

Two-Fluid Vector Equations For MHD Numerical Simulations*

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The usual derivations of reduced equations for magnetohydrodynamic (MHD) and two-fluid descriptions of plasmas are based on reduced MHD descriptions [1] in which the plasma flow velocity \mathbf{V} and the magnetic field \mathbf{B} are represented by scalar and vector potentials. In these reduced MHD descriptions the compressional Alfvén waves are eliminated analytically by the conditions that, to lowest order in k_{\parallel}/k_{\perp} , the perpendicular flow is incompressible and $\tilde{P} + \tilde{B}_{\parallel}B_0/\mu_0 = 0$.

However, the modern MHD simulation code NIMROD [2] advances the vector fields \mathbf{V} and \mathbf{B} directly — in order to avoid high-order spatial derivatives which are prone to numerical errors, to be able to consider free-boundary equilibria with embedded separatrices, and to more easily satisfy the boundary conditions at a bounding surface. In order to eliminate a time-step restriction due to compressional Alfvén waves, the NIMROD code uses a semi-implicit algorithm [3] in which the operator describing the waves embodied in the linearized MHD force operator are put on the left of the flow velocity evolution equation [4]. This procedure allows a much larger time-step — many times longer than the compressional Alfvén time scale and comparable to the shear-Alfvén wave and hence instability time scale. It also facilitates simulations of the nonlinear evolution to saturation of regular and neoclassical tearing modes which occurs on time scales that are fractions of the magnetic skin diffusion time scale and of the order of the plasma energy confinement time scale.

In order to facilitate inclusion of two-fluid effects, and in particular the diamagnetic flow effects, in the NIMROD code, we have begun developing vector field forms of the two-fluid equations that include kinetic-based closure relations (for the heat flux \mathbf{q} and parallel viscous stress π_{\parallel}) and the gyroviscosity effects [5], without eliminating the compressional Alfvén waves and without using vector and scalar potential representations of \mathbf{V} and \mathbf{B} . This poster will report progress in this regard, and illustrate how electron and ion diamagnetic flow effects enter in both the reduced MHD and vector field forms of the equations, and lead to their respective effects on pressure-gradient-driven and tearing-type current-gradient-driven instabilities.

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[1] See S.E. Kruger, C.C. Hegna, and J.D. Callen, “Generalized reduced magnetohydrodynamic equations,” *Phys. Plasmas* **5**, 4169 (1998) and references cited therein.

[2] C.R. Sovinec, D.C. Barnes, T.A. Gianakon, A.H. Glasser, R.A. Nebel, S.E. Kruger, D.D. Schnack, S.J. Plimpton, A. Tarditi, M.S. Chu and the NIMROD Team, “Nonlinear Magnetohydrodynamics Simulation using High-Order Finite Elements,” UW-CPTC 02-05 (submitted to *J. Comput. Phys.*).

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