Spectral modeling of nonlinear plasma evolution in compact toroids*

Preston Geren[1] and Loren Steinhauer[2]

[1]Geren Consulting Services, Shoreline, Washington
[2]University of Washington, Redmond, Washington

Abstract

In incompressible plasmas the magnetic and flow fields are divergence free. Therefore both **B** and **u** can be expanded in a divergence-free basis set of vectors. A complete basis set is provided by the Beltrami functions, which are solutions of the Beltrami equation $\tilde{N}\times\mathbf{Y} = \Lambda\mathbf{Y}$. For a domain with boundary condition of no normal component at the surface $\mathbf{Y}\cdot\mathbf{n} = 0$, these become discrete eigenvectors $\mathbf{Y}_k$ associated with the eigenvalues Λ_k. The equations of motion of a fluid plasma (either a single fluid or a two fluid) can be expressed as a coupled system of first order differential equations for the expansion coefficients $B_k(t)$ and $u_k(t)$. This approach can be used to model the nonlinear dynamics of plasmas in a compact geometry. The boundary condition ($B_n = u_n = 0$) is appropriate for modeling internal dynamics, i.e. the separatrix is regarded as a rigid boundary. The periodic domain can have arbitrary elongation.

For the periodic cylinder domain analytic solutions are available for the eigenvectors. Computed results will be presented for the periodic cylinder. These computations can be used to model the nonlinear development of well-known global modes such as tilting. In this way the nonlinear stability of a compact toroid plasma, including nonlinear mode-mode coupling can be investigated. To be addressed will be such important questions as spontaneous flow generation, the nonlinear stability of flowing equilibria, the ruggedness of the ideal invariants in the presence of a small dissipation, and the emergence of a quasi-steady turbulence spectrum. Also, the end-point of a relaxation can be determined. This should permit which of the several relaxation principles best represents the actual plasma behavior.

*Supported by USDOE grant No. DE-FG03-98ER54480