Semi-implicit Treatment of the Hall Term in NIMROD*

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Abstract

For nonlinear macroscopic behavior in magnetically confined plasmas, the two-fluid model has a range of time-scales that is at least $k\delta_i$ larger than in the resistive MHD model, where δ_i is the ion skin depth and k is the largest relevant wavenumber. Since the MHD system is stiff and important two-fluid effects occur at spatial scales that are smaller than the ion skin depth, modeling two-fluid behavior with numerical simulation is quite challenging. Fortunately, the macroscopic behavior does not depend on how force-balance is achieved in the electron momentum equation, so time-steps need not be limited to whistler and kinetic Alfven wave propagation times. Nonetheless, the solution at every step must respect the electron force balance at the spatial scales of the nonlinear macroscopic dynamics. With the relatively small magnetic perturbations encountered in fusion plasmas, the dominant part of the stiffness is related to linear behavior, even in nonlinear conditions. Thus, the semi-implicit method has proven effective for solving the nonlinear MHD system [1], and a similar approach has been applied to address the Hall term in an extended-MHD description [2]. Here, we describe modifications to the algorithm presented in Ref. [2] that make it suitable for a high-order finite element spatial representation with C^0 continuity [3]. The symmetric fourth-order differential operator used to stabilize the numerical advance is implemented through an auxiliary vector field, so that all terms in the weak form of the magnetic advance are integrable for the space of C^0 piecewise polynomials. The resulting algebraic system is nonsymmetric, but in the NIMROD implementation [http://nimrodteam.org], we have found the SuperLU sparse parallel linear solver library [http://crd.lbl.gov/~xiaoye/SuperLU/] to be very effective for solving the resulting matrices. The numerical algorithm and NIMROD implementation are tested on computations of simple whistler waves in a doubly periodic domain and in bounded domains. Results computed with the fourth-order semi-implicit operator are compared with numerical dispersion relations and with results from a simpler algorithm using a second-order differential operator. Initial applications to inhomogeneous equilibria are also presented.

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