Nonlinear Generalized Master Equations: building a mathematical formalism to describe transport in the presence of critical thresholds

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In the last few years, it has been often suggested that some of the following elements do probably play a role in setting the dynamics of plasma anomalous transport: collisional diffusion, turbulence, critical thresholds (gradients and/or characteristic lengths), avalanches, self-similarity, non-Gaussian probability distribution functions, long-term temporal correlations, power-laws and self-organized criticality. If this is indeed the case, it may have dramatic consequences on the reliability of the extrapolation of current plasma confinement results to larger devices. In spite of this, most of transport analysis is still done by using sets of transport equations that neglect the importance of these elements, since they are based instead on classical diffusive operators for both density and temperatures that cannot accomodate most of them in a consistent way.

In this work we will present a first step towards building a mathematical framework able of describing transport in a system in which these elements dominate the dynamics [1]. The formalism is based on a nonlinear variation of the well known Continuous-Time Random Walk (CTRW), a concept introduced in the late 50s that have been applied to many problems in Physics and Chemistry. We will show how to derive a Generalized Master Equation (GME) that provides with an ensemble-averaged description of this modified CTRW. We will also show that, in the fluid limit, this GME connects to a description of transport in terms of fractional derivative equations (FDE). The possibilities of building a new class of transport codes based on these tools will be discussed, and some applications to simple systems will be shown.

[1] B.Ph. van Milligen, R. Sánchez and B.A. Carreras, *Probabilistic transport models for transport in fusion plasmas*, in press, Physics of Plasmas (2004).