The Complete Set of Casimir Constants of the Motion in Magneto-Hydrodynamics*

Elie Hameiri
Courant Institute of Mathematical Sciences, NYU,
251 Mercer St., New York, NY 10012

Abstract

MHD is a Hamiltonian system, the evolution of which can be given a non-canonical formulation with the use of a Poisson bracket involving only Eulerian variables of the plasma fluid.[1] Casimirs are functionals for which the bracket vanishes, regardless of which Hamiltonian function is used (even non-MHD). In particular, they also constitute constants of the motion for the MHD system.

The practice so far has been to find Casimirs by formulating a variational principle for the plasma evolution, typically a Lagrangian integral using Lagrangian variables of the fluid, and to apply Noether's theorems. This procedure is problematic in a number of respects: 1. Not every Hamiltonian system has a natural Lagrangian integral; 2. The Noether approach is complicated to carry out and one does not know if all constants of the motion were found; 3. Casimirs arise in the Eulerian description and form a special class of constants of the motion, easier to find than other conserved quantities, so their special properties should be used.

Our approach is to derive from the Poisson brackets a set of functional equations all Casimirs need to satisfy. We then determine the general solution of the spatial dependence of the various functional derivatives involved. From this, a limited number of *possible* forms of every Casimir can be found. A direct check pinpoints the Casimirs. We get a full set of the Casimirs for a configuration with nested magnetic flux surfaces, as well as other configurations (which have many fewer Casimirs). We identify one velocity-dependent Casimir which exists in all configurations and can help improve a classical stability criterion.[2]

*Work supported by US Department of Energy Grant No. DE-FG02-86ER53223.

1. P. Morrison and J. Greene, Phys. Rev. Lett. **45**, 790 (1980).

2. E. Frieman and M. Rotenberg, Rev. Mod. Phys. **32**, 898 (1960).