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On electron heat transport in spherically symmetric geometry*

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Abstract

Electron heat transport plays an important role in many plasmas ranging from astrophysics to control fusion. It is well known that due to specifics of Coulomb collisions, the applicability of the classical Spitzer-Harm expression for electron heat flux is limited to very small ratios of the electron mean free path to the scale length of electron temperature variation, $\square < 0.01$. At larger $\square$ nonlocal kinetic effects play a very important role in heat transfer and alter the Spitzer-Harm expression. Numerous analytical attempts to improve the Spitzer-Harm expression have focused on the study of slab geometry. Here we report the results of our investigation of nonlocal effects of electron heat transport for the spherically symmetric case. We start with a kinetic equation for the electron distribution function $f(\mathbf{v},\mathbf{r})$ neglecting electron-ion energy exchange and assuming zero averaged ion velocity. We assume spherical symmetry in $\mathbf{r}$ space and introduce local spherical coordinates in velocity space, v and $\square$, where $\square = \mathbf{r}\ \mathbf{v}/rv$, so that $f(\mathbf{v},\mathbf{r}) = f(v,\square,r)$. For the heat flux going from the origin to infinity we notice that keeping only convection term in kinetic equation implies $f(v,\square,r) = f(v, r^2(1 \square \square^2))$, which shows that $f(v,\square,r \quad \square)$ can be strongly peaked at $\square = 1$ solely due to geometry.

To analyze the electron kinetic equation for arbitrary $\square$ we follow Ref. 1 and introduce the self-similar variable $\mathbf{w} = \mathbf{v}(m/2T(r))^{1/2}$ and use the following anzatz for electron distribution function $f(v,\square,r) = F(w,\square)(T(r))^{\square\square}$. Hereafter e, m, $T(r)$, and $n_e(r) \square (T(r))^{3/2 \square\square}$ are the electron charge, mass, effective temperature and density, and $\square$ is an adjustable parameter. For T(r) profiles such that $T^{\square+1/2}(d\ln T/dr) = \text{const.}$ we find $\square = \square T^2(d\ln T/dr)/(2\square e^4 \square n_e) = \text{const.}$, where $\square$ is the Coulomb logarithm. For such $T(r)$ profiles electron kinetic equation can be written as follows

$$\square w\left\{\square(\square F + (w/2)\square_w F) \square (\square + 1/2)(1 \square \square^2)\square_w F\right\} \square (\square_E/2)\left(\square\square_w F + (1 \square \square^2)w^{\square 1}\square_\square F\right)$$
$$= (1/4)\left\{C_{ee}(\mathbf{w},F) + Z_{eff}\, w^{\square 3}\square_\square\left((1 \square \square^2)\square_\square F\right)\right\}, \qquad (1)$$

where $C_{ee}(\mathbf{w},f) = \square_{w_\square}\square\left\{F(\mathbf{w}')\square_{w_\square}F(\mathbf{w}) \square F(\mathbf{w})\square_{w'_\square}F(\mathbf{w}')\right\}U_{\square\square}d\mathbf{w}'$, $U_{\square\square} = \left(u^2\square_{\square\square} \square u_\square u_\square\right)u^{\square 3}$, $\mathbf{u} = \mathbf{w} \square \mathbf{w}'$, $\square_E = eET/(2\square e^4 \square n_e) = \text{const.}$, and E(r) is the ambipolar electric field which can be found from a zero electron flux condition. Note that the conservation of Spitzer-Harm heat flux through a spherically symmetric surface gives $Q \square r^2 T^{5/2}(dT/dr) = \text{const.}$, resulting in $T(r) \square r^{\square 2/7}$ and corresponding to $\square = \square 4$.

We solve Eq. (1) for different regions of w and then match these solutions at the boundaries of these regions. For large w we find that unlike the slab case[1], peaking of $f(v,\square,r)$ at $\square = 1$ requires taking into account not only streaming but also pitch-scattering terms. As a result at large w we find $F(w,\square) \square w^{2(\square+3)}\exp\left([(3+2\square)2\square/(1+Z_{eff})](1 \square \square)w^4\right)$ and, like in slab case[1], the heat flux which would stay constant for Spitzer-Harm heat conductivity ($\square = \square 4$) is, actually diverges logarithmically, $Q \square \square F(w,\square)w^5 dw d\square \square \square w^{2\square+7}dw \square \square w^{\square 1}dw$.

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