Singularities and bifurcations in models for L–H transitions

R. Ball
e-mail Rowena.Ball@anu.edu.au
Department of Theoretical Physics, Australian National University
Canberra 0200 Australia

In this presentation I introduce some investigations of the structural properties of low-order models for plasma dynamical behavior using bifurcation and and stability analyses, and show how this approach can provide a guide for experimental design.

During the last decade efforts to model plasma mass dynamics around L–H transitions and concomitant oscillatory phenomena have concentrated on low-order or low-dimensional models. What drives this approach is the predictive power that a unified, low-order description of the bulk dynamics would have in designing and controlling confinement states. For example, a model that speaks of the shape and extent of hysteresis in the L–H transition would be an invaluable tool for economical management of confinement: given the number of parameters that *could* be varied around a hysteretic régime, it would be cheaper—i.e., save hundreds of cpu hours and/or expensive diagnostics—to know in advance which ones actually *do* affect the hysteresis, and which do not.

Low-order models are powerful because they are supported by well-developed theories and methodologies that give qualitative and global insight, such as bifurcation, stability, and symmetry theory. However, there are some rocky shores in the quest for a low-dimensional sub-space that captures the generic dynamics of L–H transitions [1, 2]. The heart of the matter is the relationship between the bifurcation structure of a dynamical model and the physics of the process it is supposed to represent. If we probe the nature of this relationship we find that degenerate singularities ought to reflect some matching physics—the point of onset of hysteresis, for example, or a fragile symmetry—or they are pathological. In the one case we can usually unfold the singularity in a physically meaningful way; in the other case we know that something is wrong and we can scrutinize our assumptions carefully. In both cases we have learned richly. Degenerate singularities provide opportunities to improve a model and its predictive capabilities.

Here I apply this approach to two models. The first is a model for L–H transitions [3], extended to include poloidal flow generation by ion orbit losses [4, 5]. As this rate is tuned the system passes from a hysteretic régime where stable shear flows draw energy from the turbulence via Reynolds stress decorrelation, to a qualitatively different hysteretic domain dominated by the radial electric field. The smooth path between these two extremes passes through an intermediate domain where transitions are oscillatory and hysteresis is locally forbidden. These results provide unification of previous disparate models for L–H transitions, and also suggest control and optimization strategies.

The second model highlights the role of the interaction between zonal flows and mean shear flows in the transfer of energy from the pressure gradient to the turbulence and generation of zonal flows [6]. A bifurcation analysis confirms the prediction that mean shear flow can inhibit zonal flow growth, and smooth unfoldings of degenerate singularities elicit the predictive possibilities in the model.

[1] R. Ball and R. L. Dewar. *Phys. Rev. Lett.*, 84(14):3077–3080, 2000.
[2] R. Ball and R. L. Dewar. *J. Plasma Fus. Res.*, 4:266–270, 2001.
[3] R. Ball, R. L. Dewar, and H. Sugama. *Phys. Rev. E*, 66:066408–1–066408–9, 2002.
[4] S.-I. Itoh and K. Itoh. *Phys. Rev. Lett.*, 60(22):2276–2279, 1988.
[5] K.C. Shaing and E.C. Crume. *Phys. Rev. Lett.*, 63(21):2369–2372, 1989.
[6] E-J. Kim and P. H. Diamond. *Phys. Rev. Lett.*, 90(18):185006–1–185006–4, 2003.