Extending the Peeling-Ballooning Model of ELMs: Toroidal Rotation and 3D Nonlinear Dynamics

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International Sherwood Fusion Theory Meeting

Missoula MT, 26 April 2004

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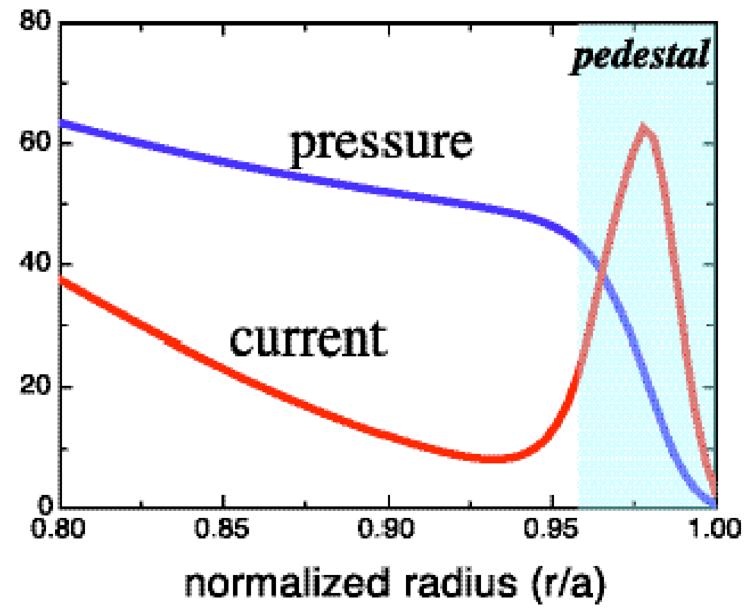
Motivation and Background

- ELMs and the edge pedestal are key fusion plasma issues
  - “Pedestal Height” controls core confinement and therefore Q
  - ELM heat pulses constrain plasma facing materials



- Peeling-Ballooning Model of ELMs - significant successes
  - ELMs caused by intermediate wavelength (n~3-30) MHD instabilities
    - Both current and pressure gradient driven
    - Complex dependencies on ν_*, shape etc due to bootstrap current and “2nd stability”
  - Successful comparisons to experiment both directly and in database studies
- Need to understand sources and transport to get profile shapes (“pedestal width”)
- Rotation and non-ideal effects to precisely characterize P-B limits, nonlinear dynamics for ELM size

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Outline

- Toroidal Flow Shear
  - Conventional ballooning theory (1D)
  - How toroidal rotation complicates the theory (1D⇒2D)
  - Eigenvalue formulation and resolution of ‘discontinuity’
  - Impact on peeling-ballooning modes in the tokamak edge region
- Initial Nonlinear ELM Simulations
  - General challenges
  - Single fluid extended MHD (NIMROD)
  - 2 fluid reduced Braginskii (BOUT)
    - Fast burst of radially propagating filaments
- Summary and Future Work

Conventional ballooning mode theory

We seek solutions $\xi \sim e^{-in\phi}$ and consider large toroidal mode number, n

the largest operators in the ideal MHD equations are then those related to field line bending:

$$\boldsymbol{B}\cdot\nabla = \frac{B_\theta}{r}\left(\frac{\partial}{\partial\theta} - inq\right)$$

In order to balance terms, we must ensure $(\boldsymbol{B}.\nabla)\xi \sim 1$, ie

$$\xi = \hat{\xi}(\psi,\theta)\mathrm{e}^{-in[\phi - q(\psi)(\theta-\theta_0)]} \qquad \frac{\partial\hat{\xi}}{\partial\theta} \sim 1$$

We can then write the full ideal MHD equations (schematically) in the form:

$$L\left(\frac{\partial}{\partial\theta} - inq, \frac{-i}{n}\frac{\partial}{\partial\psi}; \frac{\partial}{\partial t}\right)\hat{\xi}e^{-in[\phi - q(\theta-\theta_0)]} = 0$$

For large n and taking solutions $\sim e^{\gamma t}$, we then derive the ballooning equation:

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta-\theta_0); \gamma(\theta_0)\right)\hat{\xi} = 0$$

A 2nd order ODE, 1D

Higher order theory $\Rightarrow$ choose θ_0 to maximize $\gamma(\theta_0)$

Incorporate toroidal flow shear

- We introduce a toroidal flow: $\mathbf{v} = R^2\Omega(\psi)\nabla\phi$
  - introduce the ordering:

$$R\Omega / C_s \sim n^{-1} << 1 \qquad \frac{1}{q'}\frac{\partial(R\Omega / C_s)}{\partial\psi} \sim 1$$

- The principle effect is a Doppler-shift of the mode frequency, so we have

$$L\left(\frac{\partial}{\partial\theta} - inq, \frac{-i}{n}\frac{\partial}{\partial\psi}; \frac{\partial}{\partial t} - in\Omega(\psi)\right)\xi = 0$$

- To remove rapid radial variations, we introduce a time-dependent eikonal:

$$\xi(\psi,\theta,\phi;t) = \hat{\xi}(\psi,\theta;t)e^{-in[\phi - q(\theta-\theta_0)+\Omega t]} = 0$$

*Cooper, PPCF **30**, 1805 (1988)*

- Then the “ballooning” equation with flow becomes:

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta - \theta_0 + \Omega' t / q'); \frac{\partial}{\partial t}\right)\hat{\xi} = 0$$

- The presence of time in the coefficients$\Rightarrow$we can no longer assume eigenmode solutions $\sim e^{\gamma t}$: this becomes a 2-D initial value system to solve

The weak flow shear limit: relation to conventional ballooning theory

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta - \theta_0 + \Omega' t / q'); \frac{\partial}{\partial t}\right)\hat{\xi} = 0$$

- Let us define $\tau = \Omega' t/q' - \theta_0$; $\partial/\partial t \rightarrow (\Omega'/q')\, \partial/\partial\tau$ and seek Floquet-type solutions:

$$\hat{\xi}(\theta, t) = e^{\gamma t}\hat{\xi}(\theta - \tau, [\tau])$$

[…]⇒periodic function

- We then have

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta - \tau); \gamma - \frac{\Omega'}{q'}\frac{\partial}{\partial\tau}\right)F = 0$$

- For low flow shear, this has a separable solution of the form

$$\hat{\xi} = A([\tau])F(\theta - \tau)$$

- where F satisfies the conventional ballooning equation with $\theta_0 \rightarrow \tau$ and eigenvalue $\gamma(\tau)$

- The boundary condition that A be periodic provides:

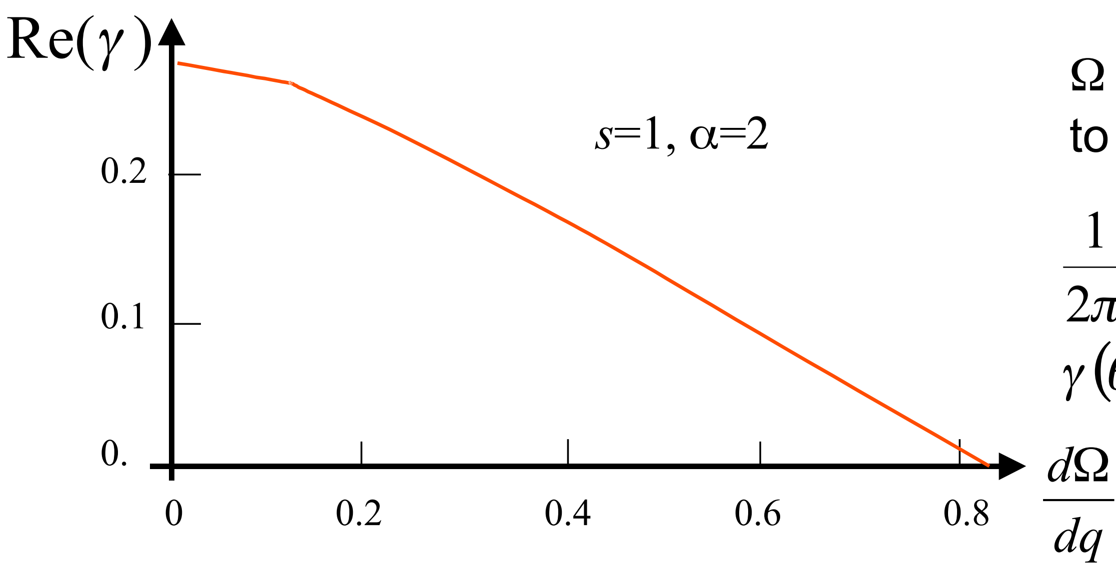
$$\gamma = \frac{1}{2\pi}\oint \gamma(\theta_0)\, d\theta_0$$

Waelbroeck and Chen Phys Fluids B3 601 (1991)

The time-dependent-eikonal results

- All previous studies of the effect of flow shear on ballooning modes have been based on this type of analysis
  - valid in the limit $n \to \infty$
  - Then, in the absence of flow we choose θ_0 to maximize $\gamma(\theta_0)$
  - For infinitesimally small flow, we average $\gamma(\theta_0)$ over θ_0
  - There is a discontinuity in the theory, which we would like to understand
  - Suggests that flow shear could in principle have a big effect on ballooning modes

- For example: Miller et al (s–α equilibrium):



Ω and γ normalised to Alfvén frequency

$$\frac{1}{2\pi}\oint \gamma(\theta_0)\, d\theta_0 = 0.31$$

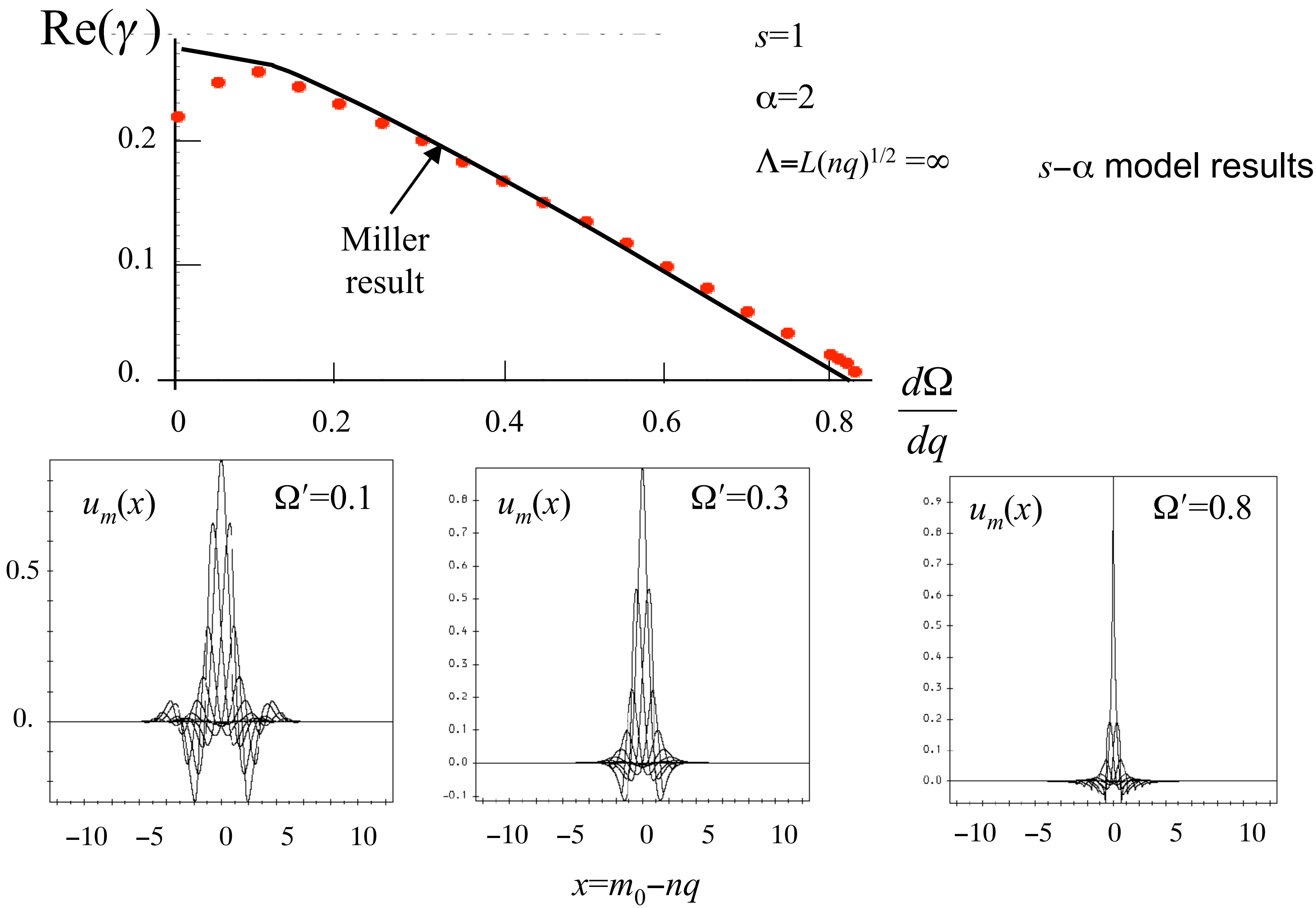
$$\gamma(\theta_0)_{max} = 0.60$$

Miller et al Phys Plasmas ***2*** *3676 (1995)*

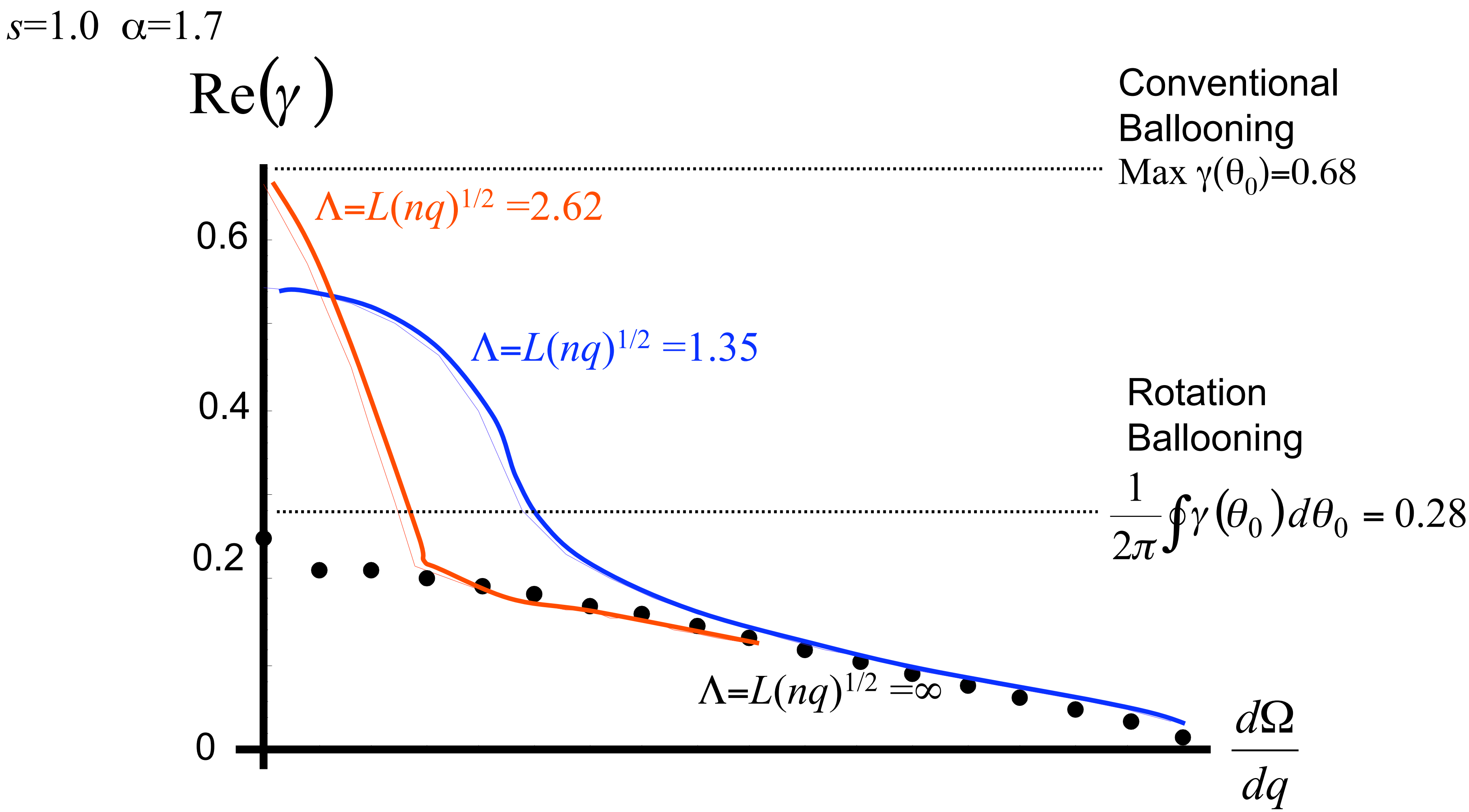
Flow shear and the Eigenmode Formalism

- We would like to develop an eigenmode formalism for the effect of flow shear on ballooning modes for a number of reasons:
  - Working with an eigenmode formalism allows us to smoothly connect to the conventional ballooning modes as $\Omega' \rightarrow 0$ and understand this ‘discontinuity’
  - It allows us to calculate the radial eigenmode structure
  - Provides an eigenmode frequency
  - Enables consideration of finite n corrections
  - Permits flow shear to be incorporated into ELITE (an eigenmode code)
    - Test impact on P-B modes in experimental equilibria

The $n \rightarrow \infty$ eigenmode growth rates agree with Miller et al;
The radial mode width reduces with increasing flow shear

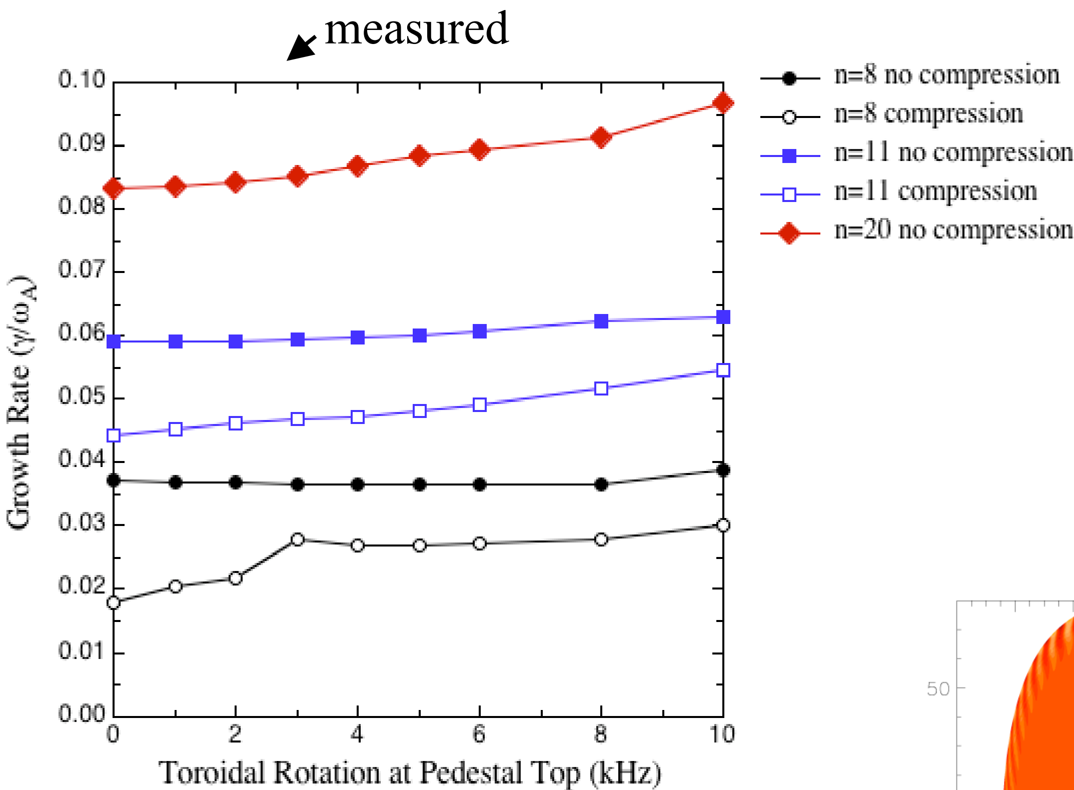


Increasing n, growth rates tend to Miller result at lower Ω' $\Rightarrow$for n$\rightarrow\infty$, rapid change in γ for infinitesimally small flow



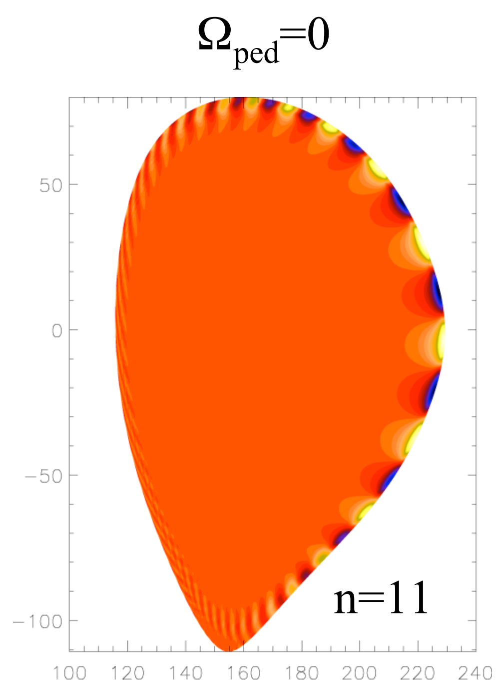
Discontinuity resolved, transition physics similar to *[Waltz 98]* case

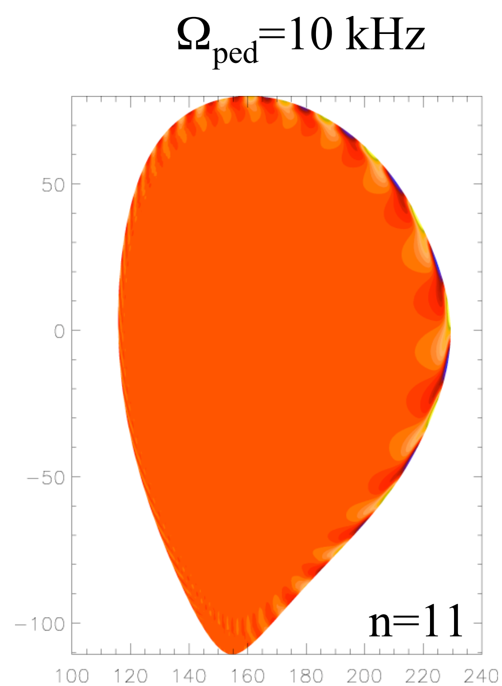
Effect on Growth Rates is Modest in Experimental Equilibria, Mode Structure Does Change



Rotation Shear on P-B Modes:

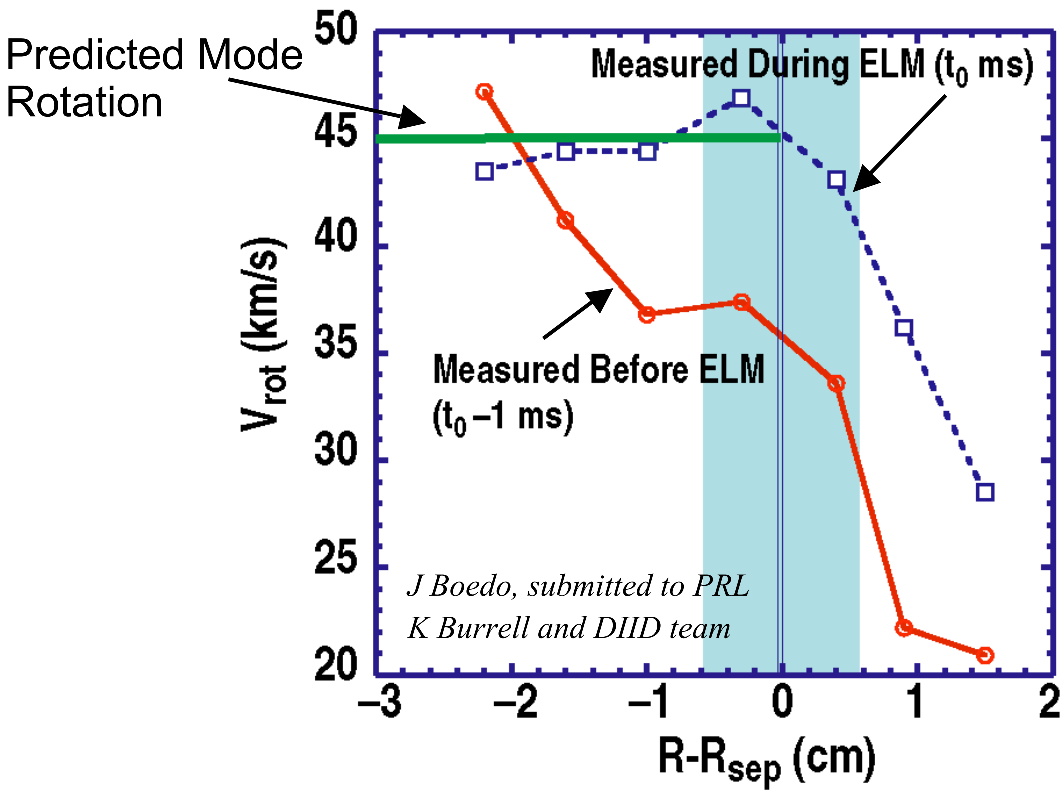
- Stabilizing near marginality
- Finite n and large γ dramatically reduce effect
- Does not measurably change expected ELM onset time

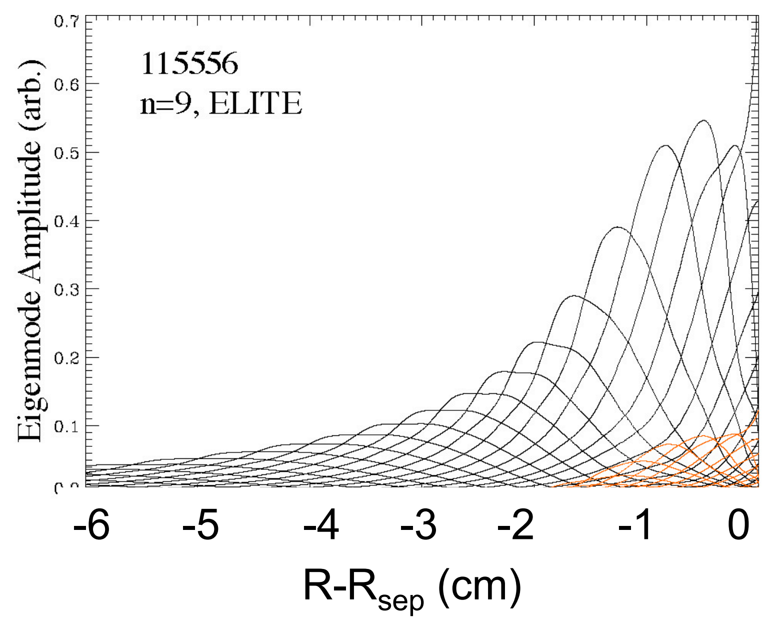




- Mode structure strongly altered
  - Narrowing and phase changes
  - May impact dynamics, ELM size

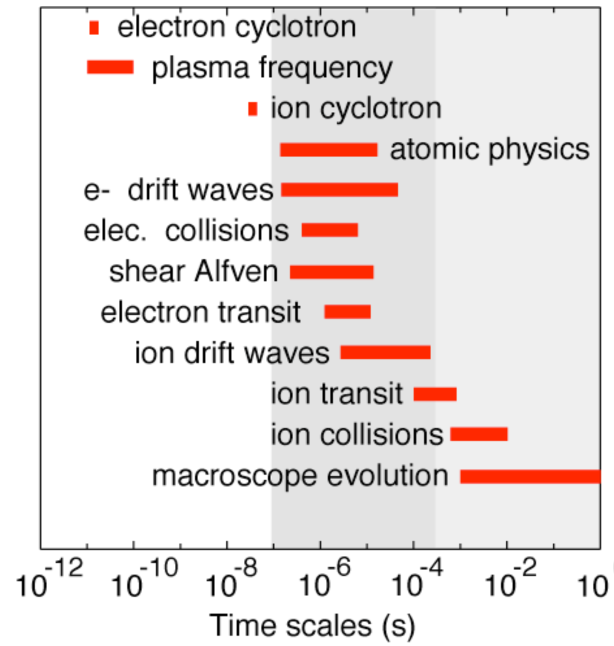
Calculated Mode Rotation Agrees with Observation during ELM



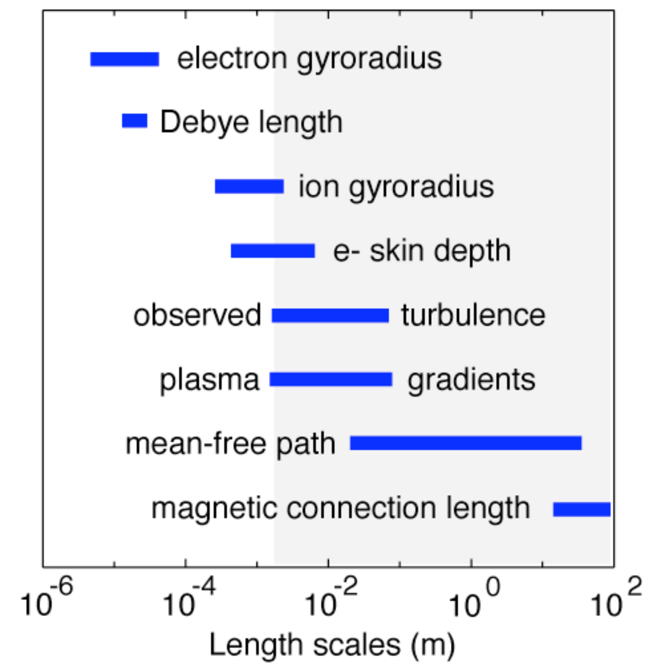


- Measured rotation profile changes from strongly sheared just before the ELM to ~flat at **~45km/s** across pedestal region at ELM onset
- Study with ELITE finds peeling-ballooning unstable just before ELM - most unstable mode (max γ/ω_*) is n=9
- Calculated frequency for this n=9 mode is ω/ω_A=0.0082, **V_{rot}=45km/s**
- Suggests “locking” of pedestal region to the mode during initial phase of ELM crash

Initial Nonlinear Edge/Pedestal Simulations



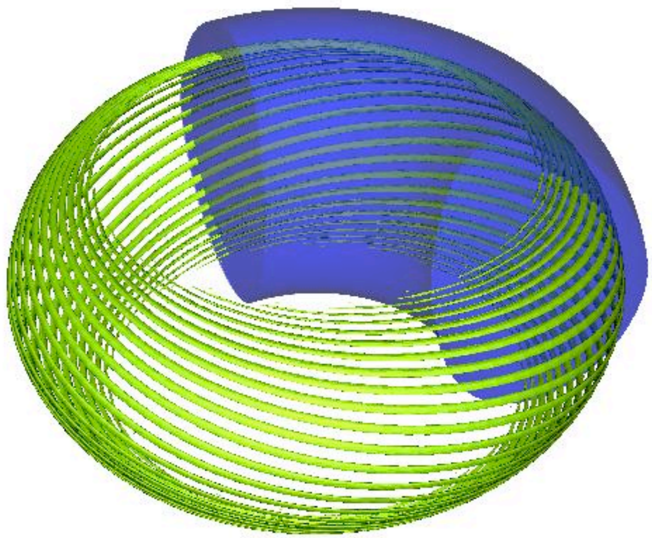
- Many challenges for nonlinear simulations of the edge region
  - Broad range of overlapping scales and physics (L-H transition, sources and transport, ELMs, density limit..)
  - Many techniques and formulations used to simplify core simulations are not applicable in edge
  - Long term goal is to unite full set of physics into massive scale simulations



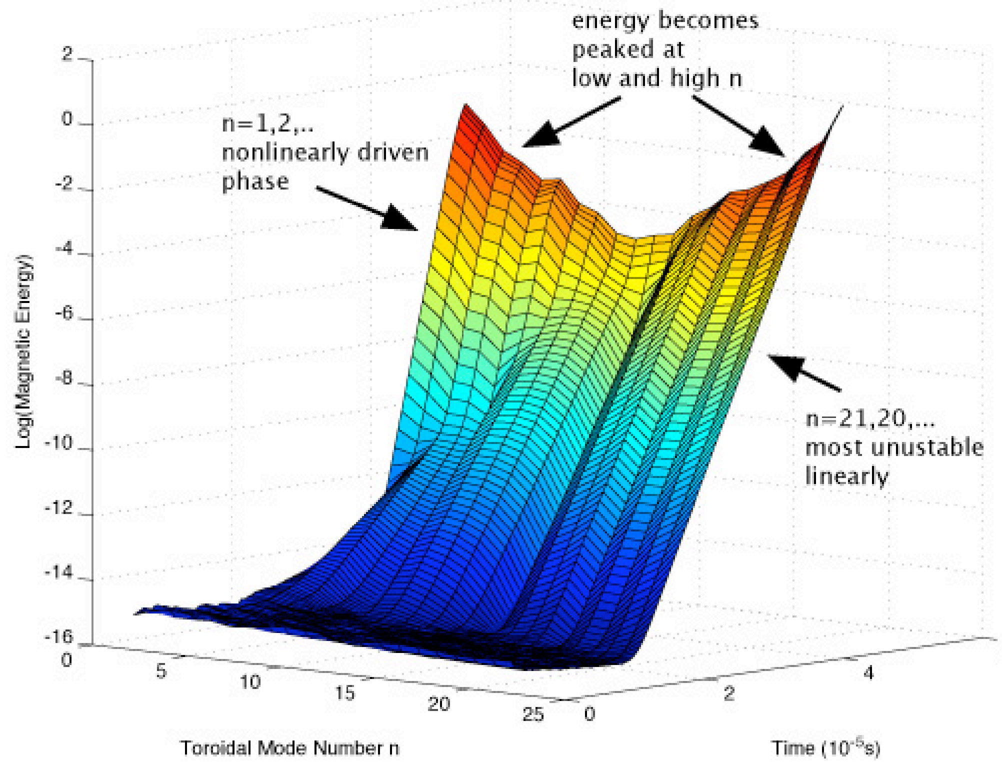
- Here we focus on the fast timescales of the ELM crash event itself
  - Goal is to understand physics determining ELM size
  - Initialize with P-B unstable equilibria, evolve dynamics on fast timescales

- Single fluid and reduced Braginskii 2 fluid simulations

Initial Nonlinear Studies with NIMROD find Linear Ballooning Structure and Nonlinear Coupling to Low n's



n=21 linear mode structure

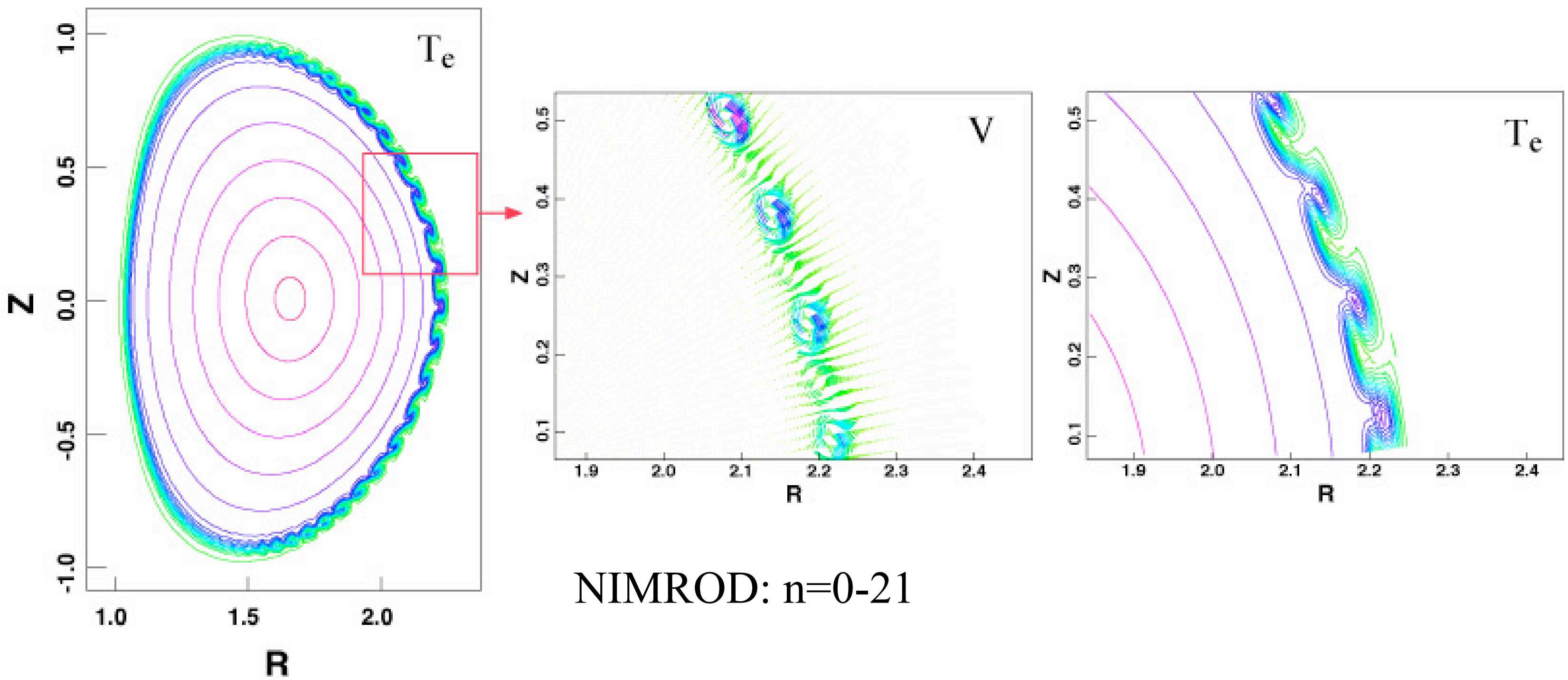


Single fluid extended MHD simulations carried out with NIMROD *[Sovinec 03],* including rapid transition to high resistivity "vacuum" region

Linear spectrum approximately consistent with expectations from ideal calculations. Low n modes initially have lower linear growth rate compared to higher n.

Using equipartition of energy as initial condition, the higher n modes grow linearly to large amplitude and their beating nonlinearly drives the lower n modes to large amplitude.

Filamentary Vortices Form in Early Nonlinear Phase



NIMROD: n=0-21

The coupled modes form complex structures in flow velocity and Temperature, among other fields.

Areas of high temperature are seen flowing out

Work ongoing to extend simulations further into nonlinear phase

“Ultraviolet catastrophe” - numerical methods and two fluid diamagnetic/FLR effects

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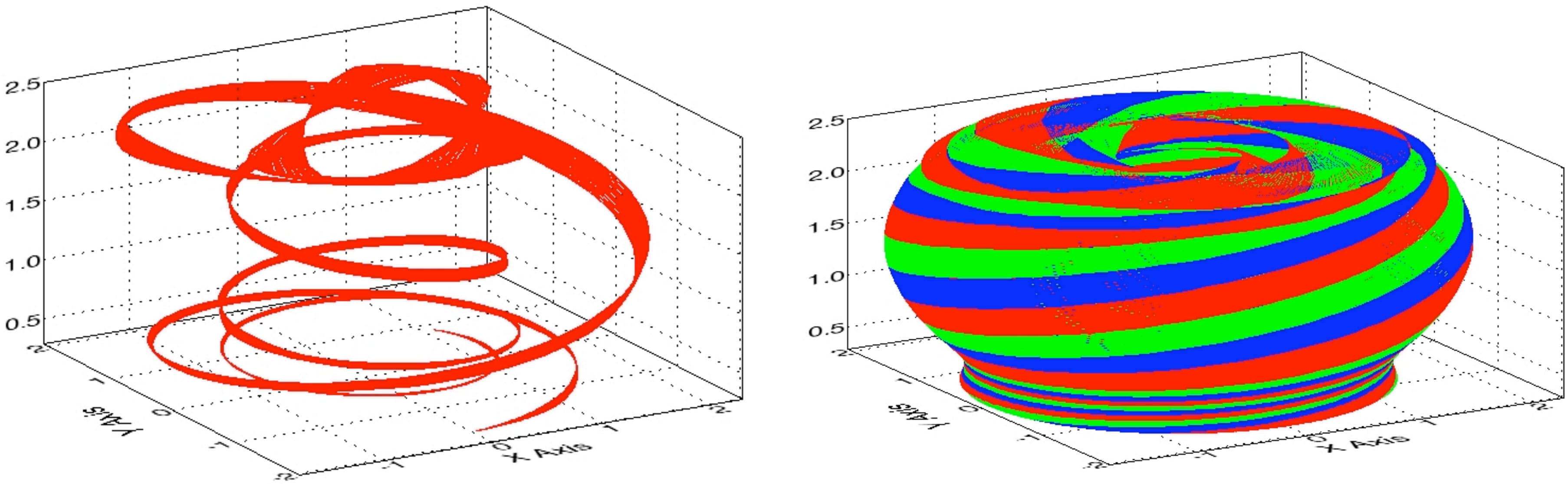
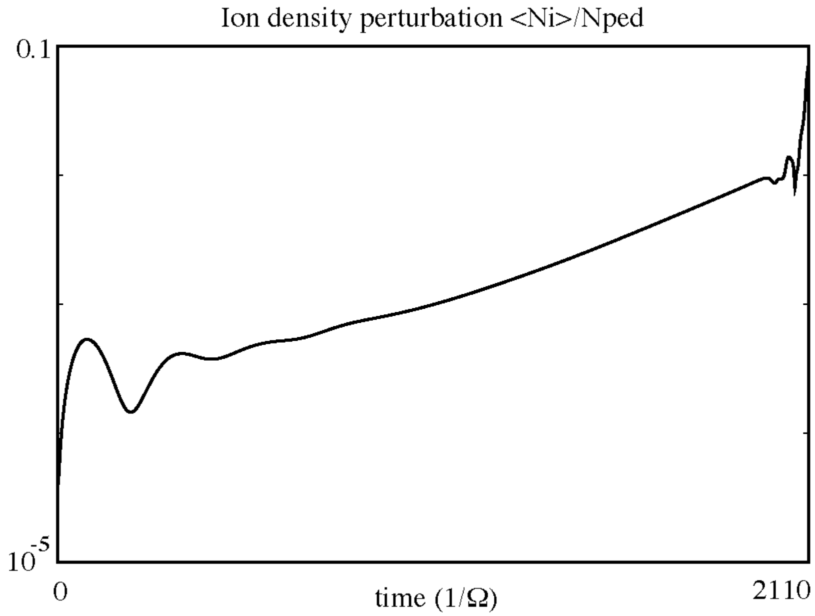
BOUT Simulation Geometry

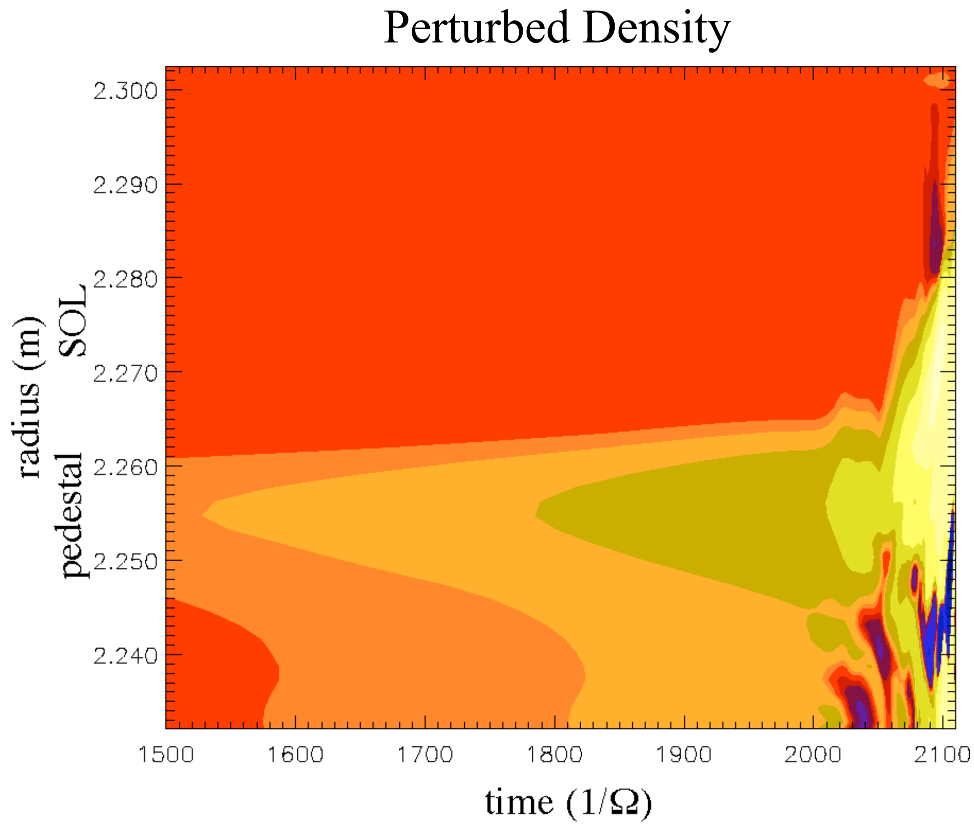


• BOUT incorporates 2 fluid/diamagnetic physics and uses field line following coordinates

- –Bundle of lines (left) wraps around 2π poloidally
- –A group of such bundles (right) spans the flux surface
- –For ELM simulations, generally go 1/5 (or 1/2) of the way around the torus, ie treat Δn=5 (or Δn=2), n=0,5,10…~105, 0.9 <~ Ψ ~< 1.1

Fast Outward Burst (ELM?) Seen in BOUT Simulations





- High density (small ELM), DIII-D LSN case, $0.9 < \psi < 1.1$
- Initial linear growth phase, then fast radial burst begins at t~2000, can see positive density (light) moving into SOL and negative density perturbations near pedestal top

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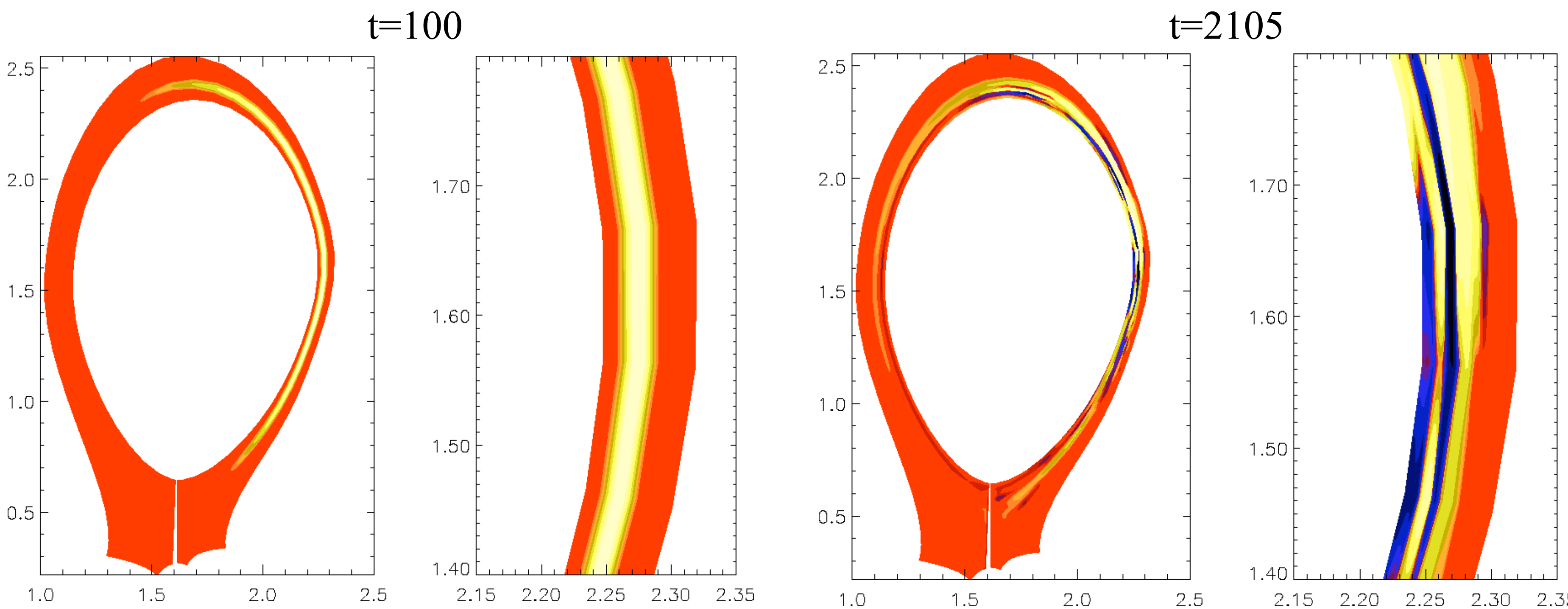
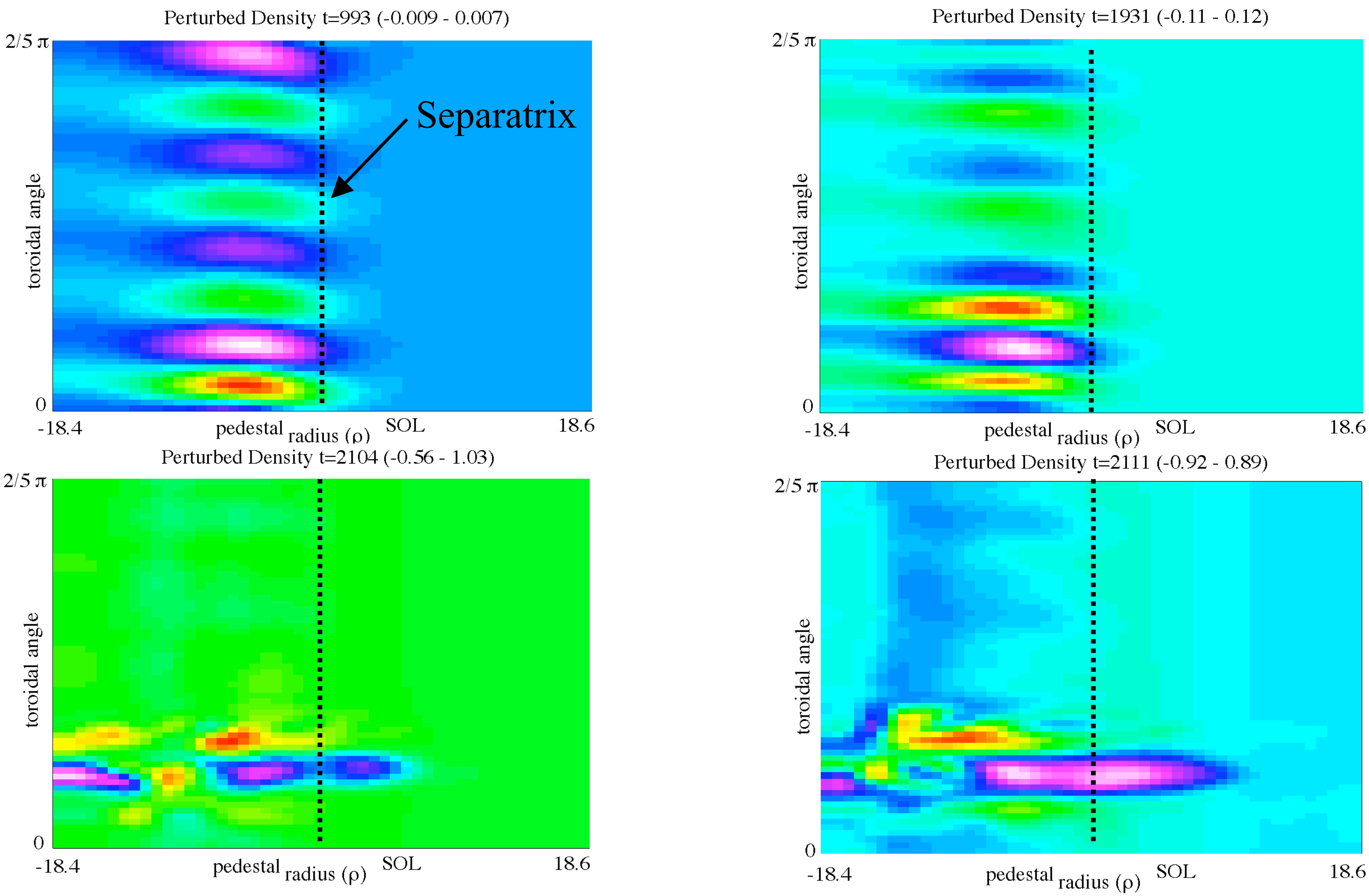
Radial Burst Extended Along the Field, But Irregular



- Plots show projections of bundles of field lines onto the RZ plane - field lines extend into and out of page (radial vs parallel)
- Linear phase: Mode has ~expected characteristics of linear mode, radial and poloidal extent, n~20, γ/ω_A~0.15
  - Reducing gradients slightly stabilizes the mode- abrupt onset near P-B boundary
- Fast Burst: Filaments extended along the field, but irregular

Fast ELM Burst Shows Toroidal Localization



- R,ϕ plots on outer midplane. Linear phase, n=20. Burst occurs asymmetrically at a particular toroidal location

Summary

- Peeling-ballooning model has achieved a degree of success in explaining ELM onset and a number of ELM characteristics
  - Extend to include rotation and nonlinear, non-ideal dynamics
- Toroidal rotation shear included in ELITE
  - Discontinuity in previous studies removed via eigenmode formulation
  - Small effect on predicted ELM onset, but significant modification of mode structure (narrowing and phase)
  - Encouraging comparisons with fast CER observations
- Initial 3D nonlinear ELM simulations carried out with NIMROD and BOUT
  - Early structure and growth similar to expectations from linear P-B
  - Radially propagating filamentary structures, similar to observations (MAST)
  - NIMROD: significant nonlinear driving of low n modes
  - BOUT: Explosive burst propagates outward, negative density and T bursts propagate in to psi~0.9, significant toroidal localization and irregular filamentary structure

Future Work

- Initial set of simulations provide insight into linear and early nonlinear phases - comparisons to experiment underway
  - Improved numerical techniques and BC's to extend duration of simulations

- Move on to larger problems:
  1) Toroidal scales – For some types of ELMs, need full torus (n=1 to $\sim\rho_i$)
  2) Radial scales – extend to wall and further into core
  3) Time scale – Include sources and drive pedestal slowly across P-B boundary

- Scale overlap and close coupling with pedestal formation (L-H) physics, inter-ELM transport and source (including atomic) physics
- Need optimal formulations, efficient numerics and large computational resources (SciDAC?)

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