# Particle Simulation of Neoclassical Pedestal Buildup and Pedestal Scaling Law

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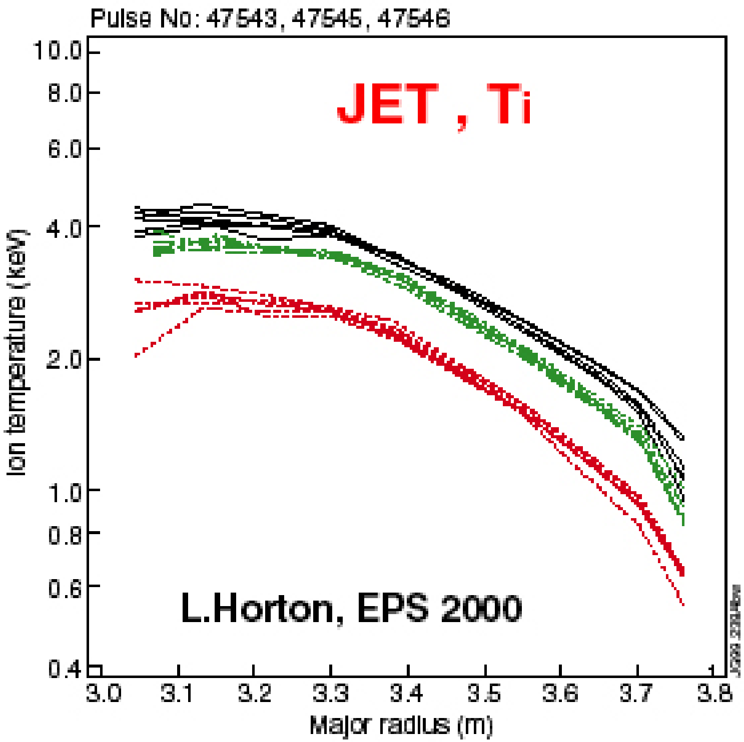
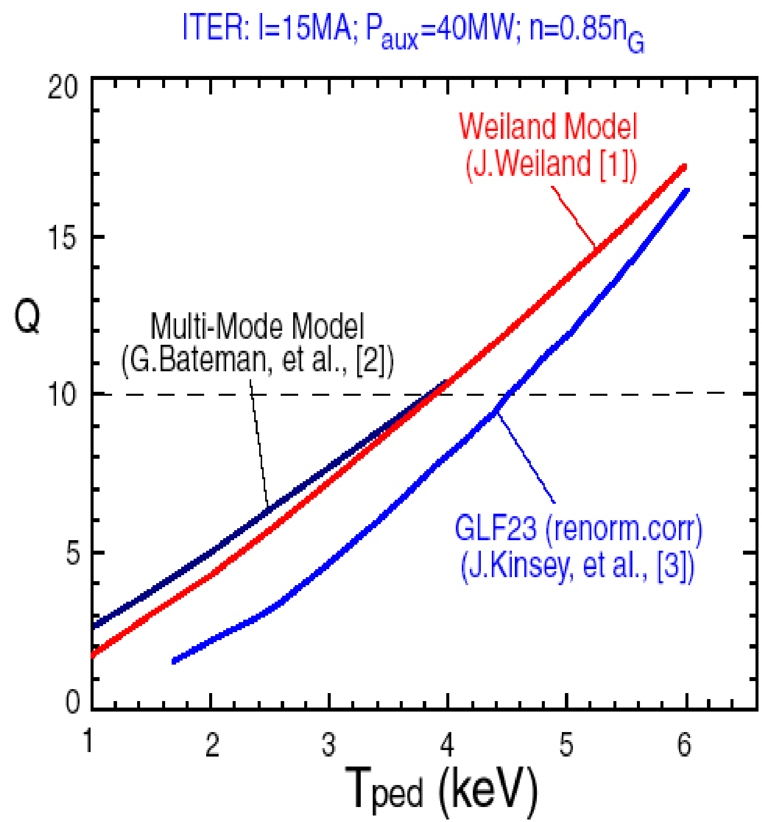
**Thanks to PPPL, GA, MIT collaborators**

# Contents

1. The XGC code and capability
2. Saturated neoclassical pedestal: core & edge
   Critical role played by self-consistent
   orbit squeezing and orbit loss
3. Neoclassical pedestal buildup in a diverted tokamak edge
4. Parallel flow and neoclassical theory
5. Pedestal scaling law

# Q-uncertainty due to uncertainty in $T_{ped}$ is a critical issue for ITER



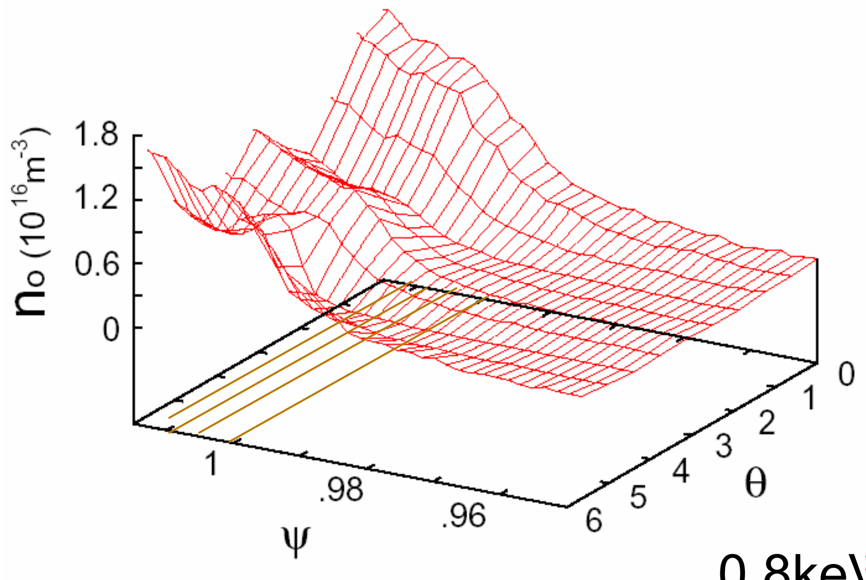




(Hubbard, TTF2002)

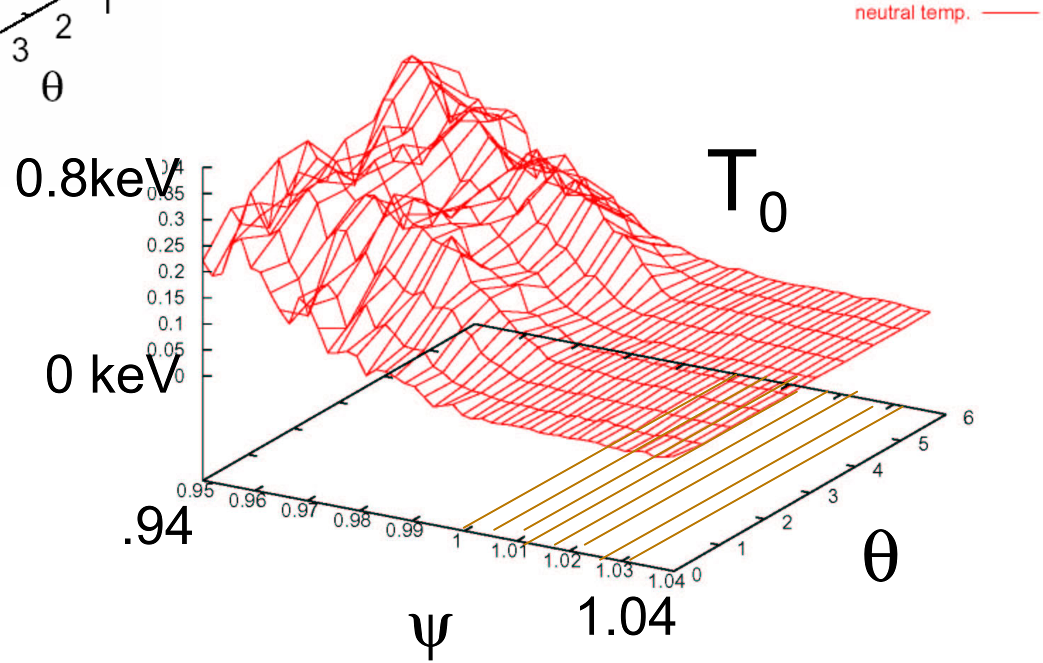
# **XGC** is a Hamiltonian ion g.c. code for self-consistent, dynamical neoclassical transport

- Massively parallel (1,028 processors on SEABORG, ~5,000 cpu hrs)
- EFIT H-mode flux surface & limiter (with X-point)
- Typical simulation range: $0.88 < \Psi < 1.03$
- Conserving MC Coulomb collision in t-evolving plasma
- Power out-flow from core
- Dynamic 2d Monte Carlo neutral transport
- Simple anomalous diffusion coefficient
- Evaluates $n_i$ , $T_i$ , $\Phi$ , $V_{\parallel}$, $V_{ExB}$, (and $V_{dia}$)profiles
- **Assumes $\Phi(\psi)$ in the present version**
- **Lacks accurate $E_r$ evaluation in scrape-off ($\Phi$=0).**

# Dynamic 2D Neutral Monte Carlo Transport




- Maxwellian Franck-Condon $D_0$ source at $\psi_o > 1$
- CX, ionization, elastic collisions in time-evolving plasma

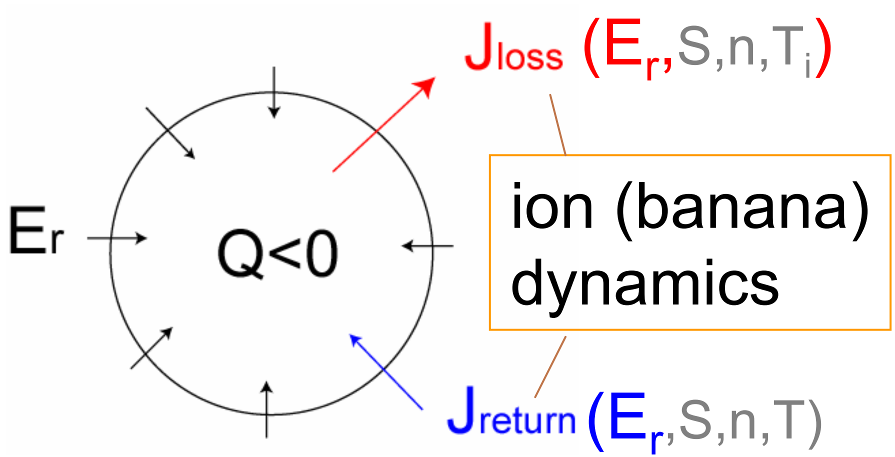


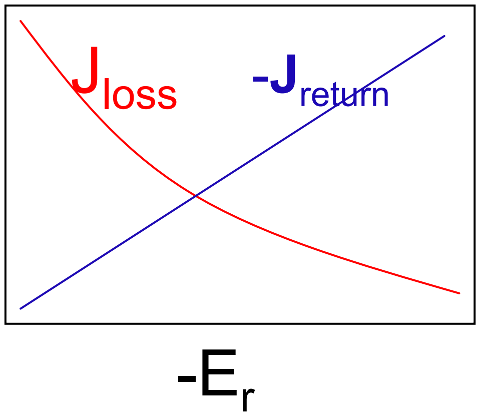

- Poloidal source distribution is an input.
- Source rate either input or from recycling

# Neoclassical $E_r$ is determined from ion dynamics ($J_{ir} \gg J_{er}$)

$$(\partial/\partial t)\, \langle \mathbf{E}\cdot\nabla\psi\rangle + 4\pi\langle(\mathbf{J}_{cp}+\mathbf{J}_{gc})\cdot\nabla\psi\rangle = c\langle\nabla\cdot\mathbf{B}\times\nabla\psi\rangle = 0$$

$$[\langle|\nabla\psi|^2\rangle + 4\pi n_i m_i c^2 \langle|\nabla\psi|^2/B^2\rangle]\ \partial^2\phi/\partial t\partial\psi = 4\pi\langle\mathbf{J}_{gc}\cdot\nabla\psi\rangle$$




The code finds $J_{net}=J_{loss}(E_r)+J_{return}(E_r) = 0$ for further evolution in quasi-equilibrium.

# Two ways to obtain a maximal pedestal

I. Buildup

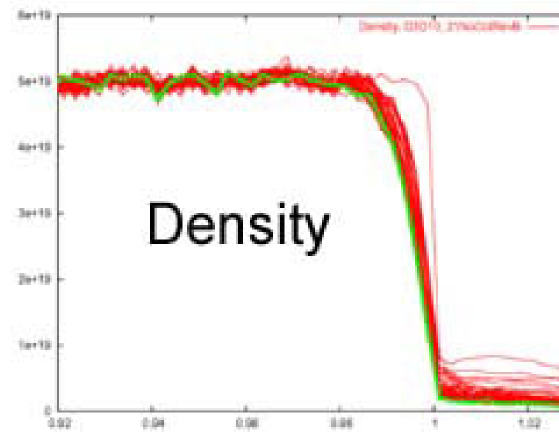
- **Physically**: More realistic, buildup demonstrations
- **Numerically**: Highly time consuming for scaling studies. Needs tedious adjustment of $n_o$, $\Gamma_i^{Ambi}$ for "standardized" steepest n & $T_i$ profiles.

II. Reduction from a steeper profile (for scaling law)

- **Physically**: a fake process to find a neoclassical quasi-equilibrium solution [Check $J_{gc}$(separatrix)=0].
  $n_o$ is not needed.
  Poisson's eq. keeps the classical polarization density.
  Use $\Gamma^{turb}$(ambipolar) or simply $n_{igc}$ for profile evolution.
- **Numerically**: Efficient way of getting a "standardized" maximal n & $T_i$ profile. Uniqueness provided by $J_{return}$.

# Quasi-neutral g.c. profile evolution (PoP, 2004)

- In neoclassical quasi-equilibrium, $J_{cp}$ and $n_{cp}$ are negligible. $\Gamma_e^{ei}$ is automatically ambipolar and very weak.
  $\Rightarrow$ Ion g.c. ($n_i = n_{i,gc}$) simulation is enough ($n_{ncp}$ included) .
- Can a transient gc evolution ($n_i = n_{i,gc}$) be quasi-neutral? Yes, equivalent to removal of net neutral plasma $\Delta n = \mathbf{n_{i.cp}}$.
- The more important $\mathbf{n_{i,ncp}}$ is included in $\mathbf{n_{i,gc}}$.
  $n_{ncp}$(banana) $\gg n_{cp}$(gyro)




- $dE_r/dt$ is evaluated with the smaller classical polarization included.

$$(\partial/\partial t) \langle \mathbf{E}\cdot\nabla\psi\rangle + 4\pi \langle (\mathbf{J}_{cp} + \mathbf{J}_{gc})\cdot\nabla\psi\rangle = c\langle \nabla\cdot\mathbf{B}\times\nabla\psi\rangle = 0$$

- From $4\pi J_{cp} = -(\omega^2_{pi}/\Omega_i^2)\, \partial\nabla_\perp\Phi/\partial t$, we get

$$[\, \langle|\nabla\psi|^2\rangle + \langle|\nabla\psi|^2\, \omega^2_{pi}/\Omega_i^2\rangle\, ]\, \partial^2\Phi/\partial t\partial\psi = 4\pi\langle \mathbf{J}_{gc}\cdot\nabla\psi\rangle$$

# (Well-known) argument for $n_{i,cp}=n_{eo}-n_{i,gc}$

- Ampere's law

  $[\,<|\nabla\psi|^2> + <|\nabla\psi|^2\,\omega^2_{pi}/\Omega_i^2>\,]\;\partial^2\Phi/\partial t\partial\psi = 4\pi<\mathbf{J_{gc}}\cdot\nabla\psi>$

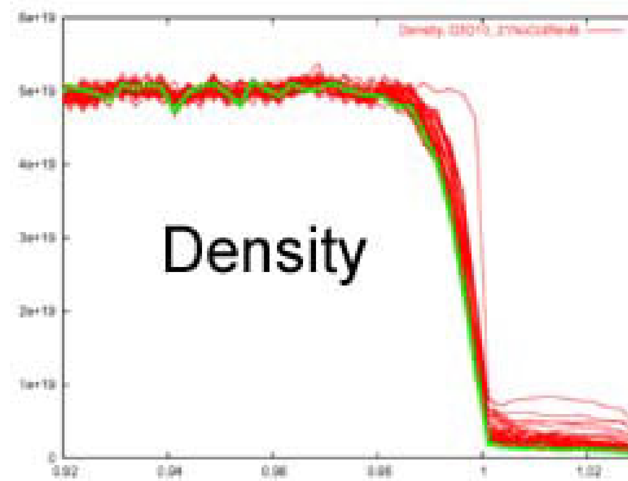
  is equivalent to the Poisson's eq.

  $\nabla_r^2\Phi + (\omega^2_{pi}/\Omega_i^2)\nabla_r^2\Phi = 4\pi e(n_{eo} - n_{i,gc})$

  $\nabla_r^2\Phi + 4\pi e n_{i,cp} = 4\pi e(n_{eo} - n_{i,gc})$

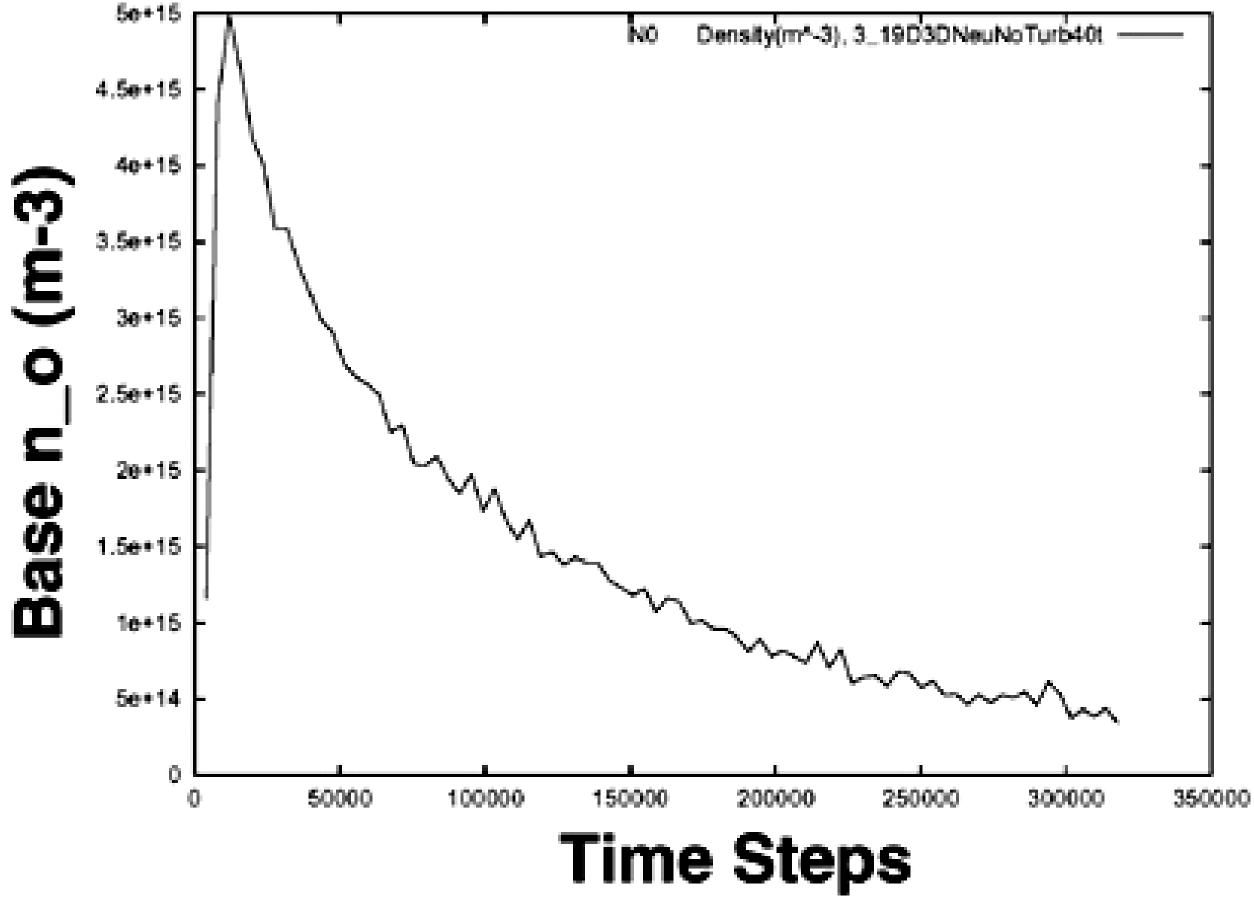
- Since $\omega^2_{pi}/\Omega_i^2 \gg 1$, we get

  $\mathbf{n_{i,cp} \cong n_{eo} - n_{igc}}$

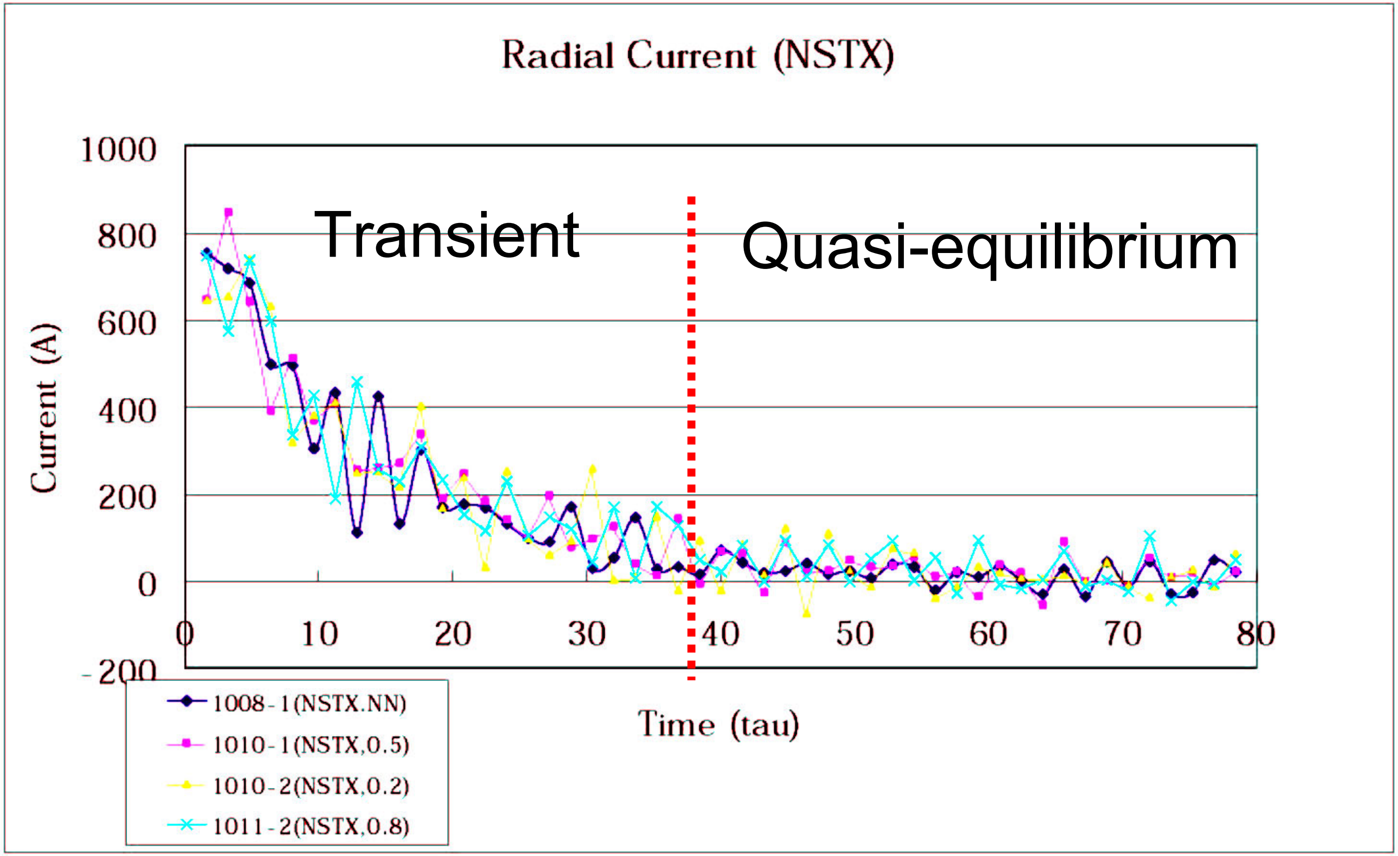



- The smaller $\mathbf{J}_{\mathbf{i},cp}$ plays an important role in quasineutral evloution (and GAM oscillation)!
- Transient profile evolution in g.c. is achieved by removal of a net neutral plasma by $\Delta n=n_{i,cp}$ until quasi-equilibrium.
- The real neoclassical simulation then starts.

# Base neutral density evolution



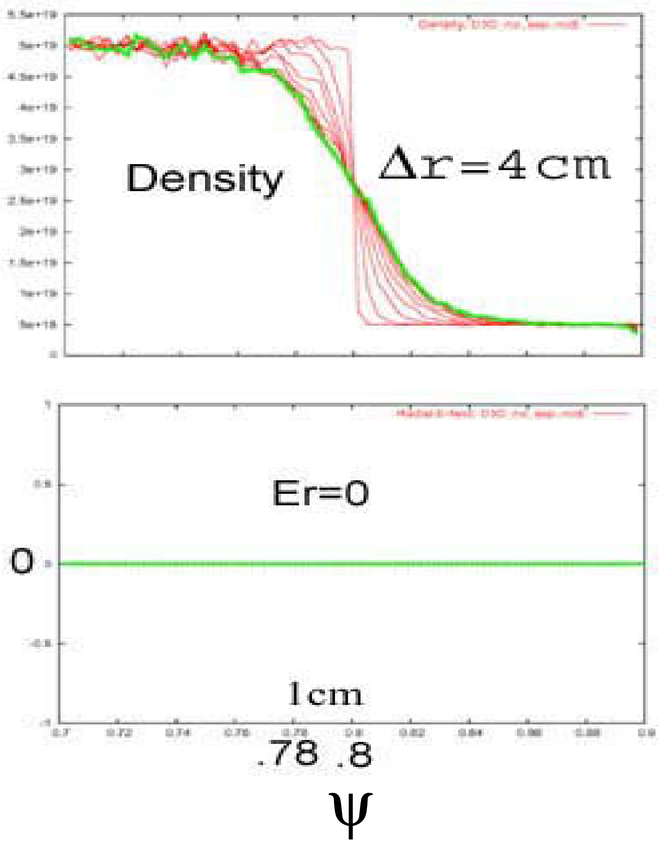
# Radial current decay



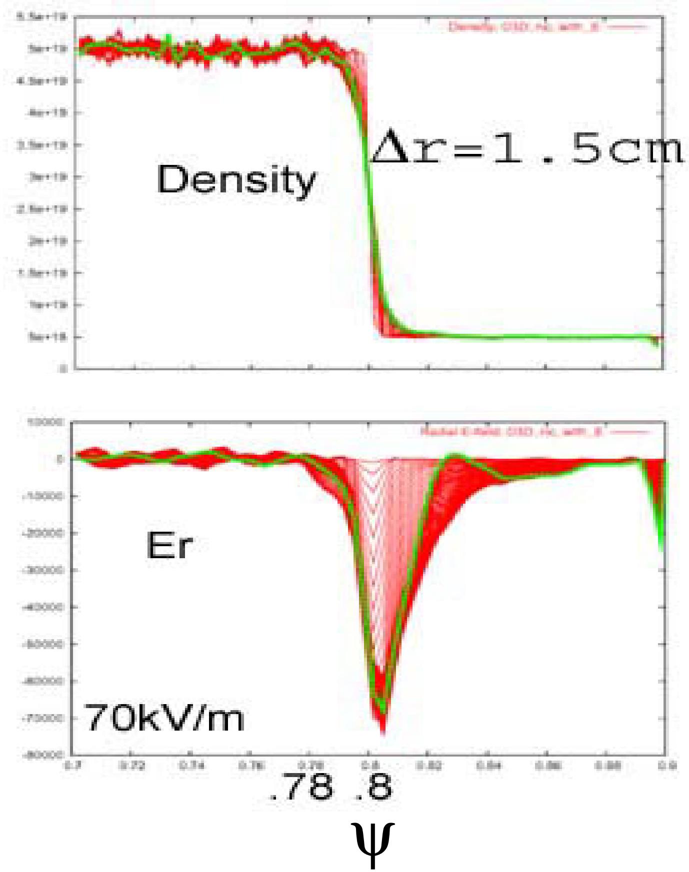

Saturation at 40-100 transit time
(7 to 15 msec)

# Steepest n-pedestal ($\nu_c$=0, DIIID 2.1T) in core: Shows importance of orbit squeezing

Steepest pedestal without $E_r$

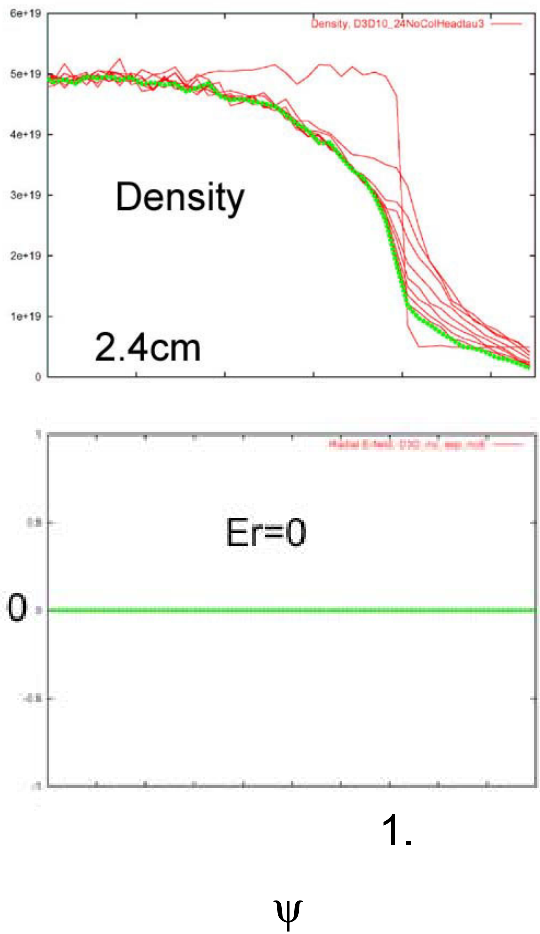



Steepest pedestal with $E_r$

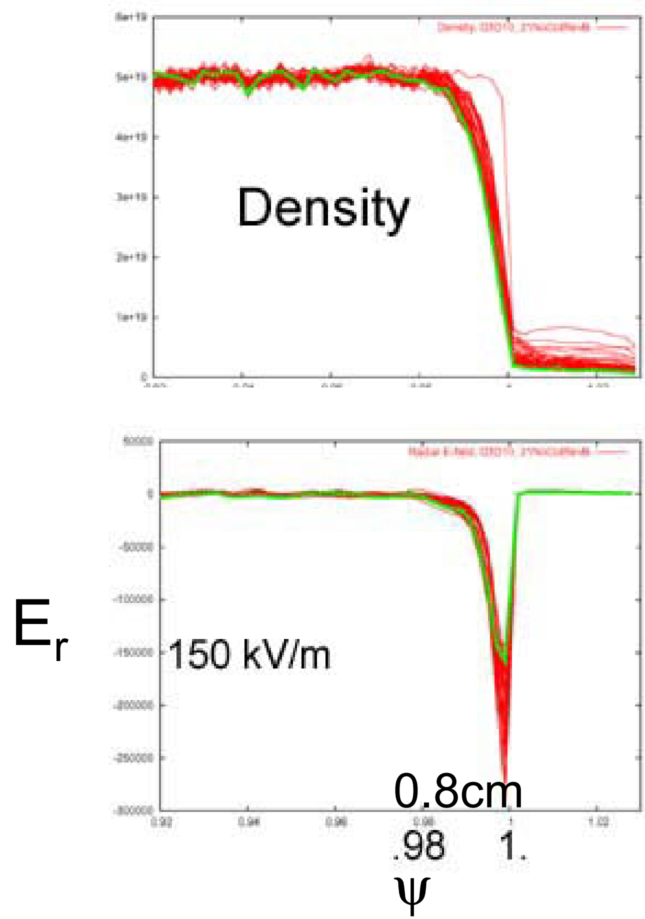


# Steepest n-pedestal at separatrix ($\nu_c$=0)

## Unsupported by Er

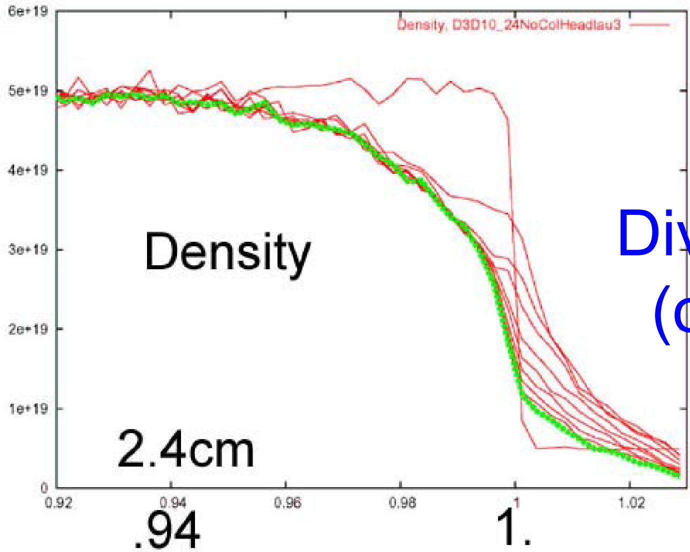



## Supported by Er

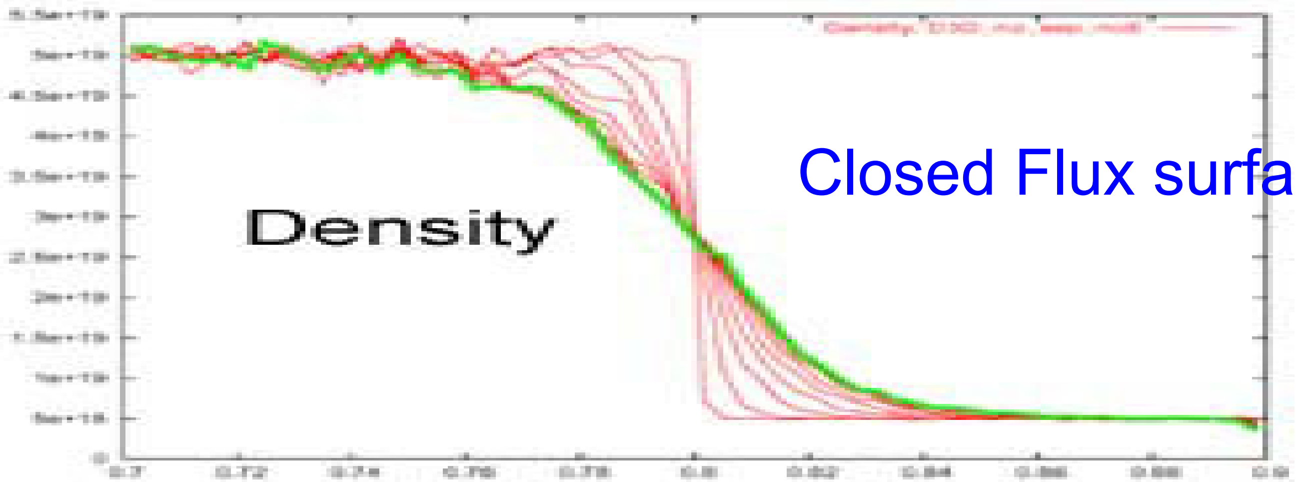


# There is loss from $\psi$<1 ($E_r$=0, $\nu_c$=0)

$E_r$=0

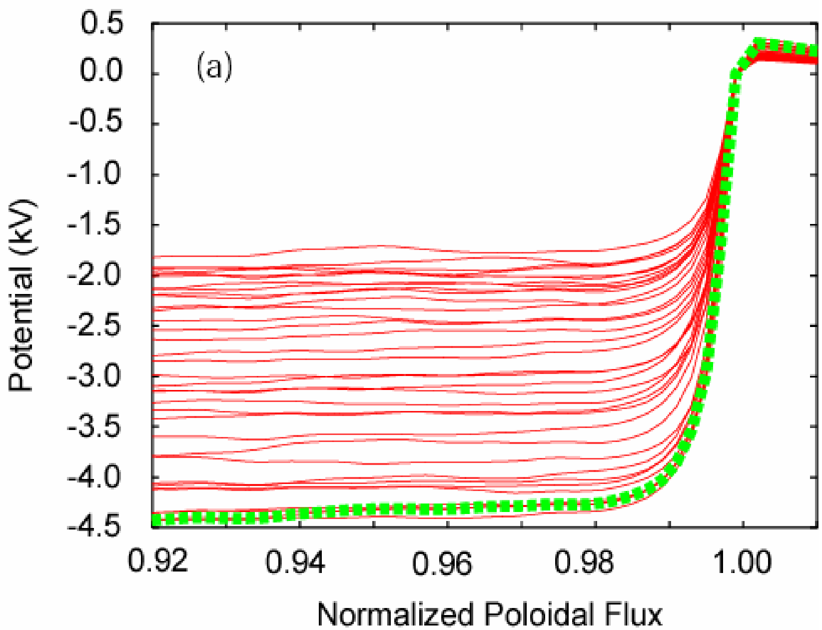


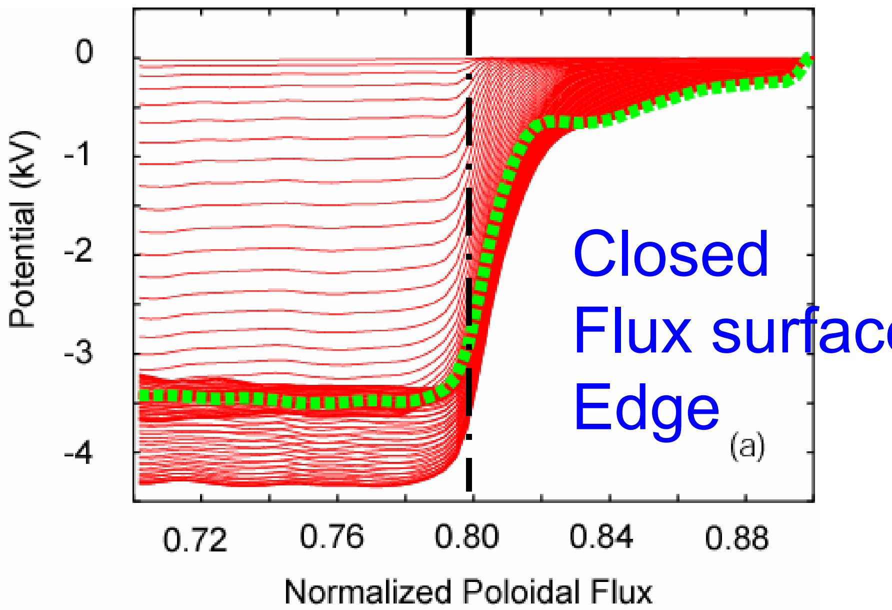

Diverted Edge
(orbit X-loss)




Closed Flux surface

# Greater $E_r$ at edge from the ion loss ($\nu_c$=0)




Diverted Edge
(Er=0 is imposed
in scrape-off)

Only a proper kinetic treatment can yield correct pedestal and E(r)!!!

# X-Transport
# = Collisional loss-cone transport due to X-point

C.S. Chang, et al, Phys. Plasmas 9, 3884 (2002)

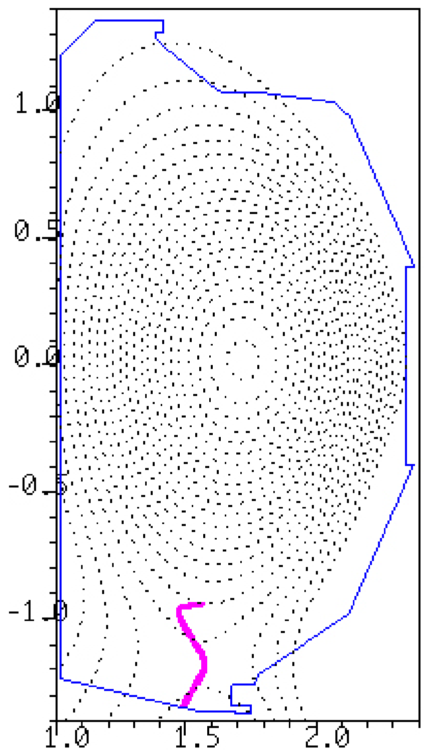



Fig. 1. X-trapping in DIII-D

**Strong effective collisions**

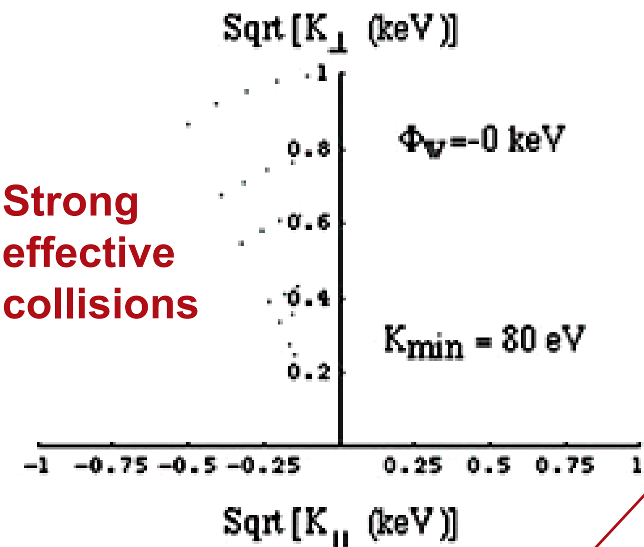
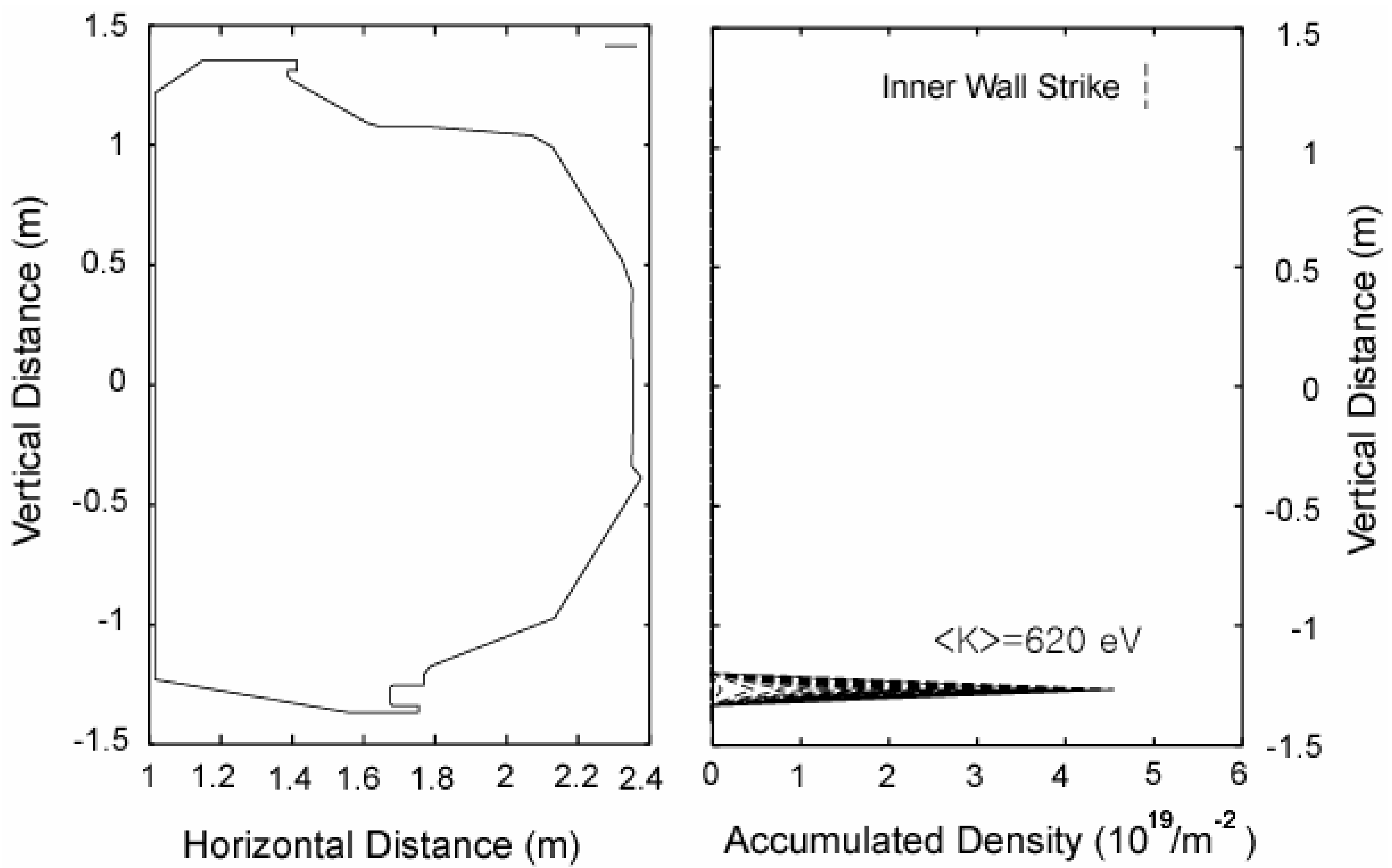



**Fig. 2.** Ion velocity space hole at 10 cm above the X-point in DIII-D.

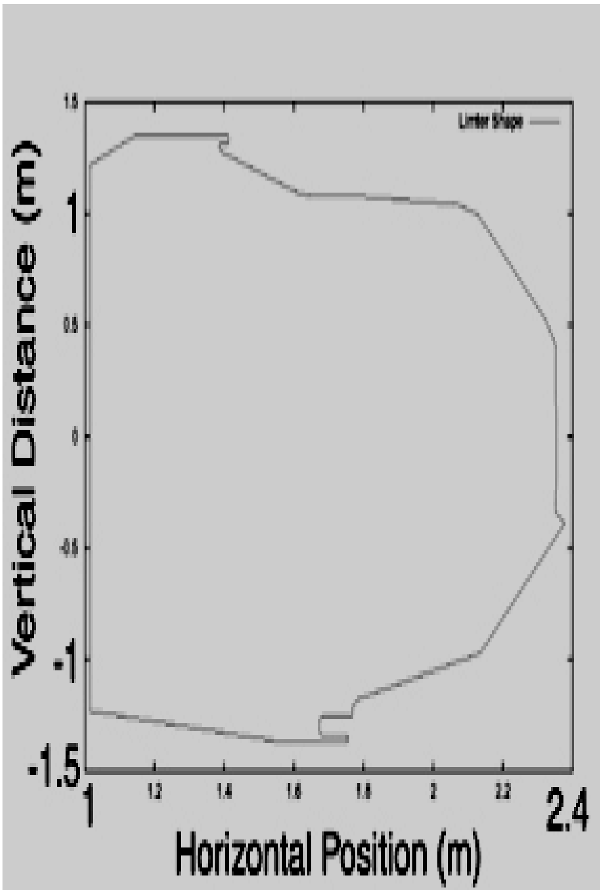
5 mm
at outside
midplane

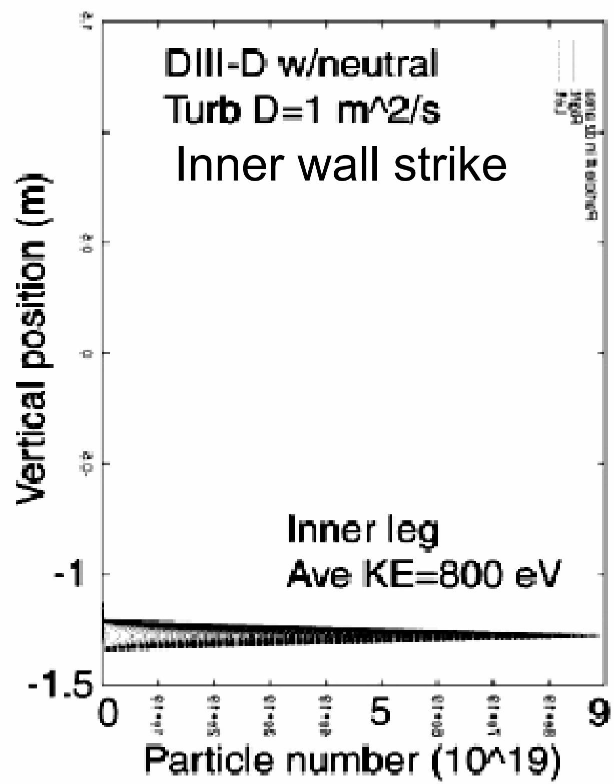
**No "Collisionless Oribt" exists in this loss.**

# Collisionless Wall-Hitting in DIII-D
## ⇒ Dominantly X-Loss



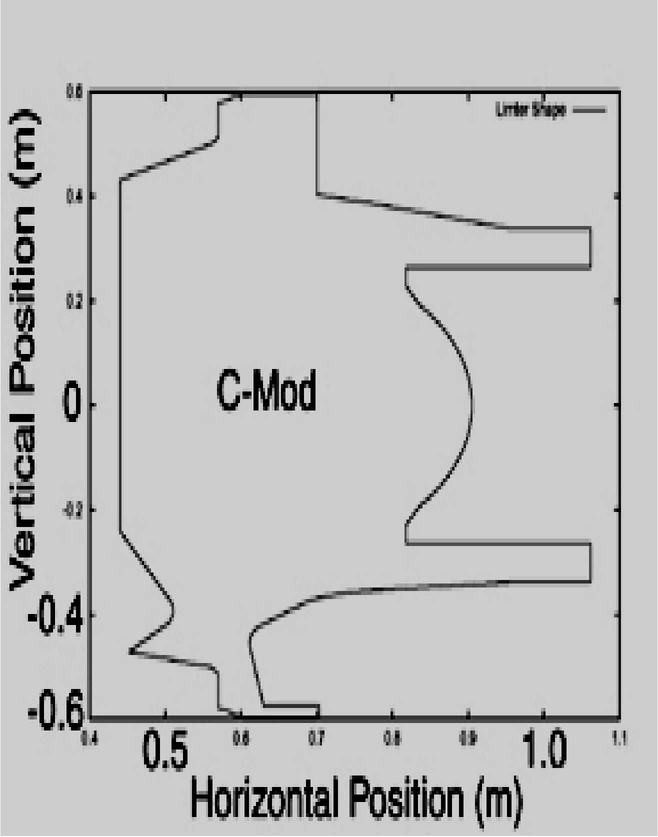
# Collisional Wall-Hitting in DIII-D
## ⇒ Dominantly X-Loss

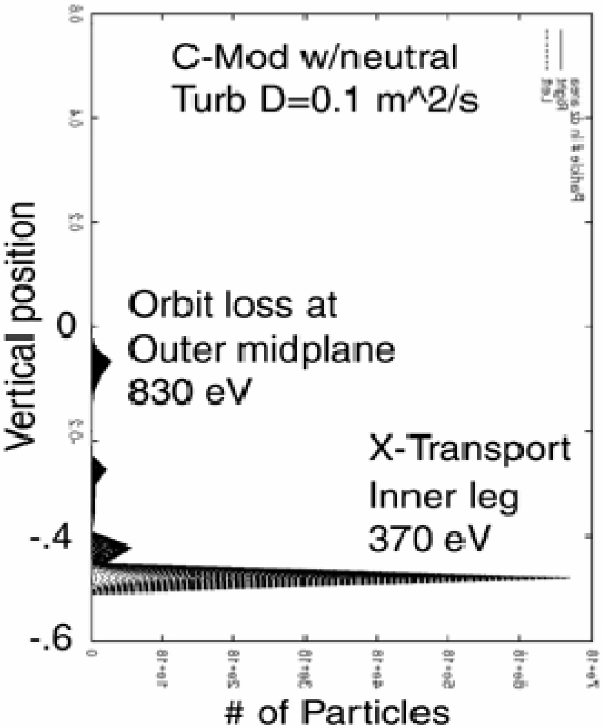


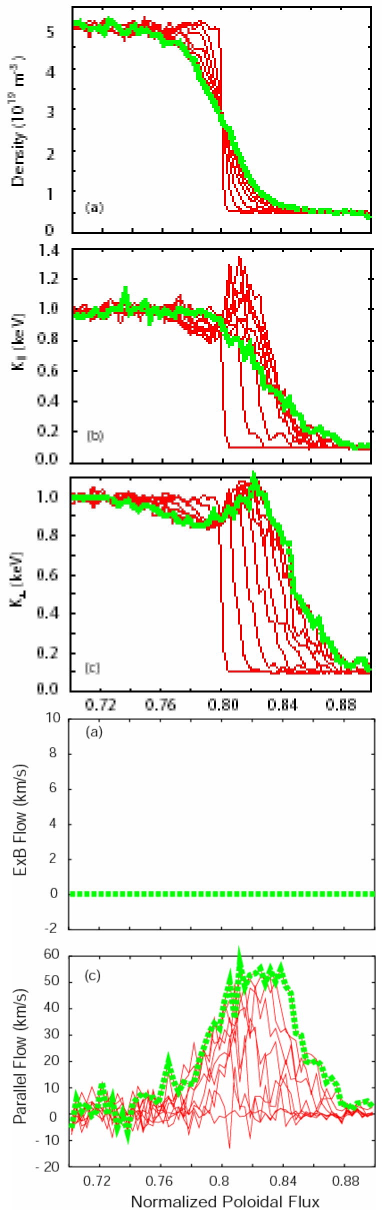
# Wall-Hitting in C-Mod

## ⇒ Dominantly X-Loss

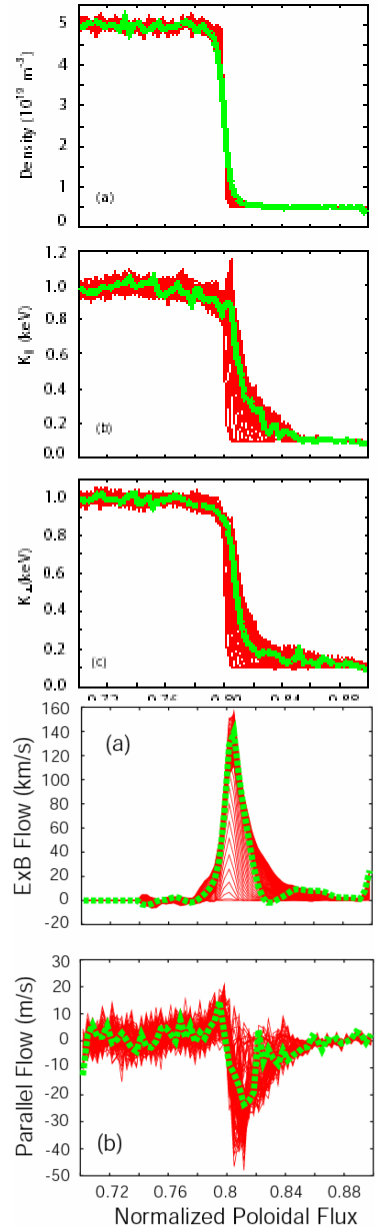


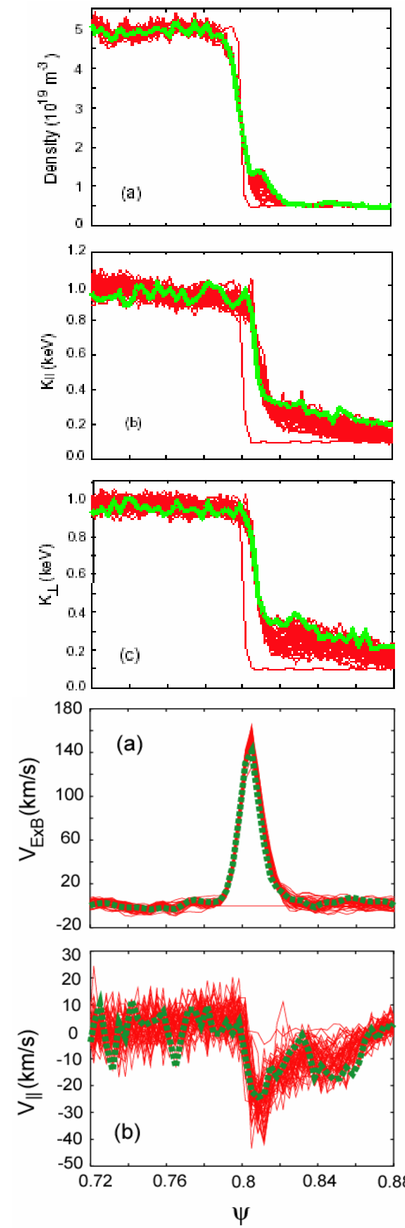
## $E_r=0$, $\nu_c=0$, core



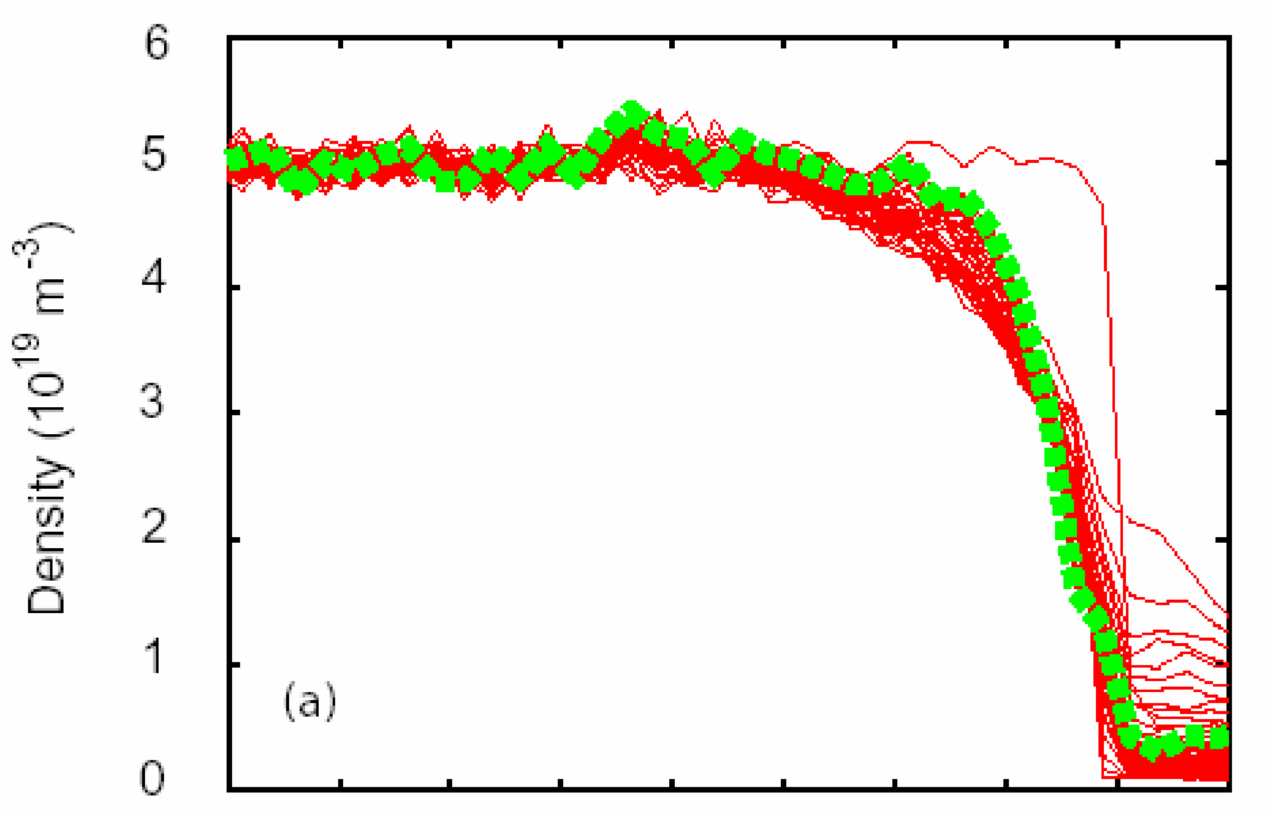

## $Er\neq0$, $\nu_c=0$ core

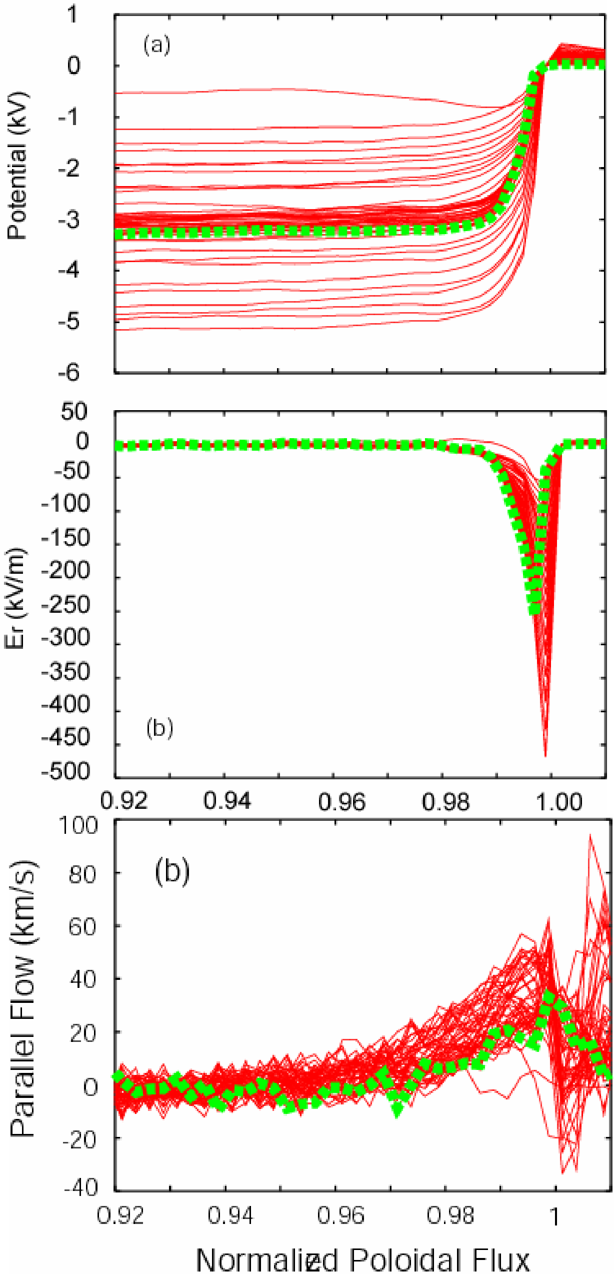



## $Er\neq0$, $\nu_c\neq0$, core

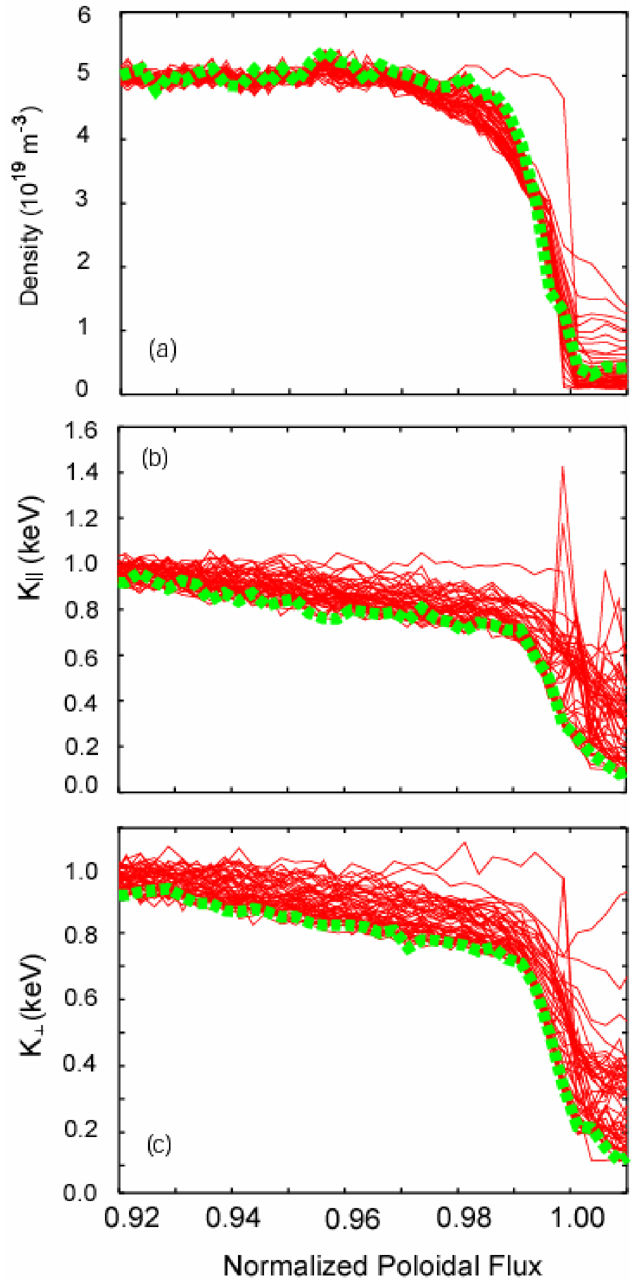


# Dynamics of pedestal buildup



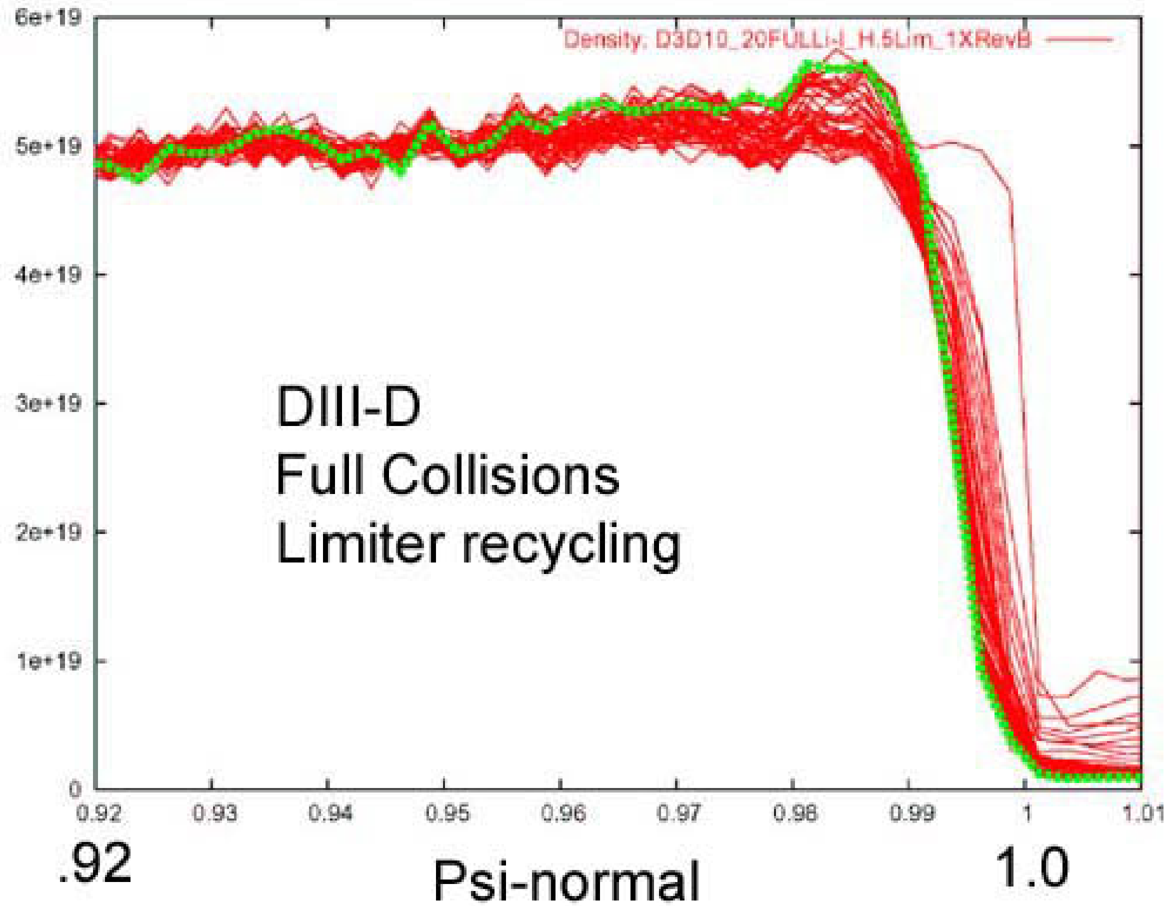


(a)
Potential (kV)
(b)
Er (kV/m)
(b)
Parallel Flow (km/s)
0.92
0.94
0.96
0.98
1.00
Normalized Poloidal Flux

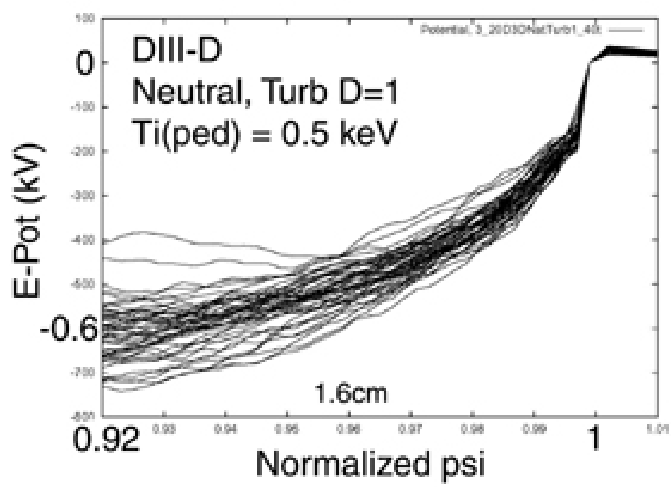



(a)
Density (10^19 m^-3)
(b)
K|| (keV)
(c)
K⊥ (keV)
0.92
0.94
0.96
0.98
1.00
Normalized Poloidal Flux

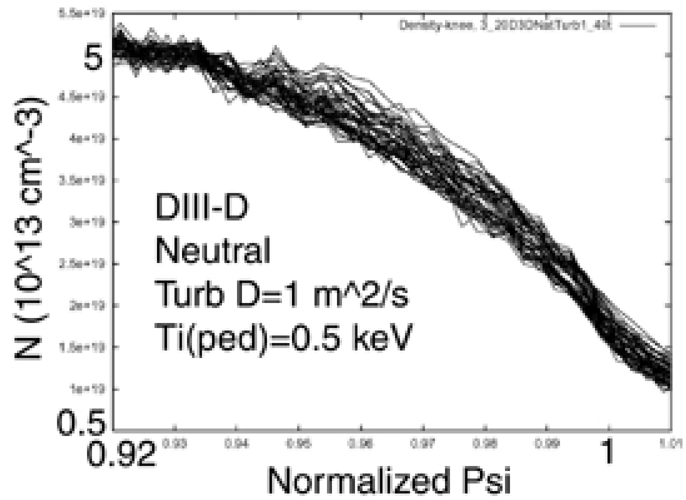
# Neutral ionization raises pedestal density

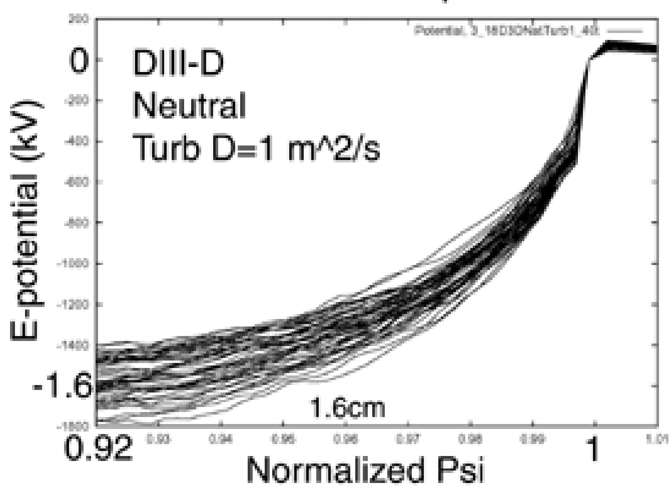


# $E_r$ increases with $T_i$ (ped) under Turbulence diffusion (D=1 $m^2/s$)

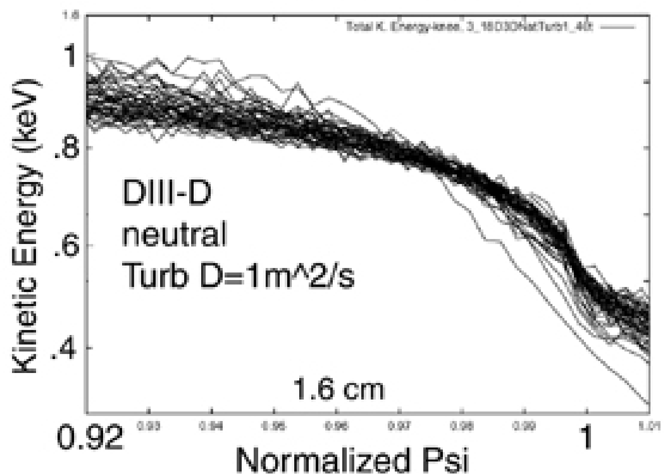



$T_i$=
0.5 keV

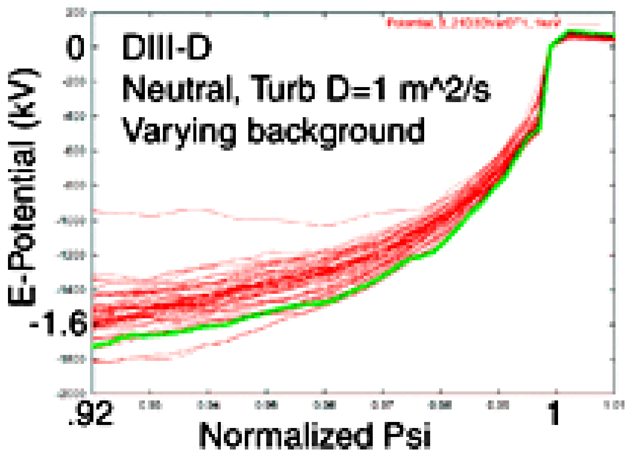


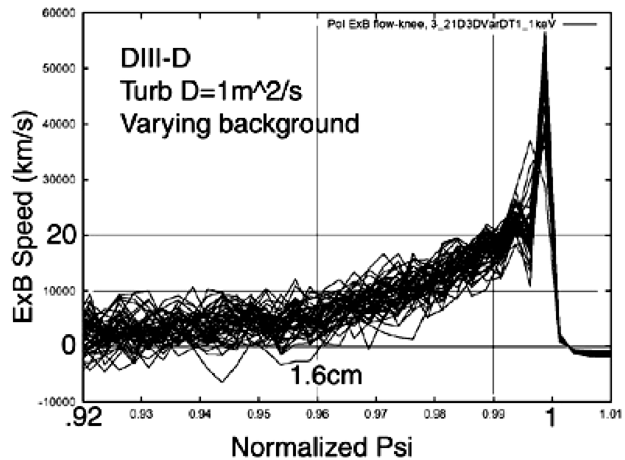



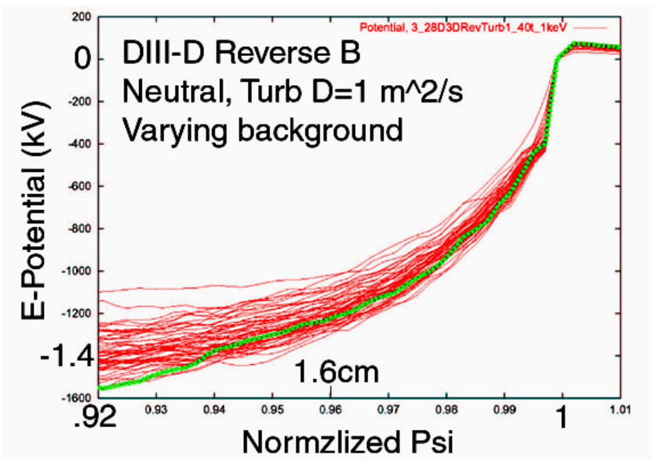

1 keV

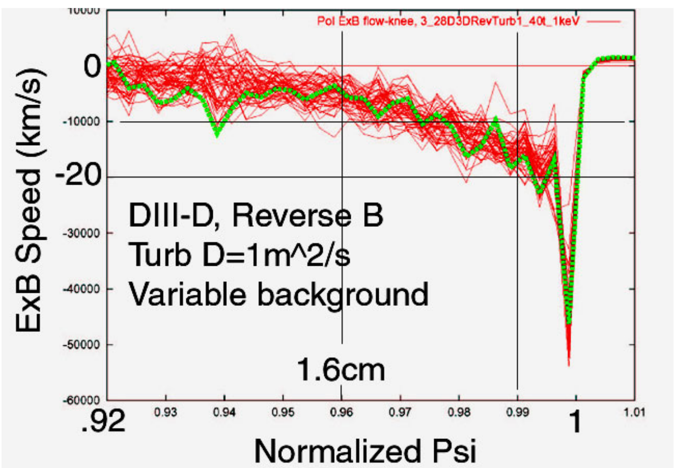


# Forward $\nabla B$ yields ≈15% greater ExB with neutrals and $D_{Turb}$



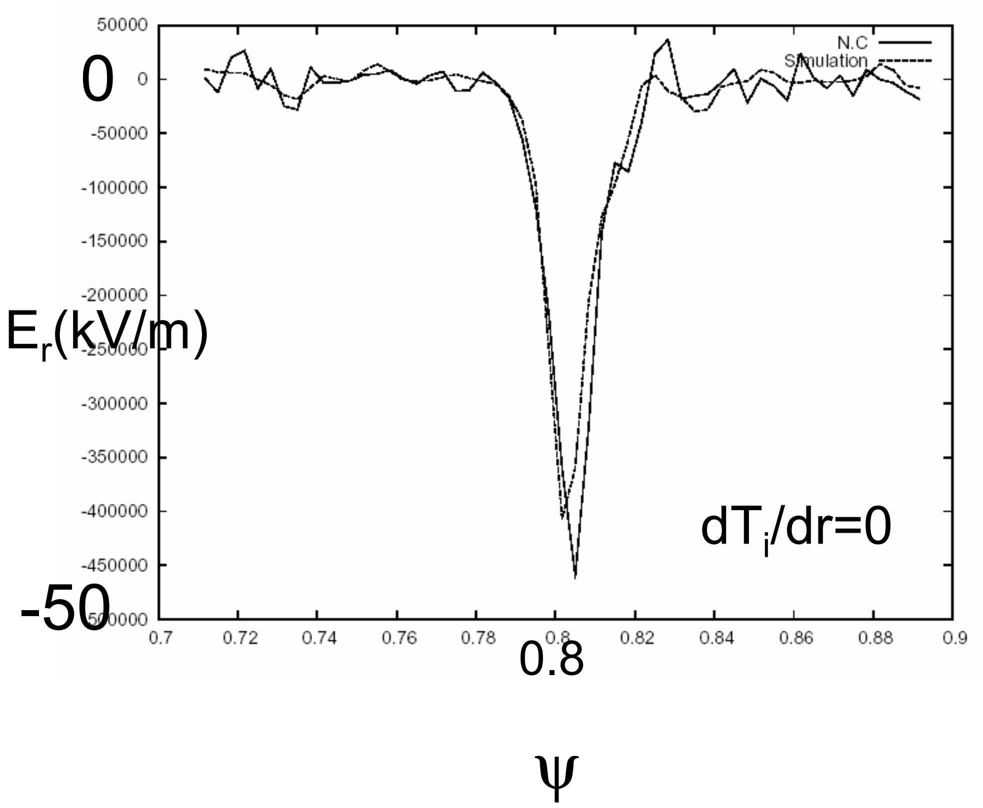



# Conventional Neoclassical flow equation in the core without the $dT_i/dr$ ambiguity

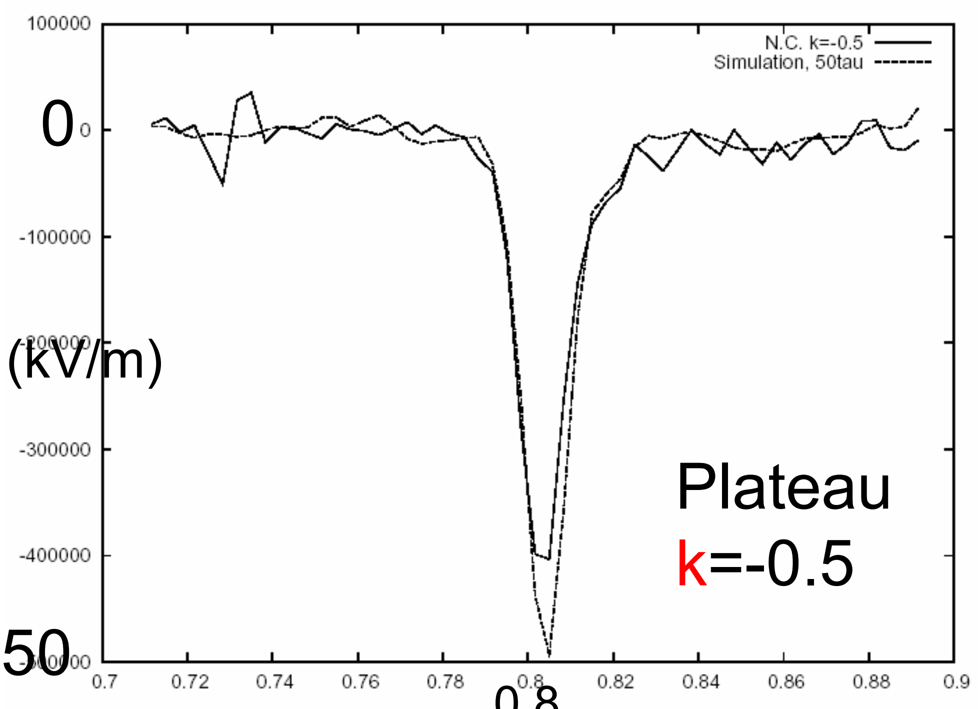
## $U_{i\,\|} = (cT_i/eB_p)(\mathbf{k}\,d\,log\,T_i/dr - d\,log\,p_i/dr - (e/T_i)d\phi/dr)$




Good within error bar

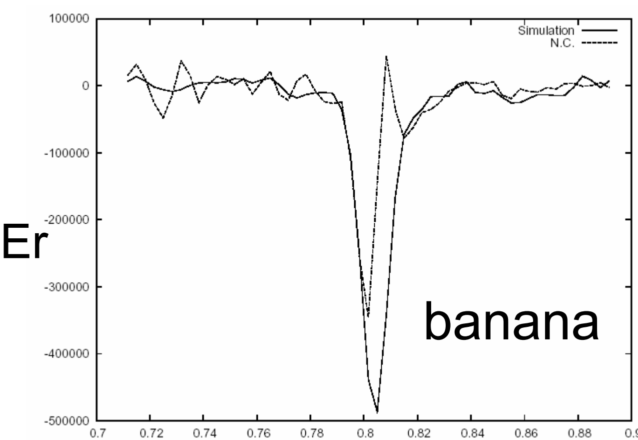
# Conventional Neoclassical flow equation

## $U_{i\,\parallel} = (cT_i/eB_p)(k\,d\,log\,T_i/dr - d\,log\,p_i/dr - (e/T_i)d\phi/dr$




Banana: k=1.17
Plateau: k=-0.5
Collisional: k=-2.1

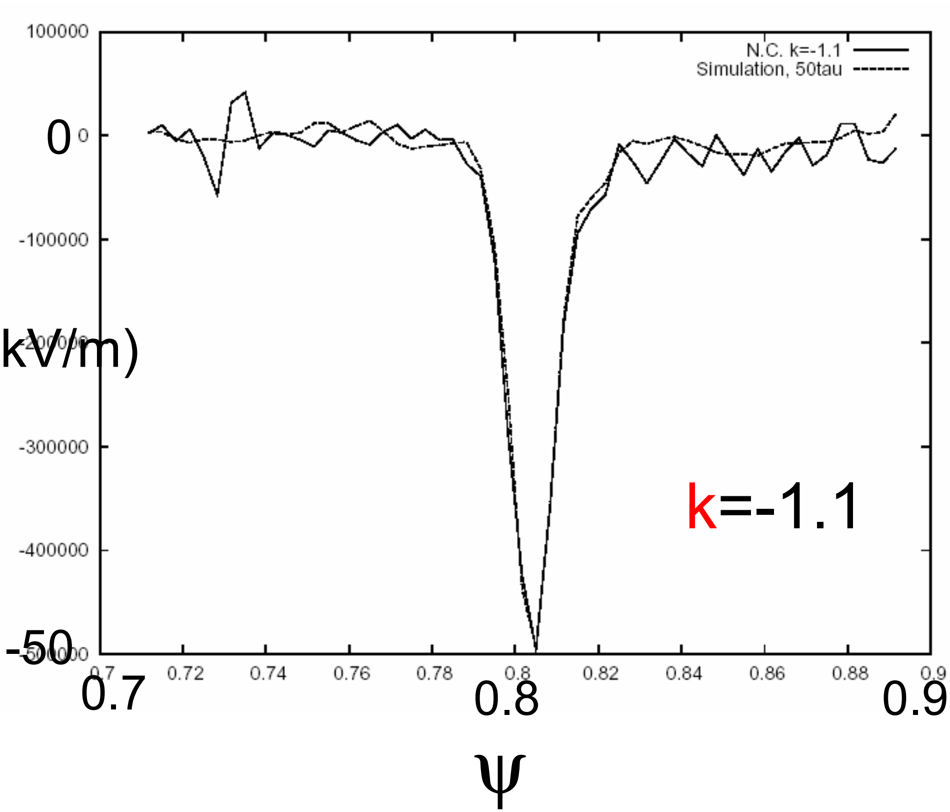
$\nu_* \sim < 1$ for $T_i$=1 keV
$n_{ped} = 5\times10^{19} m^{-3}$



# The conventional Neoclassical flow equation

$$u_{i\,\parallel} = (cT_i/eB_p)[k\,d\,log\,T_i/dr - d\,log\,p_i/dr - (e/T_i)d\phi/dr]$$

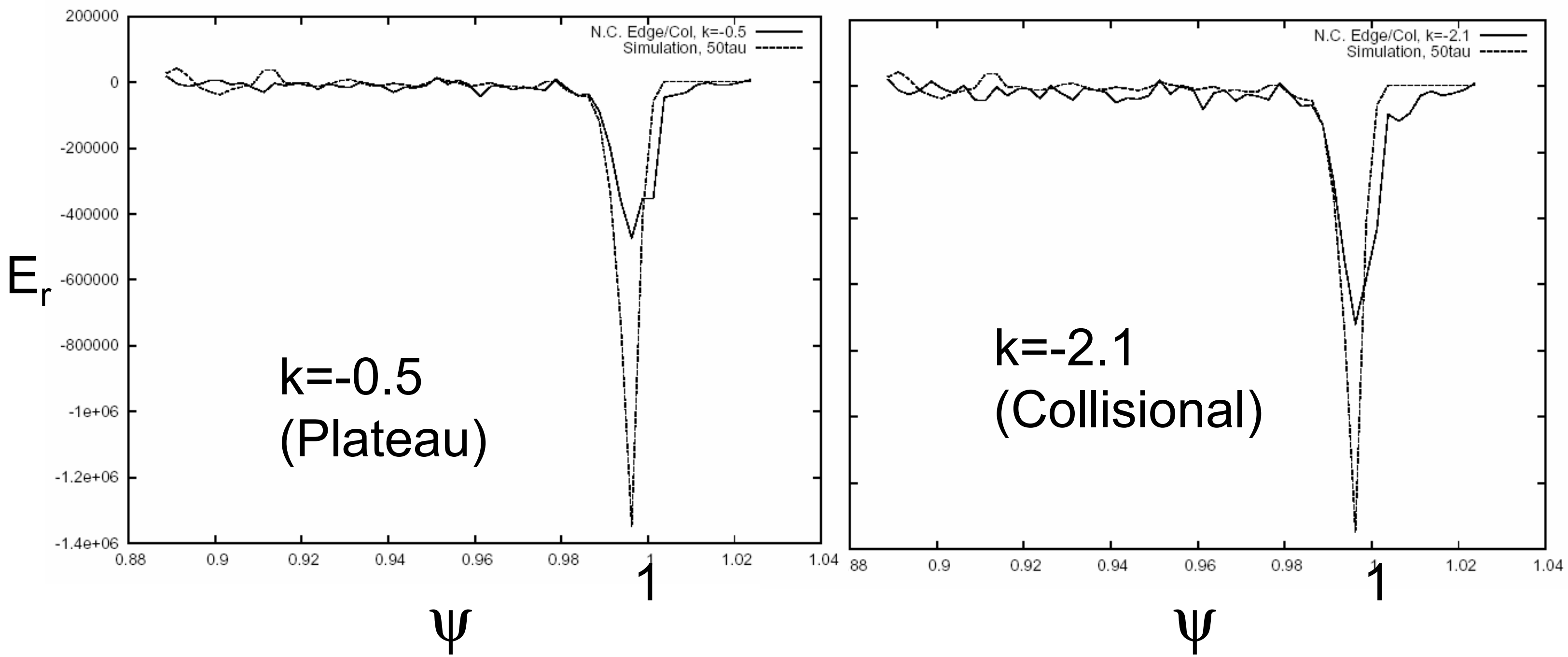
# is good for core pedestal
# if k is adjusted (finite banana correction?)




Banana: k=1.17
Plateau: k=-0.5
Collisional: k=-2.1

$\nu_* \sim 1$ for $T_i$=1 keV
$n = 5\times 10^{19} m^{-3}$

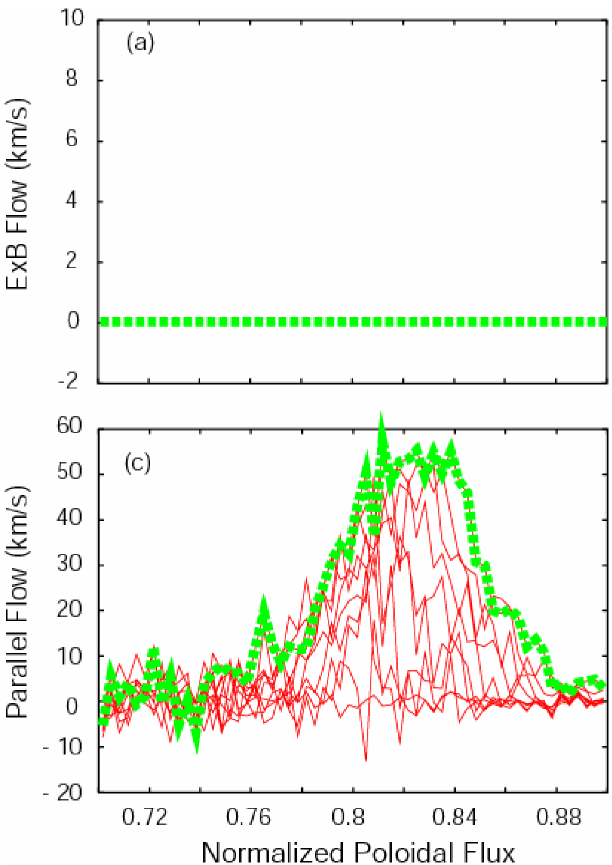
# Edge pedestal is not adequately described by the conventional neoclassical flow eqn.



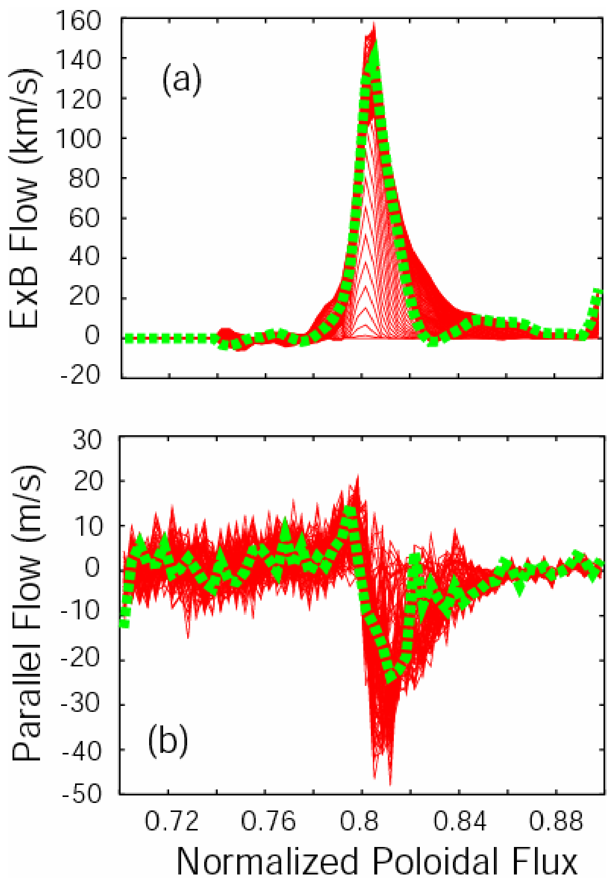

Much worse if banana, k=1.17

# In the core, $V_\zeta$ is consistent with conventional neoclassical angular momentum conservation with $\mathbf{S}_i = \langle R^2 \nabla\zeta \cdot \Pi_i \cdot \nabla r \rangle \approx 0$

$E_r=0,\ J_r \times B_p > 0$




$E_r<0,\ V_{\parallel} < 0$




E×B dominant flow,
Strong return current
$J_r \cong 0$

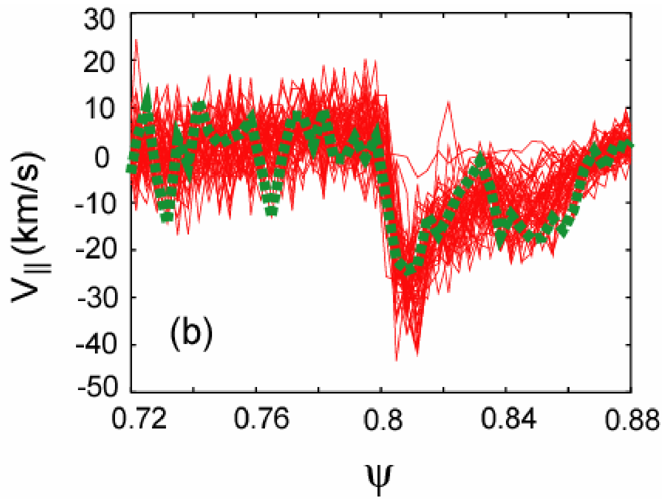
Outward banana orbits are squeezed and counter passing Ions contribute more.

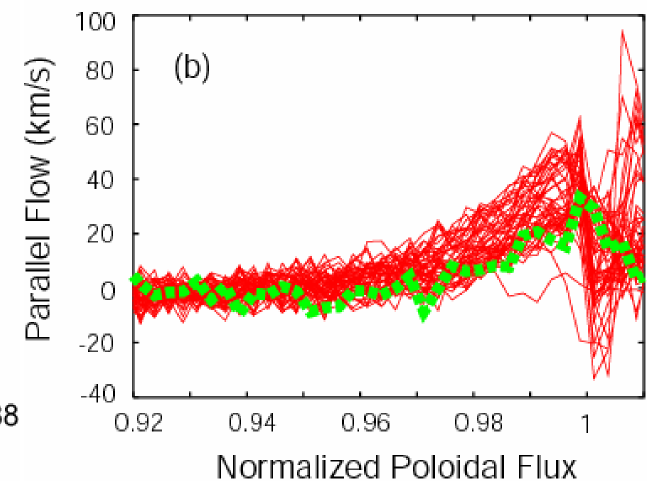
# Positive toroidal momentum source with a strong edge pedestal $\mathbf{S}_i = \langle R^2 \nabla\zeta \cdot \Pi_i \cdot \nabla r \rangle \neq 0$

Core

Edge

X-loss
of $v_{||} < 0$ ions
$\Rightarrow$ positive
momentum
source.



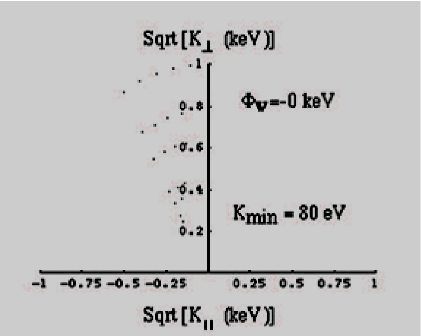
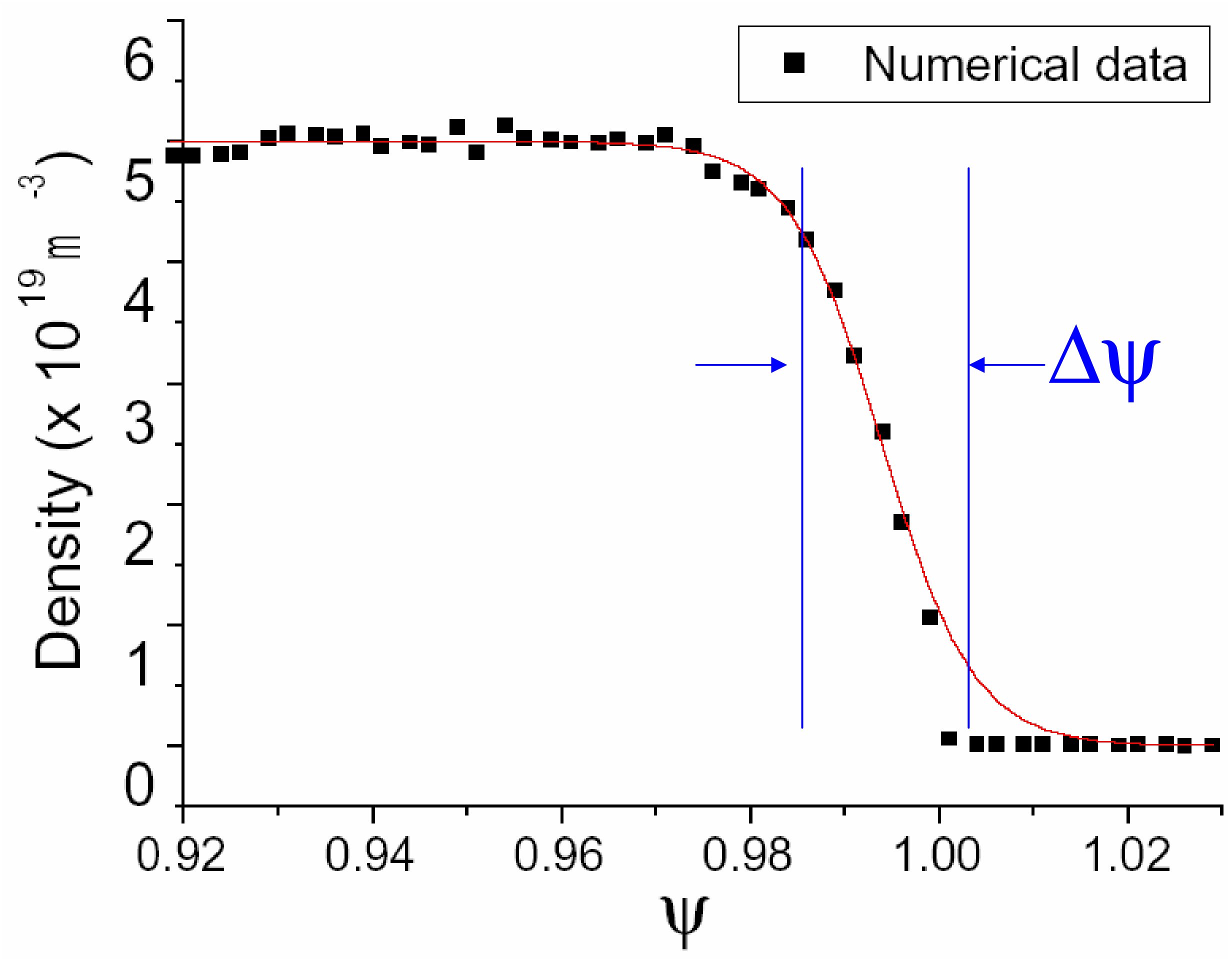
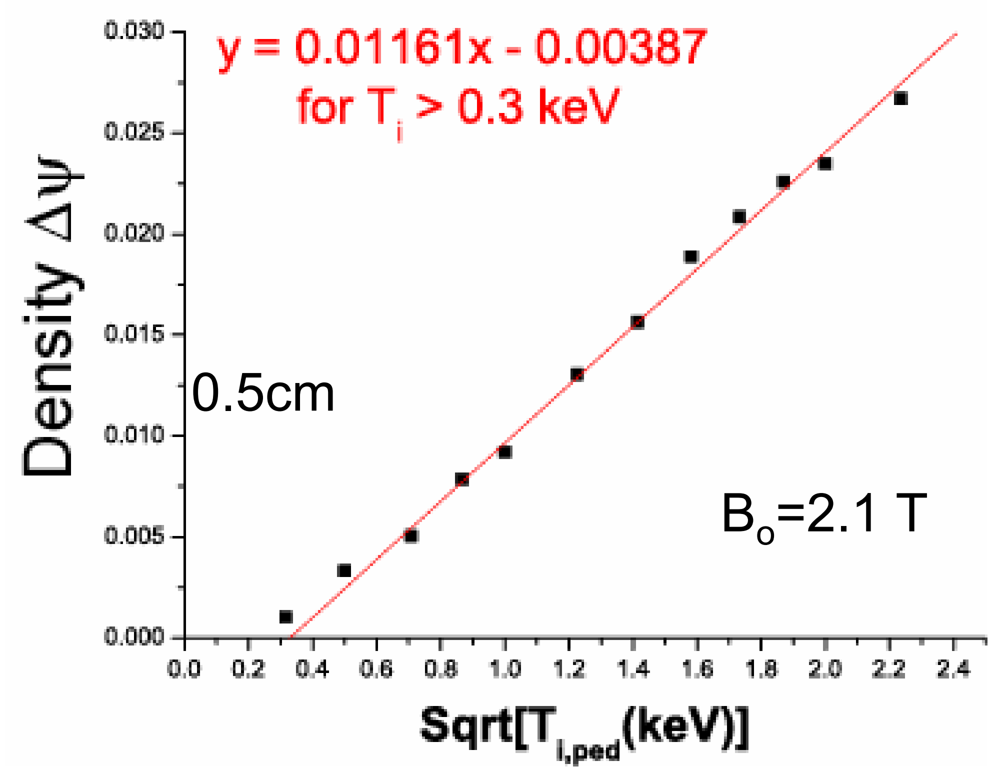




Fig. 2. Ion velocity space hole at 10 cm above the X-point in DIII-D.

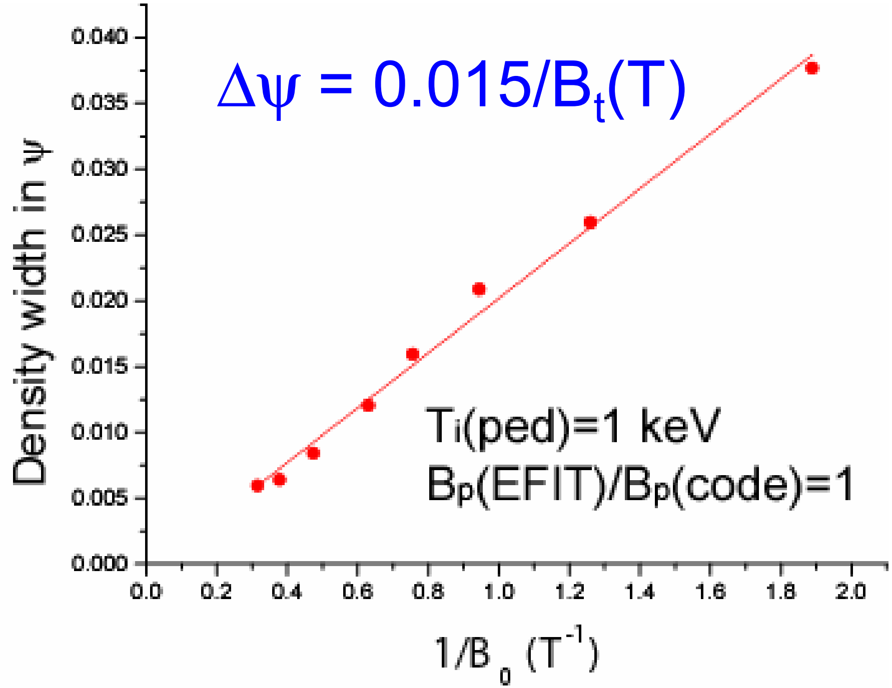
# Excellent Tanh[2(ψ–ψ$_c$)/Δψ] fit



# Density pedestal width is an Offset-Linear Function of $T_i^{1/2}$ for $T_i > 0.3$ keV



# Density pedestal width $\propto$ 1/B




**This effect is from $J_{polar}$**

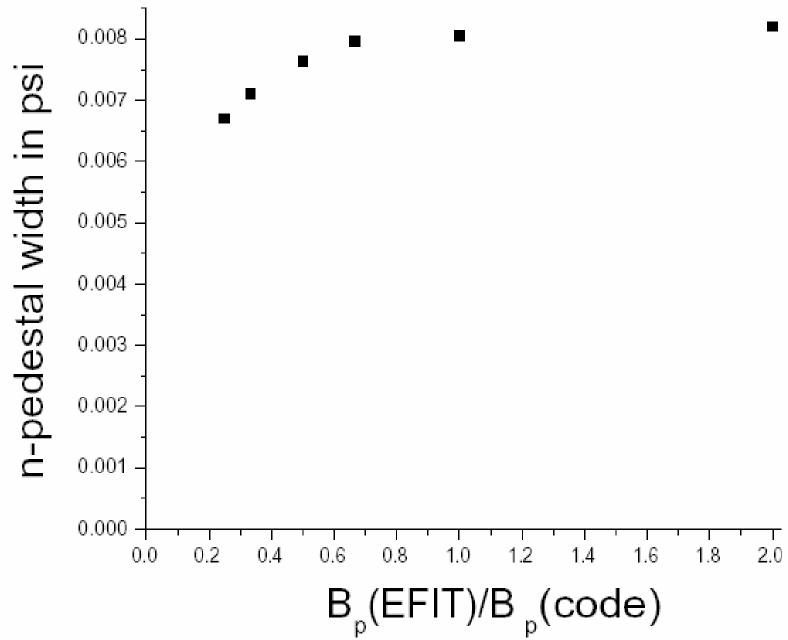
Dominance of the $J_{polar}$
$\Rightarrow dE_r/dt \propto B^2$

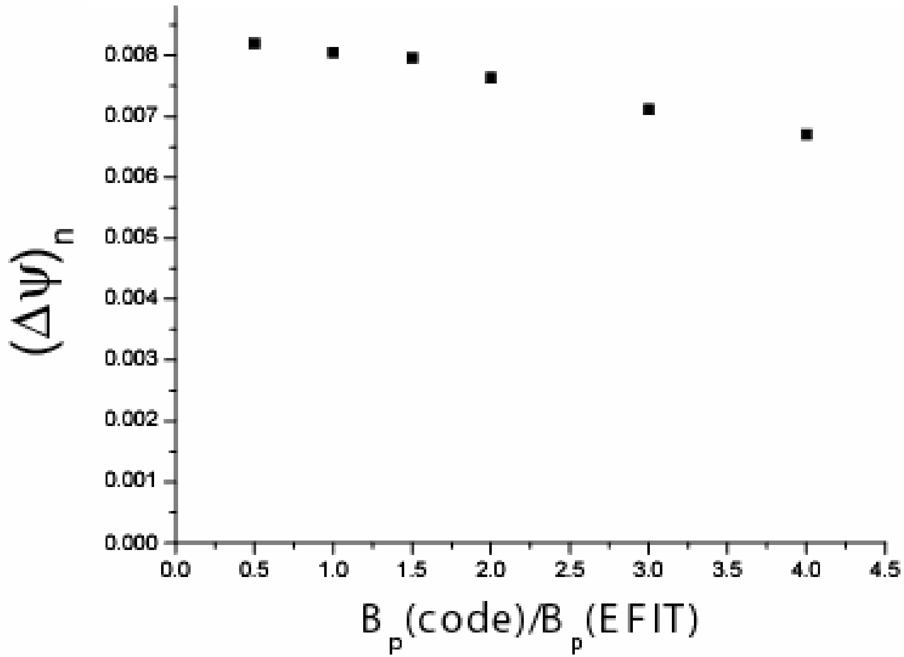
With greater B
- Faster rise of $E_r$ in a thin outer layer
- Greater orbit squeezing
- Narrower pedestal width

$$\left[\langle|\nabla\psi|^2\rangle + 4\pi n_i m_i c^2 \langle|\nabla\psi|^2/B^2\rangle\right] \partial^2\phi/\partial t\partial\psi = 4\pi \langle \mathbf{j}_{code}\cdot\nabla\psi\rangle$$

# Density Pedestal Width is not $\propto 1/B_p$

$\Delta_\psi \propto \rho_p$? No, the orbit squeezing does not allow it.
Every orbit at $\psi<\psi_{layer}$ is closed.




$B_t$(EFIT)/$B_t$(code)=1, $T_i$(ped)=1 keV

# $\Delta_n \propto 1/B_T$ agrees with a preliminary result from C-Mod

[Hughes, Mossessian, Hubbard, etc, PoP 9, 3019 (2002)]

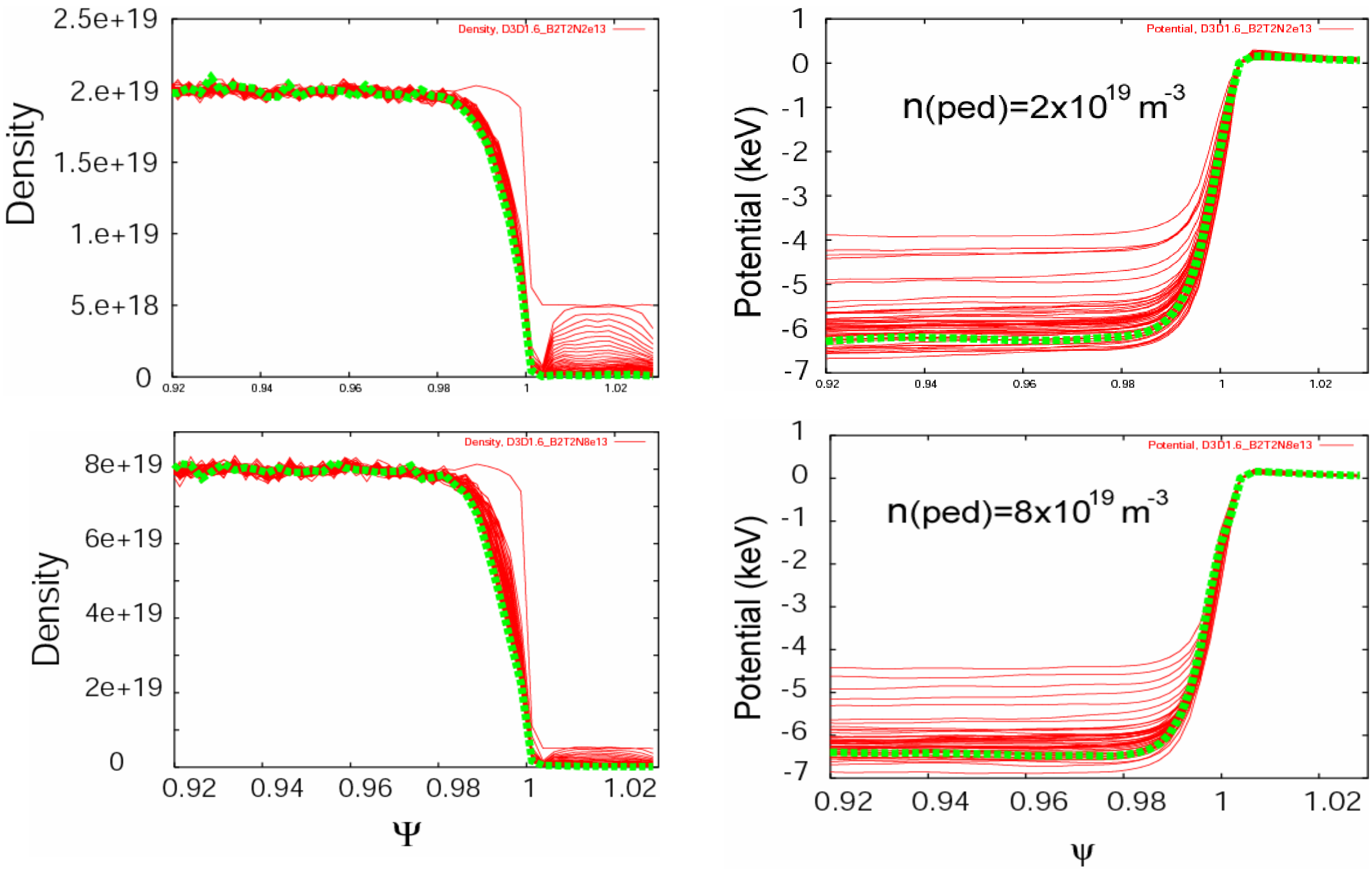
$$\Delta_n = C_0\, I_p^{\alpha}\, n_{e,L}^{\beta}\, \mathbf{B_T}^{\gamma}\, P_{sol}^{\delta}$$

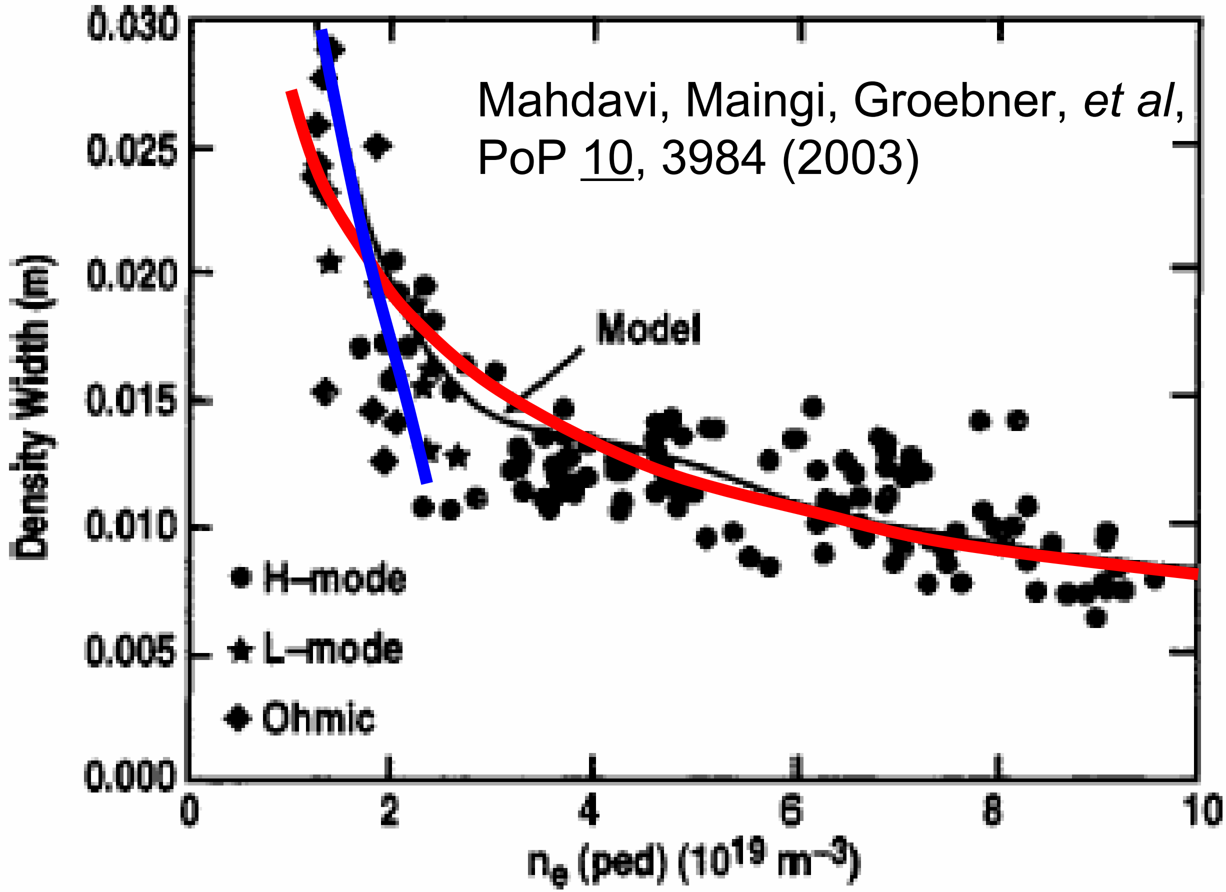
$\gamma$**= -0.99±0.29** for $4.5 \leq B_T(T) \leq 6.0$

($\alpha$=0.79±0.14, $\beta$=0.14±0.14, $\delta$=-0.01±0.08)

- $T_i$ scaling may be hidden in $I_p$, $P_{sol}$, and $n_{e,L}$?
- More experiments are planned.

# n-width & $E_r$ shows little dependence on $N_{ped}$ (not conclusive yet)






**Normally $T_i$(ped) ≈ 1/n(ped).**

**Density width ∝ $T_i^{1/2}$ ∝ 1/$n^{1/2}$ in H-mode.**

**How much of the data trend is the $T_i$ effect?**

# Conclusions

- **XGC can simulate the neoclassical Pedestal buildup and study pedestal physics.**
- **Self-consistent oribit squeezing, orbit loss (and neutral ionization) are vital to get the correct pedestal.**
- $\Delta_n^{neo} \propto T_i^{1/2}(\text{ped})\, B_T^{-1} \propto \rho_i(\text{ped})$
- **Neutrals build up pedestal, but not contribute to $\Delta_n^{neo}$**
- Neoclassical rotation physics needs revision at edge.
- $n_i$(ped) is set by neutrals and $T_i$(ped) by heating.
- $n_i$ and $T_i$ can have different pedestal shapes (neutral effect).
- **$E_r$-shear rises with $T_i$(ped) at L-mode diffusion level.**
- **Forward B yields stronger $E_{r.}$**

**In DIII-D**

- **$D_{anom}$~1 $m^2/s$ gives an L-mode pedestal shape.**
- **Residual $D_{anom}$~0.1 $m^2/s$ recovers the neoclassical shape.**