

Nested Grids in Lattice Boltzmann Representation of Resistive MHD

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Abstract

Lattice Boltzmann methods provide a novel kinetic algorithm for nonlinear macroscopic systems governed by conservation laws. The simplest form of the Lattice Boltzmann technique utilizes the linearized BGK collision operator. The algorithm to temporally evolve the particle distribution function on a discrete lattice involves (a) free streaming from spatial node to spatial node, and (b) local (macroscopically nonlinear) collisions. The strengths of lattice Boltzmann methods lie in 1) the avoidance of the nonlinear convective derivatives that typically consume most of the computation time in standard finite difference, finite volume and finite element simulations, 2) the ability to model systems in complex physical geometries, 3) the incorporation of newly developing methods in non-uniform and adaptive meshes that focus computational effort around structures of interest such as current sheets and shocks, and 4) the simple computational algorithms which lend themselves to ideal parallelization on multiple PE platforms.

Here we apply the lattice Boltzmann method to resistive MHD. Using a scalar distribution function to recover the density, velocity field and momentum flux tensor and a vector distribution function to recover the magnetic field and magnetic flux tensor decouples the viscosity from the resistivity. This scalar-vector distribution function approach also allows greater freedom in choosing expansion coefficients for the distribution functions resulting in better numerical stability. The incorporation of non-uniform grid methods resolve the regions containing shocks and current sheets with higher precision so that dynamic behavior on a fine scale can be examined.

Results for both one-dimensional and two dimensional resistive MHD simulations are presented. The one-dimensional simulations use a reduced form of the MHD equations containing a single component of the velocity field and two perpendicular components of the magnetic field. The formation of shocks and the exchange of kinetic and magnetic energies are examined. Two-dimensional simulations are presented for the Orszag-Tang vortex, a set of simple two dimensional initial conditions characterized by two alternating X and O points of vorticity and four varying current layers. These initial conditions give rise to most of the notable features of MHD turbulence such as magnetic reconnection and the formulation of jets.