A simple model of interactions between electron temperature gradient and drift-wave turbulence

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Motivation

- A current focus of plasma turbulence research is to understand possible mechanisms of anomalous electron heat transport, particularly when ion thermal and particle transport are suppressed
  - Electron temperature gradient (ETG) mode is a currently popular
  - Alternatives such as short-wavelength trapped electron modes exist
  - ETG has drastically smaller characteristic scale than ITG, but it has been argued that it may from large-scale convective cells with enhanced transport, such as streamers

- **New trend**: ETG as significant source of electron transport both in and outside of transport barriers (e.g. Horton et. al PoP 2000)
  - Since it is possible to have **relevant** instabilities on multiple length scales, should understand their interactions

Outline

1. Background and Basic Physics

2. Effectiveness of Two-Dimensional DITG Shearing of ETG Modes

3. Effect of DITG Modulations of η_e

4. Back-Reaction of ETG on DITG Turbulence

5. Extended Ecology Model for ETG – DITG – ZF Dynamics

Basic Equations for ETG / DITG

- Basic fluid model of instabilities ($x \rightarrow r,\ y \rightarrow r\theta$)

$$\frac{d}{dt}\left(\left(\delta - \rho^2 \nabla_\perp^2\right)\phi\right) \pm V_n^* \frac{\partial \phi}{\partial y} - V_B^* \frac{\partial p}{\partial y} = 0$$

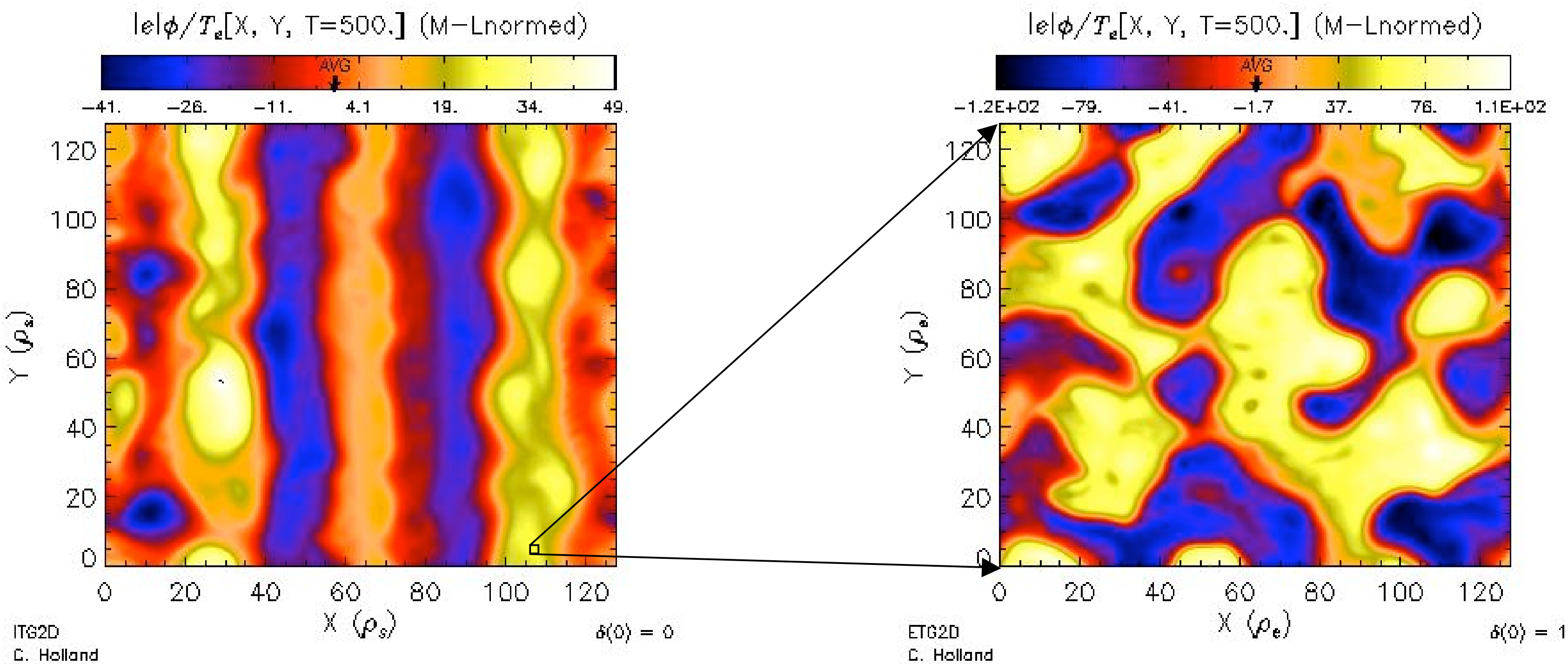
$$\frac{dp}{dt} + V_p^* \frac{\partial \phi}{\partial y} = 0$$

$$V_f^* = \frac{cT_e}{|e|B}\frac{d \ln f_0}{dx}, \quad \frac{df}{dt} = \frac{\partial f}{\partial t} - \left(\vec{\nabla}\phi \times \hat{z}\right)\cdot \vec{\nabla} f, \quad \phi = \frac{|e|\tilde{\phi}}{T_e}, \quad p = \frac{\tilde{p}}{p_0}$$

- DITG is used in this talk to denote ion gyroradius modes driven by both curvature and trapped electron effects
  - Includes both $k\rho_i < 1$ and $k\rho_i \sim 1$ modes

Schematic View of Scales of Interest

- Simple visualization of scale separation



ρ_s scale DITG
(with zonal flows)

ρ_e scale ETG

Streamers in ETG Turbulence

- Recent argument: ETG turbulence can nonlinearly from large, radially extended convective cells called "streamers" which may drive relevant experimental transport
  - Understanding their dynamics and conditions for formation still very much an open question
  - Formation of streamers or other large convective cells necessary for ETG to drive experimentally relevant transport levels

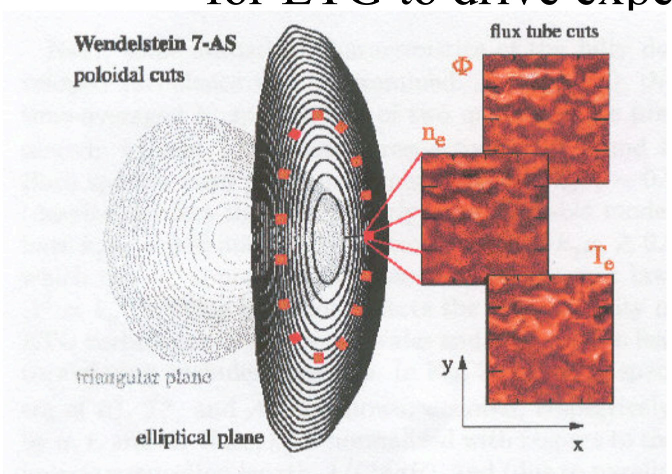


FIG. 1: Poloidal cross sections of Wendelstein 7-AS. The representative flux tube shown in red is used for the nonlinear simulation. The turbulence exhibits streamers.

(from Jenko and Kendl, NJP 2002)

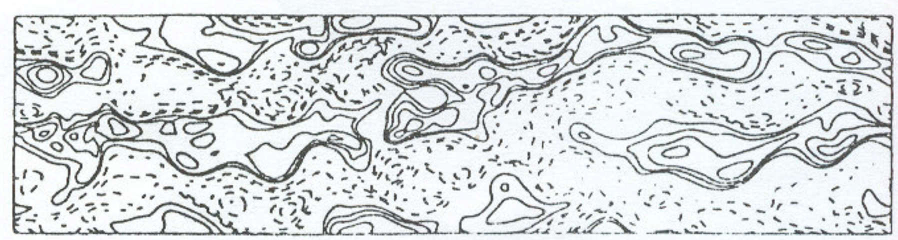


FIG. 4. Characteristic $\tilde{\phi}$ contours in the outboard x-y plane. This snapshot was taken at $t = 180$ of the run shown in Fig. 2. Positive and negative fluctuations are drawn, respectively, with solid and dashed lines.

(from Jenko et. al PoP 2000)

2D Shearing of ETG by DITG

- Most obvious effect of DITG turbulence on ETG will be the random shearing of ETG turbulence by the DITG flow
  - DITG modes will shear ETG **both radially and poloidally**
  - Shearing will be due to entire DITG spectrum, not just ρ_i scale zonal flows

- Separation of ETG, DITG time and length scales suggests that using ETG adiabatic invariants to be appropriate approach
  - Based on experience with this approach, expect shearing effects to manifest as diffusion of ETG intensity in ***k***-space

- **Key question**: will ***k***-space diffusion rate overwhelm linear growth rate?
  - If so, energy appearing at low ***k*** due to linear growth rapidly diffuses to high ***k*** where it damps ⟶ **ETG suppressed!**
  - If not, then expect that effects of DITG on ETG are weak

2D Shearing of ETG by DITG (cont.)

- Describe ETG by wave-kinetic equation for appropriate adiabatic invariant (e.g. enstrophy / roton number) $N\left(\vec{x},\vec{k}\right)$

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial \vec{k}}\left(\omega_k+\vec{k}\cdot\vec{V}_0\right)\cdot\frac{\partial N}{\partial \vec{x}}-\frac{\partial}{\partial \vec{x}}\left(\vec{k}\cdot\vec{V}_0\right)\cdot\frac{\partial N}{\partial \vec{k}}\cong 2\gamma_k N-\Delta\omega N^2$$

  - Here γ_k is the linear growth rate, $\Delta\omega$ represents nonlinear self-damping of the ETG turbulence
  - Invariant N proportional to ETG intensity $|\phi_{ETG}|^2$

- Shear flow $\vec{V}_0=-\vec{\nabla}\phi\times\hat{z}$ due to a given DITG spectrum $|\phi_q|^2$
  - Consider both $q\rho_s < 1$ curvature-driven ITG, $q\rho_s \sim 1$ (i.e. CTEM) modes
  - Again, stress that this spectrum contains **all** DITG modes, not just zonal flows, and produces a **two-dimensional strain field**

2D Shearing of ETG by DITG (cont.)

- Using quasi-linear theory, can derive evolution equation

$$\frac{\partial\langle N\rangle}{\partial t} \cong \frac{\partial}{\partial k_\alpha} \underline{\underline{D_{\alpha\beta}}} \frac{\partial\langle N\rangle}{\partial k_\beta} + 2\gamma_k\langle N\rangle - \Delta\omega\langle N\rangle^2$$

$$\underline{\underline{D_{\alpha\beta}}} = \rho_e^2 v_{te}^2 \sum_q q_\alpha q_\beta \left|\left(\vec{k}\times\vec{q}\right)\cdot\hat{z}\right|^2 R\left(\Omega_q\right)\left|\phi_q\right|^2$$

$$R\left(\Omega_q\right) = \frac{1}{2\gamma_k - i\left(\Omega_q - \vec{q}\cdot\vec{v}_g\right)}$$

2D Shearing of ETG by DITG (cont.)

- Can identify two timescales $\gamma_{lin} = 2\gamma_k$, $\gamma_D = D_{\alpha\beta} / k_\alpha k_\beta$
  - Ratio γ_D / γ_{lin} determines whether shearing of ETG by DITG is an important effect.

- To estimate γ_D, need to have an estimate of DITG spectrum- use a **ion temperature** mixing-length estimate (as an upper bound)

$$T_q = \frac{\tilde{T}_i}{T_{i0}} = \frac{1}{qL_{Ti}}$$

$$T_q = \Sigma_{Ti}\phi_q \rightarrow |\phi_q|^2 = \frac{1}{|\Sigma_{Ti}|^2} \frac{1}{(qL_{Ti})^2}$$

$$\Sigma_{Ti} \approx \frac{q_\theta V_{Ti}^*}{\omega_q + i\gamma_q} \rightarrow |\Sigma_{Ti}|^2 \approx \frac{\eta_i}{\varepsilon} = \frac{L_B}{L_{Ti}} \boxed{\rightarrow |\phi_q|^2 = \frac{1}{q^2 L_{Ti} L_B}}$$

2D Shearing of ETG by DITG (cont)

- Can now estimate γ_D / γ_{lin} as

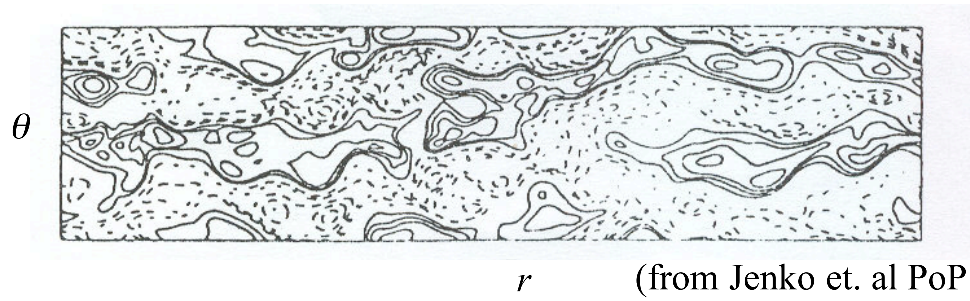
$$\frac{\gamma_D}{\gamma_{lin}} = \frac{1}{\gamma_{lin}} \frac{D_{\alpha\beta}}{k_\alpha k_\beta} \approx \frac{1}{\gamma_{lin}} \frac{\rho_e^2 v_{Te}^2}{\gamma_{lin}} \sum_q q^4 \left|\phi_q\right|^2$$

$$\rightarrow \frac{\gamma_D}{\gamma_{lin}} \approx \frac{m}{M} \left(\frac{\bar{q}\rho_s}{k_\theta \rho_e} \right)^2 \frac{\tau \eta_i}{\eta_e - \eta_e^c}$$

→ For “generic” ETG and DITG turbulence, shearing of ETG by DITG will be small because of scale separation, but…

Impact: Shearing of Streamers by DITG

- Particularly important case: large-scale ETG streamer with $k_\theta \rho_e \approx 1/10$
  - Thought to be most relevant for transport



(from Jenko et. al PoP 2000)

- Then for **short-wavelength** ($\bar{q}\rho_s \approx 1$) DITG modes

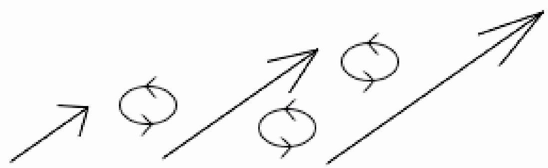
$$\frac{\gamma_D}{\gamma_{lin}} \approx 100 \frac{m}{M} \frac{\tau \eta_i}{\eta_e - \eta_e^c}$$

⟶ **Shearing by short-wave DITG may be important effect for streamers!**

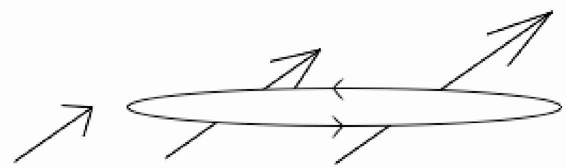
⟶ **If streamers are suppressed, is ETG relevant?**

Impact: Shearing of Streamers by DITG

- Increased spatial scale makes streamers much more susceptible to shearing by ρ_s scale DITG turbulence



ρ_e - scale eddies too small to be sheared effectively, swept along by DITG flow field



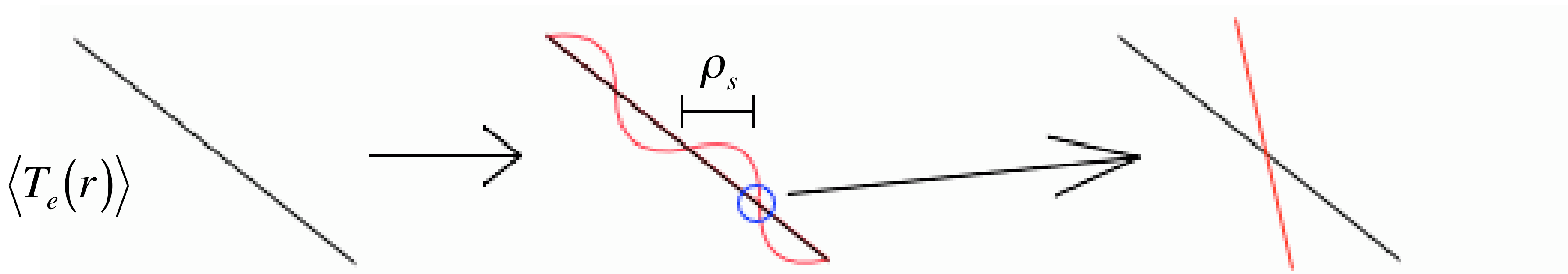
Extended radial length of streamer makes DITG shearing potentially important effect

- **Caveat for Simulations**: reduced mass ratio may distort this effect

Effects of Modulating η_e

- **Observation**: ρ_s scale fluctuations of T_e induced by larger DITG modes will appear as modulations of η_e to ETG modes

  $\longrightarrow$ what is the effect?



DITG turbulence creates ρ_s - scale features on $\langle T_e(r) \rangle$

ρ_e-scale ETG turbulence "sees" different effective background $\langle T_e(r) \rangle$

Effects of Modulating η_e (cont.)

- Assume $\eta_e = \eta_0 + \delta\eta_e$, with $\delta\eta_e / \eta_0 << 1$. Returning to wave-kinetic equation, mean-field theory predicts modulations will create a **nonlinear growth rate** γ_{NL}

$$\frac{\partial\langle N\rangle}{\partial t} \cong \frac{\partial}{\partial k_\alpha} D_{\alpha\beta} \frac{\partial\langle N\rangle}{\partial k_\beta} + 2(\gamma_k + \gamma_{NL})\langle N\rangle - \Delta\omega\langle N\rangle^2$$

$$\gamma_{NL} = 2\left(\left.\frac{\partial\gamma_k}{\partial\eta_e}\right|_{\eta_e=\eta_0}\right)^2 \sum_q R(\Omega_q)\left|\delta\eta_q\right|^2$$

Effects of Modulating η_e (cont.)

- Since $\gamma_k \propto (\eta_e - \eta_e^c)^{1/2}$, one can write

$$\left.\frac{\partial \gamma_k}{\partial \eta_e}\right|_{\eta_e=\eta_0} = \frac{\gamma_k}{2\left(\eta_0 - \eta_e^c\right)} \rightarrow \gamma_{NL} \approx \frac{1}{4}\frac{\gamma_k}{\left(\eta_0 - \eta_e^c\right)^2}\sum_q \left|\delta\eta_q\right|^2$$

$$\boxed{\Rightarrow \frac{\gamma_{NL}}{\gamma_k} = \frac{1}{4}\frac{\sum_q \left|\delta\eta_q\right|^2}{\left(\eta_0 - \eta_e^c\right)^2} = \frac{1}{4}\left(\frac{\left|\delta\eta_e\right|}{\eta_0 - \eta_e^c}\right)^2}$$

$\longrightarrow$ γ_{NL} will be important near ETG marginality, namely when the ETG fluctuation amplitude $|\delta\eta_e| \sim \eta_0 - \eta_e^c$ (i.e. η_e fluctuation amplitude is comparable to the deviation from marginality).

Mean-field Estimate of $\delta\eta_e$

- Estimate an RMS $\overline{\delta\eta_e}$ using mixing length DITG levels

$$\overline{\delta\eta_e} = \left\langle \left|\delta\eta_e\right|^2 \right\rangle^{1/2} = L_n \left\langle \left| \frac{\nabla_r \tilde{T}_e}{T_{e0}} \right|^2 \right\rangle^{1/2} = L_n \left(\sum_q \left| q_r \frac{\tilde{T}_e}{T_{e0}} \right|^2 \right)^{1/2}$$

$$\rightarrow \overline{\delta\eta_e} = \left(\sum_q \left| \frac{\Sigma_{Te}}{\Sigma_{Ti}} \frac{q_r}{q} \right|^2 \right)^{1/2} \frac{L_n}{L_{Ti}} \approx \frac{\eta_e}{2}$$

- **Upshot**: mixing-length level DITG can give $\overline{\delta\eta_e} \approx \eta_e / 2$

$\longrightarrow$ **DITG – induced fluctuations can significantly impact effective ETG growth rate**

$\longrightarrow$ **Caveat**: effects of profile modification should not be neglected when considering *linear* ETG stability bounds

Implications of η_e Modulation

- Modulation effect alters ETG growth rate, **can compete against** shearing of ETG by DITG
  - Note that the enhancement factor **does not depend upon mass ratio**

- Since effect on ETG depends upon η_e fluctuation at specific point, are interested in not just RMS level, but also **PDF of η_e fluctuations**
  - Particularly interested in “tails” of PDF
  - Should be straightforward to study numerically

- **Observation**: if η_e is near critical value, fluctuations may lead to stabilization of ETG turbulence

  $\longrightarrow$ would lead to “submarginal” ETG turbulence, “bursty” transport, on DITG space/time scales

Back-Reaction of ETG on DITG

- To gain insight into back-reaction of ETG on DITG, use a simple renormalization approach
  - Effect of slab ETG zonal flows on DITG turbulence has been investigated by Kishimoto and Li

- Model ETG, DITG as two interacting fields with different characteristic scales, each essentially described by a Hasegawa-Mima type equation
  - Ignore zonal flows in initial analysis

- **Goal**: break up effects of ETG on DITG into coherent and "noise" terms, compare against DITG self-interaction effects

Back-Reaction of ETG on DITG (cont.)

- Use < superscript for DITG, and > superscript for ETG
  - Each instability has real frequency and growth rate

$$\frac{\partial \phi_k^<}{\partial t} + \left(i\omega_k^< - \gamma_k^<\right)\phi_k^< = \sum_{k'} \Lambda_{k,k'}^< \left(\phi_{k'}^<\phi_{k-k'}^< + \phi_{k'}^>\phi_{k-k'}^>\right)$$

$$\Lambda_{k,k'}^< = \rho_s^3 c_s \frac{\hat{z}\cdot\left(\vec{k}\times\vec{k}'\right)}{1+k^2\rho_s^2}\left(\left|\vec{k}'\right|^2 - \left|\vec{k}-\vec{k}'\right|^2\right)$$

Reynolds stress on DITG from ETG

$$\frac{\partial \phi_k^>}{\partial t} + \left(i\omega_k^> - \gamma_k^>\right)\phi_k^> = \sum_{k'} \Lambda_{k,k'}^> \left(\phi_{k'}^>\phi_{k-k'}^> + \phi_{k'}^<\phi_{k-k'}^>\right)$$

$$\Lambda_{k,k'}^> = \rho_e^3 v_{Te} \frac{\hat{z}\cdot\left(\vec{k}\times\vec{k}'\right)}{1+k^2\rho_e^2}\left(\left|\vec{k}'\right|^2 - \left|\vec{k}-\vec{k}'\right|^2\right)$$

Shearing of ETG by DITG

Back-Reaction of ETG on DITG (cont.)

- Write renormalized evolution equation for DITG intensity
  - Define $I_k^< = \left\langle \left|\phi_k^<\right|^2 \right\rangle$
  - Write interactions as coherent terms, incoherent / noise terms

$$\frac{\partial I_k^<}{\partial t} = 2\gamma_k^< I_k^< \underbrace{- \nu_k^< I_k^< + \nu_k^> I_k^<}_{\text{coherent terms}} \underbrace{+ S_k^< + S_k^>}_{\text{incoherent / noise terms}}$$

- For mixing length ETG and DITG levels, can show

$$\frac{\nu_k^>}{\nu_k^<} \approx \sqrt{\frac{m}{M}} \left(\frac{\eta_e}{\tau \eta_i} \right) \qquad \frac{S_k^>}{S_k^<} \approx \sqrt{\frac{m}{M}} \left(\frac{\eta_e}{\tau \eta_i} \right)^2$$

- **Result**: self-consistent calculation suggests back-reaction of ETG on DITG is weak (relative to self-interactions)
  - Probably even less important when zonal flows are included

Extended Fluctuation Ecology Model

- Combine results into a three-field "ecology"

$$\frac{\partial E^{>}}{\partial t} = \gamma^{>} E^{>} - \beta^{>}\left(E^{>}\right)^{2} - \rho E^{<} E^{>} + \gamma^{NL} E^{<} E^{>}$$

$$\frac{\partial E^{<}}{\partial t} = \gamma^{<} E^{<} - \beta^{<}\left(E^{<}\right)^{2} + \rho E^{<} E^{>} - \alpha U E^{<}$$

$$\frac{\partial U}{\partial t} = \alpha U E^{<} - \nu U$$

- Definitions: $E^{>}$, $E^{<}$, U are ETG, DITG, ZF intensities
  - γ represents linear growth, β represents nonlinear self-damping
  - ν represents collisional damping of zonal flow
  - γ^{NL} models η_e modulation, $\rho\, E^{<}E^{>}$ terms are coherent ETG – DITG interactions, and αUE terms are DITG – ZF interactions
  - ZF is "predator," DITG is "prey," ETG is a DITG "parasite"

Extended Ecology Model (cont.)

- Nontrivial fixed point of the system is at

$$E^{<} = \frac{\nu}{\alpha} \qquad E^{>} = \frac{\gamma^{>}}{\beta^{>}} + \frac{\left(\gamma^{NL} - \rho\right)}{\beta^{>}} \frac{\nu}{\alpha}$$

- DITG saturates via zonal flow shearing (no change from previous work)
  - result is independent of ETG presence / saturation level

- ETG saturation level is set by self-interactions, growth rate modulations, and DITG shearing
  - Growth rate modulation, DITG shearing can compete against each other

- Observation: possible for ETG, DITG to nonlinearly excite each other (via profile modulation and "negative viscosity" effects)

Main Conclusions

- Two-dimensional random shearing of ETG streamers by short wavelength DITG may be significant effect

- Modulations of η_e due to DITG-induced fluctuations of T_e can lead to a strong enhancement of ETG growth rate
  - Competes against (and may **dominate**) random shearing effect
  - Simulations of ETG w/o short-λ DITG may be of limited relevance

- Cross-field effects have different mass-ratio scalings

  $\longrightarrow$ need to use care in simulations with reduced mass-ratio

- Renormalization, ecology model suggest ETG back-reaction on DITG is weak

Some Future Directions

- Computational challenge: can a **supergrid model** for ETG turbulence be developed? How to incorporate slowly varying, large-scale shear flow in ETG theory and simulations

- Interactions between shortwave CTEM and ETG
  - Especially for ETG streamers, transport barrier equilibria
  - Note: short-λ CTEM (relevant shear field source) likely to survive in ITB

- What more be said about DITG shearing of streamers?
  - How strong is DITG shearing relative to K-H breakup rate?

- More detailed analysis of η_e modulation phenomena
  - Extend to possibility of $\eta_e + \delta\eta_e < \eta_e^c$ (submarginal turbulence)
  - Probablisitic studies: intermittency and nonlocality

- Use ideas from mathematical ecology: enviornmental variability, resource competition, niche overlap, …