The discontinuous Galerkin method applied to SOL transport*

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Abstract

The discontinuous Galerkin (DG) method[1] is becoming popular for solving systems of hyperbolic conservation laws

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{F_x}}{\partial x} + \frac{\partial \mathbf{F_y}}{\partial y} = \mathbf{S}$$

because it combines some of the best features of finite element and finite volume methods. The method is distinguished by expanding the fields in basis functions that are polynomials (possibly high order) within elements and discontinuous between elements. Here a two-dimensional nodal Lagrange-polynomial basis[2] is used in which the polynomial degree of each element can be individually set from 1-10. Each element can also be individually refined with no restrictions on hanging nodes. The key to numerical stability and convergence is in the specification of the numerical flux functions which couple the solution between elements and with the boundary. In addition to a hyperbolic conservation law it is frequently useful to solve a coupled elliptic (Poisson) equation. This ability has recently been added which requires the solution of a large linear system in parallel. The DG method requires that a second order elliptic problem[3] be expressed, at least formally, in terms of first order systems. By the appropriate choice of numerical flux functions, these additional variables can be eliminated before the linear system is solved. For these flux functions the method is called a local discontinuous Galerkin method. The reduction comes at the price of creating additional shadow elements in the parallel decomposition so that neighbors and neighbors of neighbors along processor boundaries are shared. We are using PETSc for the linear solve. The time advance is currently explicit (RK) however we have tested an implicit Crank-Nicholson solver on a simple convection problem. We will describe the application this code to the transport of blobs[4][5][6] in the scrape-off-layer (SOL).

References

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