Implicit Lattice Boltzmann Schemes

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Abstract

The accurate numerical modeling of the nonlinear convective derivatives in MHD requires sophisticated treatments – as seen in the high finite element NIMROD code[1] or in a Newton-Krylov Jacobian-free algorithm[2]. Lattice Boltzmann schemes, on the otherhand, embed the macroscopic conservation equations into the higher dimensional kinetic space. For appropriate velocity-space lattice symmetries, the Chapman-Enskog limit of the lattice Boltzmann equation will recover the desired nonloinear macroscopic equations. The typical lattice Boltzmann BGK kinetic equation takes the form

$$\frac{\partial f_i}{\partial t} + \mathbf{e}_i . \nabla f_i = -\frac{1}{\tau}\left[f_i - f_i^{eq} \right] \ , \ \ \mathrm{i} = 1 \ldots \mathrm{b}$$

where $\mathbf{e}_i$ is the lattice velocity vector and τ is the relaxation rate at which f_i tends to the 'equilibrium' distribution function f_i^{eq}. The simplicity of the scheme, its replacement of the nonlinear macroscopic convective derivatives by simple linear advection and local algebraic noninearity in the f_i^{eq}, its ideal implementation on multiparallel processors are very attractive features. In particular, timings[3] on the vector *Earth Simulator* machine has resulted in not fully optimized 2D MHD lattice Boltzmann code running at over 3.6 Tflops/s.

These explicit Lattice Boltzmann schemes require small time steps and numerical stability is an issue with the standard polynomial expansion of f_i^{eq} in the macroscopic moments. Here we examine the Cao et. al[4] implicit scheme (which decouples the spatial grid from the velocity lattice)

$$f_i(\mathbf{x}, t+\Delta t) = f_i(\mathbf{x}, t) - \frac{\Delta t}{\Delta x}\left[f_i(\mathbf{x}, t) - f_i(x - \mathbf{e}_i \, \Delta x, t) \right] - \frac{\Delta t}{\varepsilon \tau}\left[f_i(\mathbf{x}, t+\Delta t) - f_i^{eq}(\mathbf{x}, t+\Delta t) \right]$$

No matrix inversions are required for this implicit scheme since the relaxed distribution function f_i^{eq} at time $t + \Delta t$ is immediately obtained from the moments of this implicit scheme.

An explicit lattice Botlzmann scheme is constructed for the collision of solitons of the KdV equation. These explicit results are compared to those arising from the implicit scheme with tests made on the allowed time steps.

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