left(E_{\varphi}+V_{\psi}B_{p}-V_{p}B_{\psi}\right)+RF_{\varphi}+R\left(\mathbf{n}_{\varphi}\bullet\mathbf{S}^{1}-mS^{0}V_{\varphi}\right)$$

- Poloidal angular momentum

$$rnm\frac{\partial V_{p}}{\partial t}+rnm\left[\left(\mathbf{V}\bullet\nabla\right)\mathbf{V}\right]_{p}+r\left(\nabla p\right)_{p}+r\left(\nabla\bullet\Pi\right)_{p}$$

$$=rne\left(E_{p}-V_{\psi}B_{\varphi}+V_{\varphi}B_{\psi}\right)+rF_{p}+r\left(\hat{\mathbf{n}}_{p}\bullet\mathbf{S}^{1}-mV_{p}S^{0}\right)$$

EXTENDED BRAGINSKII VISCOSITY TENSOR

$$W_{\alpha\beta} \equiv \hat{n}_\alpha \bullet \nabla \mathbf{V} \bullet \hat{n}_\beta + \hat{n}_\beta \bullet \nabla \mathbf{V} \bullet \hat{n}_\alpha - \frac{2}{3}\delta_{\alpha\beta} \nabla \bullet \mathbf{V}$$

$$= \left(\frac{\partial V_\beta}{\partial l_\alpha} + \sum_k \Gamma^\alpha_{\beta k} V_k \right) + \left(\frac{\partial V_\alpha}{\partial l_\beta} + \sum_k \Gamma^\beta_{\alpha k} V_k \right) - \frac{2}{3}\delta_{\alpha\beta} \nabla \bullet \mathbf{V}$$

$$\Gamma^\alpha_{\beta k} \equiv \frac{1}{h_\beta h_k} \left(\frac{\partial h_\beta}{\partial \xi_k} \delta_{\alpha\beta} - \frac{\partial h_k}{\partial \xi_\beta} \delta_{\alpha k} \right), \; \left(\xi_1 = \psi, \xi_2 = p, \xi_3 = \phi \right)$$

$$\pi_{\alpha\beta} = -\eta_0 W^0_{\alpha\beta} + \left[\eta_3 W^3_{\alpha\beta} + \eta_4 W^4_{\alpha\beta} \right] - \left[\eta_1 W^1_{\alpha\beta} + \eta_2 W^2_{\alpha\beta} \right] \equiv \pi^0_{\alpha\beta} + \pi^{34}_{\alpha\beta} + \pi^{12}_{\alpha\beta}$$

$$W^0_{\alpha\beta} \equiv \frac{3}{2} \left(f_\alpha f_\beta - \frac{1}{3}\delta_{\alpha\beta} \right) \left(f_\mu f_\nu - \frac{1}{3}\delta_{\mu\nu} \right) W_{\mu\nu}$$

$$f_\alpha \equiv B_\alpha / |B|$$

$$W^1_{\alpha\beta} \equiv \left(\delta^\perp_{\alpha\mu} \delta^\perp_{\beta\nu} + \frac{1}{2} \delta^\perp_{\alpha\beta} f_\mu f_\nu \right) W_{\mu\nu}$$

$$\delta^\perp_{\alpha\beta} \equiv \delta_{\alpha\beta} - f_\alpha f_\beta$$

$$W^2_{\alpha\beta} \equiv \left(\delta^\perp_{\alpha\mu} f_\beta f_\nu + \delta^\perp_{\alpha\beta} f_\alpha f_\mu \right) W_{\mu\nu}$$

$$W^3_{\alpha\beta} = \frac{1}{2} \left(\delta^\perp_{\alpha\mu} \varepsilon_{\beta\gamma\nu} + \delta^\perp_{\beta\nu} \varepsilon_{\alpha\gamma\mu} \right) f_\gamma W_{\mu\nu}$$

$\varepsilon_{\alpha\beta\gamma}$ antisymmetric unit tensor

$$W^4_{\alpha\beta} = \frac{1}{2} \left(f_\alpha f_\mu \varepsilon_{\beta\gamma\nu} + \varepsilon_{\alpha\gamma\mu} f_\beta f_\nu \right) f_\gamma W_{\mu\nu}$$

VISCOSITY COEFFICIENTS

- Collisional Braginskii $\eta_0 \simeq nT\tau >> \eta_{3,4} \simeq nT\tau / \Omega\tau >> \eta_{1,2} \simeq nT\tau / (\Omega\tau)^2$
- Banana-Plateau (e.g.Shaing[22])

$$\eta_0 = \frac{n_i m_i V_{thi} q R \varepsilon^{-3/2} \nu_{ii}^*}{\left(1+\varepsilon^{-3/2}\nu_{ii}^*\right)\left(1+\nu_{ii}^*\right)} \equiv n_i m_i V_{thi} q R f_i\left(\nu_{ii}^*\right)$$

LEADING ORDER PARALLEL VISCOSITY

$$\pi^0_{\alpha,\beta} = -\frac{3}{2}\eta_0\left(f_\alpha f_\beta - \frac{1}{3}\delta_{\alpha\beta}\right)H^0, \quad f_\alpha \equiv B_\alpha / B$$

3D VISCOSITY FLOW FUNCTION[25]

$$
H^{0} \equiv \left[\begin{array}{l}
\left(f_{\psi} f_{\psi} - \frac{1}{3}\right)\left\{ \frac{4}{3}\frac{\partial V_{\psi}}{\partial l_{\psi}} - \frac{2}{3}\left(\frac{\partial V_{p}}{\partial l_{p}} + \frac{\partial V_{\phi}}{\partial l_{\phi}} \right) + 2\left(\frac{1}{h_{\psi}}\frac{\partial h_{\psi}}{\partial l_{p}} V_{p} + \frac{1}{h_{\psi}}\frac{\partial h_{\psi}}{\partial l_{\phi}} V_{\phi} \right)\right\} + \\
\left(f_{p} f_{p} - \frac{1}{3}\right)\left\{ \frac{4}{3}\frac{\partial V_{p}}{\partial l_{p}} - \frac{2}{3}\left(\frac{\partial V_{\psi}}{\partial l_{\psi}} + \frac{\partial V_{\phi}}{\partial l_{\phi}} \right) + 2\left(\frac{1}{h_{p}}\frac{\partial h_{p}}{\partial l_{\psi}} V_{\psi} + \frac{1}{h_{p}}\frac{\partial h_{p}}{\partial l_{\phi}} V_{\phi} \right)\right\} + \\
\left(f_{\phi} f_{\phi} - \frac{1}{3}\right)\left\{ \frac{4}{3}\frac{\partial V_{\phi}}{\partial l_{\phi}} - \frac{2}{3}\left(\frac{\partial V_{p}}{\partial l_{p}} + \frac{\partial V_{\psi}}{\partial l_{\psi}} \right) + 2\left(\frac{1}{h_{\phi}}\frac{\partial h_{\phi}}{\partial l_{\psi}} V_{\psi} + \frac{1}{h_{\phi}}\frac{\partial h_{\phi}}{\partial l_{p}} V_{p} \right)\right\} + \\
2 f_{\psi} f_{p} \left\{ \frac{\partial V_{\psi}}{\partial l_{p}} + \frac{\partial V_{p}}{\partial l_{\psi}} - \left(\frac{1}{h_{p}}\frac{\partial h_{p}}{\partial l_{\psi}} V_{p} + \frac{1}{h_{\psi}}\frac{\partial h_{\psi}}{\partial l_{p}} V_{\psi} \right)\right\} + \\
2 f_{p} f_{\phi} \left\{ \frac{\partial V_{\phi}}{\partial l_{p}} + \frac{\partial V_{p}}{\partial l_{\phi}} - \left(\frac{1}{h_{p}}\frac{\partial h_{p}}{\partial l_{\phi}} V_{p} + \frac{1}{h_{\phi}}\frac{\partial h_{\phi}}{\partial l_{p}} V_{\phi} \right)\right\} + \\
2 f_{\psi} f_{\phi} \left\{ \frac{\partial V_{\phi}}{\partial l_{\psi}} + \frac{\partial V_{\psi}}{\partial l_{\phi}} - \left(\frac{1}{h_{\psi}}\frac{\partial h_{\psi}}{\partial l_{\phi}} V_{\psi} + \frac{1}{h_{\phi}}\frac{\partial h_{\phi}}{\partial l_{\psi}} V_{\phi} \right)\right\}
\end{array}\right]
$$

VISCOUS DAMPING OF TOROIDAL ANGULAR MOMENTUM

- FSA of toroidal viscous torque

$$\left\langle R^2\nabla\phi\bullet\nabla\bullet\Pi\right\rangle = \frac{1}{V'}\frac{\partial}{\partial\psi}\left(V'\left\langle R^2\nabla\phi\bullet\Pi\bullet\nabla\psi\right\rangle\right) = \left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^2 h_p \pi_{\psi\phi}\right) + B_p\frac{\partial}{\partial l_p}\left(\frac{R\pi_{p\phi}}{B_p}\right)\right\rangle$$

- $\pi_{p\phi}$ term vanishes on FSA because of single-valuedness

- the parallel viscosity term

$$\left\langle R^2\nabla\varphi\bullet\nabla\bullet\Pi_0\right\rangle = \left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^2 h_p \pi_{\psi\phi}\right)\right\rangle = -\frac{3}{2}\left\langle \frac{1}{Rh_p}\frac{\partial\left(R^2 h_p \eta_0 f_\psi f_\varphi H^0\right)}{\partial \ell_\psi}\right\rangle$$

vanishes in axisymmetric tokamaks $f_\psi \equiv B_\psi/|B| = 0$,leaving the gyroviscous Π_{34} term dominant. However, in 3D non-axisymmetric tokamaks $f_\psi \equiv B_\psi/|B| \neq 0$ and the leading order parallel term survives.

MAGNITUDE OF 3D VISCOUS TOROIDAL DAMPING RELATIVE TO AXISYMMETRIC NEOCLASSICAL VALUE

- For the circular model $\left(R = R_0(1+\varepsilon\cos\theta), B = B_0(r)/(1+\varepsilon\cos\theta)\right)$ in which $\left(f_\varphi \simeq 1, f_\theta \simeq O(\varepsilon), f_r \simeq O(\varepsilon^3)\right)$ and V_r , the Shafranov shift, any poloidal dependence and 2nd derivatives are neglected, the 3D viscous toroidal angular momentum damping term reduces to

$$\left\langle R^2 \nabla\phi \cdot \nabla \cdot \Pi_0 \right\rangle \approx \left\langle \frac{3}{r} f_r \left[\begin{array}{l} \overline{V}_\phi(r,\phi)\, f_r \varepsilon \left(\eta_0 + \dfrac{r}{2}\dfrac{\partial \eta_0}{\partial r} \right) + \overline{V}_\theta f_r f_\theta \left(2\eta_0 \dfrac{\varepsilon}{2} + \dfrac{\partial \eta_0}{\partial r} R_0 \right) \\ + \dfrac{\partial \overline{V}_\phi(r,\phi)}{\partial \phi} \dfrac{2}{27} \left(\eta_0 + r \dfrac{\partial \eta_0}{\partial r} \right) - \dfrac{\partial \overline{V}_\phi(r,\phi)}{\partial r} f_r R_0 \left(\eta_0 + r \dfrac{\partial \eta_0}{\partial r} \right) \end{array} \right] \right\rangle$$

- By comparison, the axisymmetric neoclassical gyroviscosity damping term is[20]

$$\left\langle R^2 \nabla\varphi \bullet \nabla \bullet \Pi_{34} \right\rangle \simeq -\left\langle \left(\frac{1}{rR} \right) \frac{\partial}{\partial r} \left(R^3 \eta_4 \frac{\partial \left(V_\varphi / R \right)}{\partial \theta} \right) \right\rangle$$

- Because $\gtrsim$ $\eta_0/\eta_{34} \approx \Omega_i \tau_{ii} \approx 10^3 - 10^4$ the $\gtrsim$ viscous damping due to 3D toroidal asymmetries $\left\langle R^2 \nabla\varphi \bullet \nabla \bullet \Pi_0 \right\rangle$ can be the gyroviscous damping due to poloidal asymmetries $\left\langle R^2 \nabla\varphi \bullet \nabla \bullet \Pi_{34} \right\rangle$, even for small $f_r = B_r/B$.

POLOIDAL VISCOSITY & ROTATION

- The poloidal angular momentum balance determines poloidal rotation.

$$rnm\frac{\partial V_p}{\partial t}+rnm\left[\left(\mathbf{V}\bullet\nabla\right)\mathbf{V}\right]_p+r\left(\nabla p\right)_p+r\left(\nabla\bullet\Pi\right)_p$$

$$=rne\left(E_p-V_\psi B_\varphi+V_\varphi B_\psi\right)+rF_p+r\left(\hat{\mathbf{n}}_p\bullet\mathbf{S}^1-mV_pS^0\right)$$

- The poloidal viscous damping rate for 3D viscosity has the same flow function H^0 as the 3D toroidal viscous damping[26].

$$\left[\nabla\bullet\Pi\right]_p^0=-\frac{3}{2}\left[\begin{array}{l}\left\{\frac{\partial\left(\eta_0 f_\psi f_p H^0\right)}{\partial\ell_\psi}+\frac{\partial\left(\eta_0(f_p^2-\frac{1}{3})H^0\right)}{\partial\ell_p}+\frac{\partial\left(\eta_0 f_\varphi f_p H^0\right)}{\partial\ell_\varphi}\right\}+\\ \eta_0 H^0\left\{\begin{array}{l}f_\psi f_p\left(\frac{1}{h_p}\frac{\partial h_p}{\partial\ell_\psi}+\frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial\ell_\psi}\right)-\left(f_\psi^2-\frac{1}{3}\right)\left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_p}\right)+\\ \left(f_p^2-\frac{1}{3}\right)\left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_p}+\frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial\ell_p}\right)-f_p f_\varphi\left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_p}\right)+\\ f_\varphi f_p\left(\frac{1}{h_p}\frac{\partial h_p}{\partial\ell_\varphi}+\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_\varphi}\right)-\left(f_\varphi^2-\frac{1}{3}\right)\left(\frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial\ell_p}\right)\end{array}\right\}\end{array}\right]$$

RADIAL ION FLUX

- The radial ion flux is determined by the radial component of the momentum balance

$$nm\frac{\partial V_\psi}{\partial t}+nm\left[\left(\mathbf{V}\bullet\nabla\right)\mathbf{V}\right]_\psi+\left(\nabla p\right)_\psi+\left(\nabla\bullet\Pi\right)_\psi=ne\left(E_\psi-V_\varphi B_p+V_p B_\varphi\right)+F_\psi+\left(\hat{\mathbf{n}}_\psi\bullet\mathbf{S}^1-mV_\psi S^0\right)$$

- The 3D radial viscous force is[26]

$$\left(\nabla\bullet\Pi\right)_\psi=\left[\frac{1}{h_p h_\varphi}\frac{\partial\left(h_p h_\varphi \pi_{\psi\psi}\right)}{\partial\ell_\psi}+\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_p}\pi_{\psi p}+\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_\varphi}\pi_{\psi\varphi}\right]+$$

$$\left[\frac{1}{h_\psi h_\varphi}\frac{\partial\left(h_p h_\psi \pi_{p\psi}\right)}{\partial\ell_p}-\frac{1}{h_p}\frac{\partial h_p}{\partial\ell_\psi}\pi_{pp}\right]+\left[\frac{1}{h_\psi h_p}\frac{\partial\left(h_p h_\psi \pi_{\varphi\psi}\right)}{\partial\ell_\varphi}-\frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial\ell_\psi}\pi_{\varphi\varphi}\right]$$

- The radial ion flux must be corrected for ion orbit loss and compensating return current.

ION ORBIT LOSS & INTRINSIC ROTATION

- The fraction of “thermalized” ions that are able to access loss orbits which remove them from the plasma increases as the ions are transported radially outward. The minimum energy at which ions are “ion-orbit-lost” can be calculated at each radius from conservation of canonical angular momentum, magnetic moment and energy[27,28]. A radially increasing fraction of the outward fluxes of ion particle, momentum and energy are “ion-orbit-lost”. (ζ_0 is the direction cosine with respect to B, $V_{0\min}=\sqrt{2E_{0\min}/m}$ and $\varepsilon_{\min}=E_{0\min}/kT_{ion}$.

$$F_{orb}\left(\rho\right)\equiv\frac{N_{loss}}{N_{tot}}=\frac{\int_{-1}^{1}\left[\int_{V_{0\min}(\zeta_0)}^{\infty}V_0^2 f(V_0)dV_0\right]d\zeta_0}{2\int_0^{\infty}V_0^2 f(V_0)dV_0}=\frac{\int_{-1}^{1}\Gamma\left(3/2,\varepsilon_{\min}(\rho,\zeta_0)\right)d\zeta_0}{2\Gamma\left(3/2\right)}$$

- These ion orbit losses can be represented in the radial momentum balance equation by $S_{\psi}^{1}=-2\left(\partial F_{orb}/\partial\ell_{\psi}\right)nV_{\psi}$, where the “2” accounts for the compensating return current , or the calculated radial particle flux can be reduced by $\left(1-2F_{orb}\left(\psi\right)\right)$. Γ is the gamma function.

- Predominantly counter-current directed ions are lost, resulting in a co-current intrinsic rotation of the remaining ions[29,30].

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BALANCE & CLOSURE RELATIONS

- The ion densities are determined from the continuity equations
$$\frac{\partial n}{\partial t}+\frac{1}{h_p h_\varphi}\frac{\partial\left(h_p h_\varphi n V_\psi\right)}{\partial \ell_\psi}+\frac{1}{h_\psi h_\varphi}\frac{\partial\left(h_\psi h_\varphi n V_p\right)}{\partial \ell_p}+\frac{1}{h_p h_\psi}\frac{\partial\left(h_p h_\psi n V_\varphi\right)}{\partial \ell_\varphi}=S^0$$
- The ion and electron pressures are determined from the energy balance equations.
- The ion and electron heat conduction are represented by the Fourier heat conduction closure relations $\mathbf{q}=-n\chi\nabla T$

RADIAL ELECTRIC FIELD

- Solving the ion and electron momentum balance equations using an interspecies friction term of the form $\mathbf{F}_{ei} = -nm\nu_{ei}\left(\mathbf{V}_e - \mathbf{V}_i\right) = -\eta n^2 e^2\left(\mathbf{V}_e - \mathbf{V}_i\right) = ne\eta\mathbf{j}$ leads to

$$\mathbf{E} = \eta\mathbf{j} - \mathbf{u}\times\mathbf{B} - \frac{\nabla p_e}{ne} + \frac{\mathbf{j}\times\mathbf{B}}{ne}$$

where $\mathbf{u} = \left(n_i m_i \mathbf{V}_i + n_e m_e \mathbf{V}_e\right) / n_i m_i + n_e m_e$, with a sum over ion species "i" being understood.

- Making use of the leading order plasma force balance $\mathbf{j}\times\mathbf{B} = \nabla p = \nabla p_i + \nabla p_e$ yields another form of Ohm's Law for the determination of the electric field in a rotating plasma

$$\mathbf{E} = \eta\mathbf{j} - \mathbf{u}\times\mathbf{B} + \frac{\nabla p_i}{ne}$$

- The second equations is a convenient form for the determination of the radial electric field that is produced by the rotation velocities and the ion pressure gradient discussed above.

- However, the first equation provides an insight into how the radial electric field (and hence the quantities such as ion orbit loss, radial ion flux and rotation that depend upon it) could be controlled by creating a poloidal or toroidal current in the plasma, most likely in the edge plasma. Further pursuit of this obviously interesting and potentially important topic is beyond the scope of this poster, but is definitely recommended for future investigation.

FUTURE WORK

- The 3D Viscosity Flow Function H^0 must be reduced to a computationally practical form.
- Relevant 3D B-radial and viscosity coefficient representations must be incorporated to represent specific experimental conditions.
- Practical computation models, first for momentum damping rates, ultimately for radial particle fluxes, rotation velocities, electric field, etc. must be developed.
- The Braginskii flow-stress viscosity tensor should be extended to include the heat flux[31].
- The possibility of external control of poloidal and/or toroidal rotation in the plasma edge should be investigated.
- Comparisons with experiment must be carried out.

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