— 1979 Sherwood Meeting —

Theoretical Aspects of Controlled Thermonuclear Research

Mount Pocono, Pennsylvania
April 18-20, 1979

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General Information

All sessions will be held at the Pocono Manor. The morning oral sessions will be in the Terrace Ballroom; the afternoon (or evening) poster sessions will be in the Plymouth Meeting Center. Coffee and other refreshments will be available during both the oral and poster sessions.

There will be two consecutive poster sessions on Wednesday afternoon and Thursday evening and one on Friday afternoon. Thursday afternoon is free.

A Cocktail Hour will be held in the Horizon Lounge Wednesday at 5:30. Two drinks are included in the registration fee.

The Registration and Travel desks are in the Fountain Room. If you need assistance in planning transportation out, check with Travel early.

There were 255 papers submitted of which 21 were chosen for oral presentation. Provisions have been made for these authors to present the details of their work in a subsequent poster presentation.
SCHEDULE

Tuesday, p.m.
5:00 - 7:00  Registration & Complementary Punch
7:00 - 8:00  Dinner
8:00 - 11:00 Registration
9:00 - 11:30 Snack for late arrivals

Wednesday
7:30 - 9:00  Breakfast
8:30  Welcome
8:45  Oral Session 1A
10:25 - 10:45 Coffee
10:45  Oral Session 1A
12:00 - 1:00  Lunch
1:30 - 3:15  Poster Session 1B
3:00 - 4:00  Refreshments
3:30 - 5:15  Poster Session 1C
5:30 -  Cocktails
6:30 - 8:00  Dinner

Thursday
7:30 - 9:00  Breakfast
8:45  Oral Session 2A
10:25 - 10:45 Coffee
10:45  Oral Session 2A
12:00 - 1:00  Lunch
FREE AFTERNOON
6:30 - 8:00  Dinner
7:30 - 9:00  Poster Session 2B
8:45 - 9:30  Refreshments
9:15 - 10:45 Poster Session 2C

Friday
7:30 - 9:00  Breakfast
8:45  Oral Session 3A
10:25 - 10:45 Coffee
10:45  Oral Session 3A
12:00 - 1:00  Lunch
1:30 - 3:00  Poster Session 3B
1979 SHERWOOD MEETING
THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR RESEARCH

TUESDAY APRIL 17

5:00-7:00 REGISTRATION AND COMPLIMENTARY RUM PUNCH Fountain Room
7:00-8:00 DINNER Main Dining Room
8:00-11:00 REGISTRATION Fountain Room
9:00-11:30 SANDWICHES FOR LATE ARRIVALS Sam's Place

WEDNESDAY APRIL 18

7:30-9:00 BREAKFAST Main Dining Room
8:15-12:15 REGISTRATION
(Registration and Travel Desk open during all sessions)
8:30 Welcome M. B. Gottlieb
Announcements J. L. Johnson

8:45 ORAL SESSION 1A Terrace Ballroom
D. Ross and H.R. Lewis Chairman

1A2. Finite Beta sub e Universal Mode Turbulence and Alcator Scaling. K. Molvig.

10:25-10:45 COFFEE BREAK


12:00-1:00 LUNCH (Dining Room doors close promptly at 1:00)

1:30-3:00 POSTER SESSION 1B
(All Poster Sessions are in the Plymouth Meeting Center)

ORAL Papers 1A1-1A4 will be given in Patrick Henry C

Jefferson Room

1820. A Finite Element Solution of a Reduced Fokker-Planck Equation. I. Bernstein, A. Weiser, S. Eisenstat, and M. Schultz.
1823. Coupling of Lower Hybrid to Acoustic Modes. E. J. Valeo and L. Chen.
1831. Spectrum and Eigenfunctions for a Field Equation With Stochastic Ray Trajectories. S. W. McDonald and A. N. Kaufman.


Monroe Room


Patrick Henry A


Patrick Henry B


3:00-4:00 REFRESHMENTS Manor Grill

3:30-5:15 POSTER SESSION 1C

Oral Papers 1A5-1A7 will be given in Patrick Henry C
Manor Hall Auditorium


1C2. Effects of Ion Dynamics on Tearing Modes. X. S. Lee, S. M. Mahajan, and R. D. Hazeltine.


1C10. Simulations of DCLC Modes Near Linear Marginal Stability. B. I. Cohen and N. Maron.


Jefferson Room


1C35. Mathematical Problems Arising in Adiabatic Compression of Plasma. G. Vigfusson.


Patrick Henry A

1C37. Reduced Set of Resistive MHD Equations in Toroidal Geometry. B. Carreras, H. R. Hicks, and J. A. Holmes.

1C38. Free and Forced m = 0 Oscillations of a Sharp-Boundary Vlasov-Fluid Screw Pinch. T. E. Cayton and H. R. Lewis.

1C39. Particle Orbits in Field-Reversing Ion Rings: Ergodic or Not?. D. A. Larrabee and R. V. Lovelace.


1C42. Wave Particle Transport From Electrostatic Instabilities: An Overview. S. P. Gary.

1C43. Studies of Current Due to RF Induced Runaway in the DIIA Lower Hybrid Experiment. R. W. Harvey, J. C. Riordan, J. L. Luxon, and K. D. Marx.


Patrick Henry B

1C45. Transient Amplification of Shear Alfvén Waves. Y. Y. Lau.

1C46. Numerical Simulation of Plasma Confinement and Heating by Field-Reversed Ion Rings. A. Mankofsky, R. N. Sudan, and J. Denavit.


5:30-6:30 COCKTAILS Horizon Lounge

6:30-8:00 DINNER Main Dining Room

THURSDAY, April 19

7:30-9:00 BREAKFAST

8:45 ORAL SESSION 2A Terrace Ballroom

A.H. Glasser and K. Tsang Chairman


10:25-10:45 COFFEE BREAK


12:00-1:00 LUNCH (Dining Room doors close promptly at 1:00)

FREE AFTERNOON

6:30-8:00 DINNER Main Dining Room

7:30-9:00 POSTER SESSION 2B

Oral Papers 2A1-2A4 will be given in Patrick Henry C Manor Hall Auditorium


Monroe Room


Patrick Henry A


Patrick Henry B

8:45-9:30 REFRESHMENTS Manor Grill

9:15-10:45 POSTER SESSION 2C
Oral Papers 2A5-2A7 will be given in Patrick Henry C

Manor Hall Auditorium

    M. Mahajan, R. D. Hazeltine, and H. R. Strauss.
2C5. Lower Hybrid Heating and Current Generation in Versator II. R. Engelke, T. Antonsen, and M.
     Porkolab.
2C7. The Distribution of and Classical Transport by Alpha Particles in a Thermonuclear Plasma. J. D.
     Gaffey and R. S. Schneider.
2C16. Electron Cyclotron Resonance Heating of Tokamaks at Omega = 2 Omega sub ce. B. H. Hui, K. R.
     Chu, E. Ott, and T. M. Antonsen.
     Pellat.
     T. Mizoguchi and T. Kammash.

Jefferson Room

     Killeen, and A. A. Mirin.
     J. B. Taylor.
     Tsang.
2C27. Anomalous Loading of RF Antennae Due to Near Field-Particle Interactions. G. J. Morales.
2C29. Turbulent Model of Magnetic Braiding II: Pressure Correlation Function and Self-Consistency. P.
     Diamond, D. Tetreault, and T. Dupree.
2C30. The Electric Sheath and Pre-Sheath in a Collisionless Finite Ion Temperature Plasma. G. A. Emmert, R.

Monroe Room


Patrick Henry A

2C38. Finite Beta Trapped Particle Modes. T. M. Antonsen.
2C42. The Role of the Continuous Spectrum in Ideal MHD Ballooning Mode Theory. A. H. Glasser.

FRIDAY, APRIL 20

7:30-9:00 BREAKFAST Main Dining Room
8:45 ORAL SESSION 3A Terrace Ball Room
B. Cohen and O. Manley Chairmen


10:25-10:45 COFFEE BREAK


12:00-1:00 LUNCH (Dining Room doors close promptly at 1:00)
1:30-3:00 POSTER SESSION 3B

Oral Papers 3A1-3A4 will be given in Patrick Henry C
Oral Papers 3A5-3A7 will be given in Patrick Henry B
Manor Hall Auditorium


3B

OPR: - 7500 DOWN APPROX 30 MNS - TINA


Jefferson Room


Monroe Room


Patrick Henry A

3B44. Fueling of a Long-Pulse Divertor Tokamak. H. C. Howe.

Patrick Henry B


3:00 WE HOPE YOU ENJOYED THE MEETING
Treve, Y. M. - 1B24
Tripathi, V. K. - 1845, 2B1
Tsang, K. T. - 2B11, 2C26, 3B26
Uckan, N. A. - 1848
Uckan, T. - 1B46
Ueno, C. - 1B33
Ueno, C. - 1B39
Uo, K. - 1839
Vahala, G. - 2B44, 3B36
Valeo, E. J. - 1B22, 1B23
van Rij, W. I. - 1C33
Vella, M. C. - 3B16
Vigfusson, G. - 1C35
Villalon, E. - 2B8
Wakatani, M. - 1B39
Walstead, A. E. - 3B45
Waltz, R. E. - 1B44, 1C9, 2B17, 2C36
Wang, T. S. - 1B34
Ware, A. A. - 3A1
Watanabe, T. - 1B33
Weiser, A. - 1B20
Weitzner, H. - 1C7
Welter, H. - 1C21
Wen, S-L. - 3B30
Wersinger, J. M. - 1B3, 3B4
White, R. B. - 1A1, 1B32, 2B12
Whitson, J. C. - 1C18, 2B11
WieLand, R. M. - 2C30
Wiley, J. C. - 1C11, 3A1
Winske, D. - 1C47
Winsor, N. - 1B4
Winsor, N. K. - 2B37
Wong, S. K. - 2A19
Woo, J. T. - 2B32
Wu, C. C. - 2C11, 2C12
Yoshikawa, S. - 1B32, 1B33
Yoshioka, T. - 1B39
CALCULATION OF THE KOLMOGOROV ENTROPY FOR MOTION ALONG A STOCHASTIC MAGNETIC FIELD*

A. B. Rochester
Bell Laboratories, Murray Hill, New Jersey, 07974

M. N. Rosenbluth
Institute for Advanced Study, Princeton, New Jersey, 08540

R. B. White
Plasma Physics Laboratory, Princeton University, Princeton New Jersey 08540

We have developed a statistical theory for stochastic magnetic fields. A formula for the Kolmogorov entropy has been derived. Excellent agreement between a probability description and direct dynamical computations has been found.

* This work was supported in part by DoE contracts No. EY-76-C-02-3073, and No. EY-(76-S)-3237.
A self-consistent resonance broadening theory for finite $\beta_e$ universal mode turbulence is presented. Saturation results from resonance broadening of the electron response due to radial diffusion in combination with streaming along the lines in the presence of magnetic shear, as described in the preceding paper. Electron diffusion, for $\beta_e > m_e/m_i$, is due to the magnetic part of the fluctuations. The island width exceeds the rational surface spacing at fluctuation levels of order $\bar{B}_{rmn}/B \sim 10^{-7}$, giving ergodic behavior of the lines on a fine scale. Accordingly, the theory constitutes an example of a self-consistent theory of stochastic magnetic fluctuations. The anomalous electron thermal conductivity at saturation,

$$\chi_e = 0.1\left[\frac{T_e}{T_e + T_i}\right]^{4/3} \left[\left(\frac{m_e}{m_i}\right)^{-1/3}\right] \left(\frac{L_s}{L_n}\right)^2 v_i \rho_i^2 / L_n$$

has many similarities with experimental observations, including absolute magnitude, and scaling with density, electron and ion temperatures, magnetic field, aspect ratio, and ion mass.


*This research was sponsored in part by the Office of Fusion Energy (ETM), U.S. Department of Energy, under contract No. W-7405-eng-26 with the Union Carbide Corporation and the U.S. Energy Research and Development Administration Grant No., EG-77-G-01-4108.
The importance of magnetic field fluctuations in producing anomalous cross-field energy transport has been recognized recently and, in particular, the temperature-gradient driven drift-tearing mode is likely to be the source of these magnetic fluctuations. An investigation of the nonlinear interaction of these drift-tearing modes has been carried out which demonstrates that wave energy cascades from long to short wavelength. The temperature gradient is of crucial importance in producing this energy flow. The quantity \( \sum_{k} |A_k|^2 \) is conserved in the nonlinear interaction. Damping of the long wavelength \( (\omega < \nu_{ei}) \) by the short wavelength modes \( (\omega > \nu_{ei}) \) leads to a saturation of the instability when \( |E|/B = 0.3 \nu_{ei}/\nu_{Te} \). The results are in good agreement with recent measurements of magnetic field fluctuations on the macrotor tokamak in which the spectrum of \( |E|/B \) extended to \( \omega \approx 2 - 3 \nu_{ei} \) and the amplitude \( |E|/B \) was independent of the density.

*Research supported by the Department of Energy.


Renormalized Induced Scattering and Nonlinear Damping of Collisionless Drift Waves

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Princeton, NJ 08544

A kinetic theory of the turbulent damping of collisionless drift waves is presented. The Direct Interaction Approximation for the nonlinear dielectric function is reduced to a renormalized version of induced scattering. In contrast to classical resonance broadening theory, the theory reduces correctly to the weak turbulence limit and is energetically consistent; it includes both propagator broadening by turbulent collisions as well as turbulent corrections to the mean distribution function. Explicit calculations are given for shear-free geometry in the approximation which reduces to Compton scattering on the ions; these systematize, correct, and extend to finite ion gyroradius the earlier calculations of Dupree and Tetreault. For long wavelengths where the Markovian approximation \(|k| \rightarrow 0\) is valid, the nonlinear ion "growth" rate is large and positive, proportional to \(k^2D_i\). Nevertheless, energy conservation between the waves and particles is demonstrated explicitly by eschewing the Markovian approximation and summing \(<\delta j \cdot \delta E>\) over all modes. The net power flow into the ions is small, proportional to the square of a typical parallel wavenumber. Extensions of the theory which describe sheared geometry and electron nonlinearities are discussed.


1 D.F. Dubois and M. Espedal, Plasma Phys. 20, 1209 (1978);

THEORETICAL STUDIES FOR THE ELMO BUMPY TORUS (EBT) DEVICE*

D. A. Spong, D. B. Batchelor, C. L. Hedrick, and E. F. Jaeger
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

ABSTRACT

The ELMO Bumpy Torus (EBT) is a closed line device consisting of a core plasma confined within 24 toroidally linked mirror sectors and heated by microwaves. In the toroidal or T-mode of operation, rings of hot electrons are formed in the midplane at each mirror and produce an average minimum in the magnetic field, thus stabilizing the toroidal plasma against flute and interchange modes. Energy and particle loss rates in the toroidal core plasma are predominately due to random scattering of particles onto drift orbits with greater displacements from the plasma center (neoclassical diffusion). The toroidal plasma is heated by the strong damping of extraordinary mode microwaves at the fundamental cyclotron resonant surfaces. Theoretical understanding of this device has rapidly evolved during recent years in a number of areas. These include: equilibria, particle orbits, MHD and drift wave stability, heating (microwave and neutral injection), transport, and ring physics. Work in a number of these areas will be described.

ENHANCEMENT OF TANDEM MIRROR PLUG POTENTIALS BY THERMAL PARTICLE PUMPOUT

David E. Baldwin and B. Grant Logan
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Livermore, California 94550

ABSTRACT

The solenoid ions of a tandem mirror are confined by the potential between the solenoid and the more dense plugs which is generated by Maxwellian electrons. This potential barrier increases only logarithmically with plug density, although (classically) the power to maintain the plugs increases as their density squared. The high peak magnetic fields and neutral injection energies formally seen as required for a tandem mirror reactor are direct consequences of these confinement characteristics.

In principle, plugs could alternatively be formed by magnetically confined electrons. However, in the face of electron scattering rates, to do so in the presence of Maxwellian ions filling the resultant negative potential leads to densities having prohibitive power requirements.

Provided the neutralizing, oppositely charged species can be maintained non-Maxwellian, dramatic reductions can be achieved in the density (and thus power) required to maintain either ion (I) or electron (E) plugs. Versions of bounce resonance heating or transit-time magnetic-pumping appear promising in the role of a pump-out mechanism. The requirements for frequency, penetration, coupling, and side effects differ for I and E plugs and ultimately will determine the superiority of one type.

This paper will cover a number of topics related to the pump-out, energetics, and stability of such plugs with enhanced potentials. The physics issues have important antecedents in both the Mirror and ELMO-EBT Programs, so that credible cases can be described which require little extrapolation from past experiments. In general, this technique appears to offer promising means for significant reduction in the technology requirements on fields and beams here-to-for thought necessary for tandem mirrors and to improve Q at higher power density.

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-Eng-48.
Effects of Toroidicity on the Nonlinear Interaction of Tearing Modes*

H. R. Hicks, B. Carreras**, and S. J. Lynch
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

The nonlinear evolution of tearing modes proceeds much the same in toroidal geometry as in cylindrical.\(^1\) The most significant difference is that, in our toroidal calculations, about 90\% of the plasma volume is encompassed by a stochastic magnetic field line region when pre-disruption profiles\(^1\) \((q > 1)\) are studied. The time for the development of this region corresponds to the growth time of the linear 2/1 tearing mode. The negative voltage spike has a similar character in both cylindrical and toroidal cases. A survey of profiles shows that the extent of the stochastic region depends on the profile assumed. In the toroidal case, the set of profiles which evolves to a large stochastic region may be somewhat larger than in the cylindrical case.

The results are obtained with a new computer program, Lobeto, which advances the low B toroidal reduced resistive MHD equations.\(^2\) When a flux coordinate system is employed, the equations are formally very similar to the cylindrical ones. Consequently, we have used numerical techniques similar to our cylindrical code RSF.\(^3\) The three-dimensional functions are expanded in a trigonometric series in poloidal and toroidal angles, thus converting the problem to a large number of coupled one-dimensional partial differential equations.


**Visitor from Junta de Energia Nuclear, Madrid, Spain.


\(^{2}\)B. Carreras, H. R. Hicks, abstract submitted to this conference.

\(^{3}\)H. R. Hicks et al., Computational Plasma Physics Meeting, Monterey, CA, June 1978, CONF-780614.
Charge Exchange as an Impurity Recombination Mechanism

R. A. Hulse, D. E. Post, and D. R. Mikkelsen
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Princeton, New Jersey 08544

Relatively large (~ $10^{-14}$ cm$^2$) cross sections for charge exchange between neutral hydrogen atoms and highly stripped impurity ions can yield an important recombination mechanism for such ions present in tokamak plasmas. The result can be a marked alteration of the ionization balance, along with enhanced radiative losses in some circumstances. The linear dependence of the conventional electron-ion recombination processes on electron density (together with the typically decreasing penetration of neutrals into the plasma interior with increasing density) makes the charge exchange recombination process particularly important for low density, intensely neutral beam heated plasma environments. Results are presented from zero-dimensional time independent (coronal equilibrium) and time dependent atomic physics codes in which charge exchange recombination has been included. Particular attention is given to modeling the recent PLT high temperature experiments, where the behavior of the iron impurity at the center is calculated to be dramatically altered by this process. The implications for other tokamak experiments are also discussed.

*Work supported by U. S. DoE Contract No. EY-76-C-02-3073.
RADIAL SCALING IN THE QUASILINEAR MODEL OF DRIFT CYCLOTRON LOSS CONE (DCLC)

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ABSTRACT

Linear theory of DCLC predicts that, as the plasma radius (relative to the Larmor radius) increases, stability requires less warm plasma 'dwelling in the uncontained hole in velocity space. To realize this reduction the temperature of this plasma must also drop. However, prior attempts to simulate this effect with the quasilinear code failed in that in all cases marginal stability occurred where the distribution function filled the entire velocity space hole as was the case for the small plasma radius configuration. This behavior was due to the fact that the stable point typically requires more phase space density in the hole as its temperature climbs. In the first quasilinear models the self generated turbulence affected the stream only by heating, consequently reducing its ability to stabilize; as a result, the simulation ran away to the regime where the turbulence was high enough to spill particles out of the trapped part of phase space (2XII-B scaling). The latest calculations improve upon this model by incorporating the effect that a heated plasma stream (due to the turbulence) increases the penetration over the ambipolar barrier thus raising the supply of warm plasma. For the tandem mirror configuration we adopt the model

\[ J_{\text{stream}} \sim n_c \left( \frac{n_c}{n_p} \right)^{T_e/T_{\text{stream}}} \]

where \( n_c(n_p) \) is the density in the central cell (plug) and the exponent is the standard Boltzman factor. The quasilinear code has been run to compare steady state parameters relevant to TMX and the proposed MFTF B configuration. For the latter, typical plug parameters at 80 kev neutral injection are \( T_e \sim 1 \text{ keV}, T_i \sim 50 \text{ keV}, \beta \sim .5 \) and \( \delta_v \sim 30-50 \text{ ev} \).

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.*
Toroidal Effects on the Accessibility of Lower Hybrid Waves*

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We use a lower hybrid wave, toroidal ray tracing code which includes electromagnetic effects, thermal effects, and damping to study the accessibility and energy deposition of lower hybrid waves. Comparisons are made with the limiting case of a cylindrical plasma. In toroidal geometry the poloidal wave number is no longer a conserved quantity. As a result the wave number parallel to the magnetic field $k_\parallel$ changes. This modifies considerably the straight cylinder picture of accessibility and electron Landau resonance. In general the picture of accessibility is quite different than for a straight cylinder. For example, the lower hybrid wave can mode convert to a fast wave and the fast wave can mode convert back to a lower hybrid wave several times before (due to toroidal changes in $k_\parallel$), becoming accessible. Results showing energy deposition for typical tokamak situations of interest (e.g., Alcator C) will be presented, and the implications of different ion damping mechanisms will be discussed (e.g., ion cyclotron and unmagnetized ion Landau damping).

A Fully Two-Dimensional Transport Model*

by

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Niels Winsor and Jay Boris
Naval Research Laboratory
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A fully two-dimensional Eulerian-Lagrangian computer simulation model of tokamak discharges has been developed. The model is based on the quasi-static evolution of force balance in the plasma and steady-state flow along the flux surfaces. The coordinate system incorporates a general connectivity triangular grid which can simulate non-circular flux surfaces, multiple magnetic axes (including a separatrix) and limiters.

The Lagrangian dynamics$^1$ are based on the assumption that the force-balance equation is always satisfied which permits the spatial motion of the flux surfaces to be tracked directly.

The transport and diffusion equations are solved in an Eulerian fashion. Since the magnetic fluxes diffuse through the surfaces, the separatrix remains rigorously defined. The perpendicular velocity is found from Ohm's law and the parallel velocity is found from the momentum equation assuming steady-state flow. The triangular grid structure allows the current densities to be determined in closed form.

Results will be presented illustrating the dynamical evolution and transport of a circular tokamak discharge. Both high and low poloidal beta discharges will be considered and the resulting differences in the induced currents will be discussed.

*Work supported by U. S. Department of Energy.
†Present address: NRL, Code 6020, Washington, DC 22101
*M. Emery, et al., NRL Memorandum Report 3744.
LOWER HYBRID ELECTRON LANDAU DAMPING AND CURRENT DRIVE IN THE PRESENCE OF AN APPLIED DC ELECTRIC FIELD AND TRANSPORT LOSSES*

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National Magnetic Fusion Energy Computer Center
Lawrence Livermore Laboratory
Livermore, California

R. W. Harvey, V. S. Chan and J. M. Rawls
General Atomic Company
San Diego, California

An applied dc electric field $E$ can give rise to a distortion of the electron velocity distribution $f_e(v)$ sufficient to significantly modify the (quasilinear) electron Landau damping of lower hybrid waves. In particular, the electron tail will be enhanced in the direction antiparallel to $E$ and depleted in the direction of $E$. This asymmetric distortion of $f_e(v)$ will result in different absorption rates of the two LH ray channels arising from a symmetric standing wave antenna.

In addition, if the electron energy loss channels are velocity dependent, further deviations from a Maxwellian electron velocity distribution will result. In some instances, a portion of the LH energy which is deposited on the electrons will not thermalize but instead will be lost directly by transport processes.

The effects on LH heating and current drive of both an applied dc electric field and electron transport due to braided magnetic fields\(^1\) are examined by means of a Fokker-Planck code containing a quasilinear electron Landau diffusion term. The key issues addressed are:

1. Electron Landau damping of LH waves propagating in the parallel and antiparallel directions with respect to $E$.
2. Transport losses of the absorbed LH energy relative to the energy thermalized on the bulk of $f_e(v)$.
3. Effects of transport on LH current drive.

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.

CURRENT PROFILE STABILIZATION OF D-SHAPED TOKAMAKS
TO IDEAL MHD MODES

L. C. Bernard, D. Dobrott, F. J. Helton, and R. W. Moore

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ABSTRACT

A complete numerical study of ideal MHD modes, including both internal and external modes, has been carried out. A fixed D shape has been chosen (aspect ratio 2.4, elongation 1.7) in order to concentrate on current profile optimization. The stability analysis of internal modes is done with the interchange and ballooning mode criteria. External modes are studied with the global code ERATO including the effect of a conducting wall arbitrarily located. The axisymmetric mode is shown to be easily stabilized by an external wall. By contrast, it is shown that it is difficult to stabilize external kink modes by an external wall. No wall stabilization is used for the external kink mode. Instead the current profile is varied to obtain stability. Under these conditions, plasma equilibria with beta above 8% are found which are stable to all modes. The n = 1 external mode appears to be the most restrictive, where n is the toroidal wave number. The optimal current profile is rather flat and has a poloidal beta less than unity.

Work supported by Department of Energy, Contract No. EY-76-C-03-0167, Project Agreement No. 38.
A COMPACT FORM OF THE INTEGRAL EQUATION FOR WAVES IN AN INHOMOGENEOUS PLASMA

Steven P. Auerbach

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ABSTRACT

Consider an electrostatic wave propagating in a one-dimensional plasma confined by an external field. The electric field of the wave, $E(x) \exp(i\omega t)$, obeys an integral equation

$$E(x) = \int \! dx' \, K(x,x';t) E(x').$$

The kernel $K(x,x';\omega)$ is determined by the unperturbed distribution function $f_0(x,v)$. By making use of Liouville's theorem, which states that phase space is conserved, a simple compact form of $K$ can be derived:

$$K(x,x';\omega) = -\omega_p^2 \int_0^\infty \! d\tau \exp(i\omega \tau) \frac{\partial}{\partial v} f_0(x,\nu(x,x,\tau))$$

where $\nu(x,x,\tau)$ is the velocity required for a particle which starts at $\hat{x}$, at time $t = 0$ to arrive at $x$ at $t = \tau$. (There may be multiple values of $\nu$; a sum over such values is implied.) One noteworthy feature of this form is that $\partial f_0/\partial v$ does not appear. In words, this states that the influence of point $\hat{x}$ on point $x$ is an integral over $\tau$ of the number of particles which can travel from $\hat{x}$ to $x$ in time $\tau$, weighted by the appropriate wave phase $\exp(i\omega \tau)$. The proof of this result, applications, and generalizations to three-dimensional magnetized plasma will be presented.

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-Eng-48.

NONLOCAL HYBRID-KINETIC STABILITY ANALYSIS OF THE MIRROR DRIFT-CONE INSTABILITY*

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Richard E. Aamodt
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This paper develops a fully self-consistent nonlocal theory of the mirror-drift-cone instability with emphasis on the influence of large ion Larmor radius and axis-encircling orbits on stability behavior. The analysis is carried out within the framework of a hybrid Vlasov-fluid model. The electrons are described as a macroscopic, cold ($T_e \rightarrow 0$) fluid immersed in a uniform axial magnetic field $B_{0z}$. On the other hand, we adopt a fully kinetic model for the ions in which the ions are described by the Vlasov equation. This allows for the possibility of large ion orbits with characteristic thermal Larmor radius ($r_{Li}$) comparable to the radius of the plasma column ($R_p$). The stability analysis assumes electrostatic flute perturbations about a cylindrical ion equilibrium $f_i^0(H_i-\omega_i P_\theta, v_z)$ where $\omega_i = $const.$= \text{angular velocity of mean rotation.}$

The radial eigenvalue equation for the potential amplitude $\hat{\phi}(r)$ is solved exactly for the particular choice of loss-cone equilibrium, $f_i^0 = (n_0^e m_i^e/2\pi) \times \delta(H_i-\omega_i P_\theta - \hat{T}_i) G(v_z)$, which corresponds to a sharp-boundary (rectangular) density profile and a parabolic temperature profile. The resulting dispersion relation for the complex eigenfrequency $\omega$ is an algebraic equation of order $\ell + 2$, where $\ell$ is the azimuthal mode number. The dispersion relation is solved numerically for a broad range of system parameters including the important influence of large ion orbits and ion thermal effects. It is found that the nonlocal growth rate exhibits a sensitive dependence on $r_{Li}/R_p$, $R_p/R_c$, etc. Moreover, the stability growth rate is typically more severe for fast rotational equilibria ($\omega_i = \hat{\omega}_i$) with axis encircling orbits than for slow rotational equilibria ($\omega_i = \hat{\omega}_i$). Stability results are presented for the entire range of $r_{Li}/R_p$ allowed by the equilibrium model ($0 < 2r_{Li}/R_p < 1$).

* Research was supported by the Department of Energy. The research by one of the authors (H.S.U.) was supported in part by the Office of Naval Research under the auspices of a Joint Program with the Naval Research Laboratory.
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STABILITY PROPERTIES OF A FIELD-REVERSED ION LAYER IN A BACKGROUND PLASMA*

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Stability properties of an intense proton layer (P-layer) immersed in a background plasma are investigated within the framework of a hybrid model in which the layer ions are described by the Vlasov equation, and the background plasma electrons and ions are described as macroscopic, cold fluids. Moreover, the stability analysis is carried out for frequencies near multiples of the mean rotational frequency \( \omega_\| \) of the layer. It is assumed that the layer is thin, with radial thickness \((2a)\) much smaller than the mean radius \((R_0)\). Electromagnetic stability properties are calculated for flute perturbations \((\delta z=0)\) about a thin P-layer described by the rigid-rotor equilibrium distribution function \(f_0^p=(m_{\perp}/2\pi)\delta(h_{\perp}-\omega_\|P_{\perp}-\hat{T})G(v_z)\), where \(n_0\), \(\omega_\|\) and \(\hat{T}\) are constants. Moreover, it is assumed that the background plasma has a step-function density profile. Stability properties are investigated including the important effects of (a) the equilibrium magnetic field depression produced by the P-layer, (b) transverse magnetic perturbations \((\delta B_0\neq0)\), (c) small (but finite) transverse temperature of the layer ions, and (d) the dielectric properties of the background plasma. All of these effects are shown to have an important influence on stability behavior. A detailed analysis of the radial eigenvalue equation is carried out for eigenfrequencies near multiples of the mean P-layer rotational frequency, i.e., \(|\omega-\omega_\||<<\omega_r\), where \(\omega\) is the complex eigenfrequency, \(\ell\) is the azimuthal harmonic number, \(\omega_{\|}\) is the mean rotational frequency of the P-layer, and \(\omega_r\) is the radial betatron frequency of the layer ions. It is found that the instability growth rate exhibits a sensitive dependence on layer density \(n_p\), background plasma density \(n_b\), the degree of magnetic field depression \(\eta=B_0^0(r=0)/B_{ext}\), and the transverse temperature of the beam ions. For example, for a dense background plasma, the system can be easily stabilized by a sufficiently large transverse temperature of the layer ions. Moreover, for \(-1<\eta<1\), the instability growth rate is significantly reduced whenever the background plasma density is sufficiently large that \(\omega_{\perp}R_0/c>>1\).

* Research was supported by Department of Energy. The research by one of the authors (H.S.U.) was supported in part by the Office of Naval Research under the auspices of a Joint Program with the Naval Research Laboratory.
† Naval Surface Weapons Center, Silver Spring, Maryland, 20910.
This paper investigates the thermal equilibrium properties of an intense ion beam that has rotational as well as axial motion in an externally applied guide field $B_0$. The ion beam propagates through a background plasma that provides partial charge neutralization, and the ion beam equilibrium is described by the thermal equilibrium distribution function $f_0 = \frac{\hat{n}_b}{(2\pi m_i T)^{3/2}} \exp(-m_1 v^2_{\perp}/2T) \times \exp[-(H+\omega_b \phi - V P_z)/T]$. Here, $H = \frac{p^2}{2m_i} + e\phi$ is the energy, $P_\theta = r[p_\theta + eA^s_z(r)/c]$ is the canonical angular momentum, $P_z = p_z + eA^s_z(r)/c$ is the axial canonical momentum, $\omega_b$ and $T$ are constants, $-\omega_b = \text{const.}$ is the angular velocity of mean rotation, and $V = \text{const.}$ is the mean axial velocity. Introducing the effective potentials, $\psi_\theta(r) = -m_1 \omega_b r^2/2T + e\omega_b A^0_\theta(r)/cT$ and $\psi_z(r) = (e/c)[\phi^0(r) - \beta A^s_z(r)]$, yields the coupled nonlinear equations for $\psi_\theta$ and $\psi_z$,

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi_\theta}{\partial r} \right) = \frac{2}{\delta^2} \exp[-(\psi_\theta + \psi_z)], \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi_z}{\partial r} \right) = \frac{\delta}{b^2} \exp[-(\psi_\theta + \psi_z)], \quad (2)$$

where $n_b(r) = \hat{n}_b \exp[-(\psi_\theta + \psi_z)]$ is the density, $\delta = \frac{2 \omega_b}{\beta^2_b} \frac{c^2 v^2_1}{1}$, and $b = \frac{2 \omega_b}{\beta^2_b} |\beta^2_z - (1-f)|/4v^2_1$. Here $v^2_1 = 2T/m_i$, $\beta = v/c$, $\beta^2_z = 4\pi v_n e^2/m_i$, $f = \text{const.}$ is the fractional charge neutralization, and $\epsilon = \text{sgn}[^2_z - (1-f)]$. The solutions to Eqs. (1) and (2) are investigated analytically and numerically for $0 \leq \delta^2/b^2 \leq \infty$, and the necessary and sufficient conditions for the existence of radially confined equilibrium solutions $[n_b(r=0) = 0]$, and for the onset of field reversal $B^0_z(r=0)/B^\infty_z < 0$ are derived. As a general remark, for $\delta^2 < b^2$, the inequality $|\psi_\theta| > |\psi_z|$ is satisfied, and the axial motion and equilibrium space charge fields play the dominant role.
GEOMETRIC OPTICS IN INHOMOGENEOUS ISOTROPIC AND ANISOTROPIC PLASMAS AND ON THEIR BOUNDARIES*

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This paper exploits a general approach to geometric optics in inhomogeneous plasmas based on the properties of the local dielectric tensor $\varepsilon$. We express $\varepsilon$ in terms of its eigenvalues $\varepsilon_j$ and eigenvectors $\hat{e}_j$. Then to zero order in the geometric optics approximation the determinant $D=\varepsilon_1\varepsilon_2\varepsilon_3$ vanishes giving in general three branches of the dispersion relation. The possibility of branching makes the formulation of the geometric optics equations different in an anisotropic plasma, where only one eigenvalue vanishes, from that in an isotropic (degenerate) plasma with more than one zero eigenvalues. In the nondegenerate case, one can trace the rays by solving equations

$$\begin{align*}
\hat{r} &= -\frac{D_k}{D_w} ; \\
\hat{k} &= \frac{D_\varepsilon}{D_w}
\end{align*}$$

(1)

These equations, however, are singular in the degenerate plasma. Here one can use the fact that the sum $F=\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_3$ of the second order minors of $\varepsilon$ also vanishes and, therefore, can be used in defining nonsingular ray equations

$$\begin{align*}
\hat{r} &= -\frac{F_k}{F_w} ; \\
\hat{k} &= \frac{F_\varepsilon}{F_w}
\end{align*}$$

(2)

The transition from (2) to (1) on the boundary between degenerate and nondegenerate regions presents numerical difficulties, since $D_k=D_\varepsilon=D_w=0$ on the boundary. It will be shown that l'Hopital's rule applied to (1) as one approaches the boundary from the nondegenerate side, gives in general two values for $\hat{k}$, corresponding to different branches of the dispersion relation. On using these derivatives, one can split the rays on the boundary, make a small step into the nondegenerate region, and then follow each of the modes by solving (1). We will demonstrate this method in a case where radiation from a vacuum region enters an inhomogeneous magnetized plasma. The details of our general geometric optics code, which traces the rays and finds the amplitudes, polarization and absorption of the waves along the rays, will also be reported.

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HIGHER ORDER CHAPMAN-ENSKOG THEORY FOR ELECTRONS:
APPLICATION TO TEMPERATURE GRADIENT-DRIVEN MODES*

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The Chapman-Enskog expansion is carried to second order for the electron fluid. The resulting equations, an order more accurate than those of Braginskii, are employed to obtain a class of temperature gradient-driven modes. These modes were heretofore not derivable from fluid theory. In particular, the collisional\textsuperscript{1,2} and semi-collisional\textsuperscript{2} versions of the tearing mode are recovered. Likewise, the higher-$m$ temperature gradient-driven drift and drift-tearing modes\textsuperscript{3} are also considered.

*Research supported by a fellowship from the Center for Theoretical Physics, University of Maryland.
3) N. T. Gladd, J. F. Drake, C. L. Chang, and C. S. Liu, this meeting.
We have investigated the collisional electrostatic drift wave in slab geometry and have demonstrated both numerically and analytically that this mode can be driven unstable by a positive electron temperature gradient \( \eta_e = \frac{d \ln T_e}{d \ln n} \). Collisions have been included with a velocity-dependent Lorentz collision operator.\(^1\) The temperature gradient produces "wells" on either side of the rational surface which localize the radial eigenmode. For a reasonable shear \( (1/1^\sim 20) \), the dissipative drift waves have a threshold \( \eta_e > 3 \). As the collisionality is reduced, these \( \eta_e \) driven modes become stable, making a smooth transition to previous collisionless results.\(^2\) Analytic expressions for the growth rate have been obtained and are in good agreement with the numerical results.

\(^*\)Research supported by the Department of Energy.

MICROTEARING MODES AND ANOMALOUS TRANSPORT IN TOKAMAKS*

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A numerical and analytic study of the temperature-gradient-driven tearing mode\(^1\) has been carried out which demonstrates that this mode is unstable in a slab model for realistic tokamak parameters. Collisions have been included with a velocity-dependent Lorentz collision operator. Modes with \(\omega_e < \nu_{ei} \) are found to be unstable with the growth rate peaking around \(\omega_e \approx \nu_{ei} / 5\). Details of growth rates, eigenfunctions, and the stability boundaries will be presented. Radial magnetic field fluctuations \(|\mathbf{B}|^2 / B^2\) associated with this instability lead to enhanced electron thermal transport by effectively converting \(\chi_{ei}^\parallel\) to \(\chi_{ei}^\perp\). In a nonlinear calculation, we have shown that this instability saturates by transferring energy from unstable long wavelengths to stable short wavelengths, a form of eddy viscous damping. We find that the resulting perpendicular transport \(\chi_{ei}^\perp\) scales as \(T_e^{1/2}/n\), which is consistent with Alcator scaling, and is of the same order of magnitude as the experimentally observed transport coefficient.

*Research supported by the Department of Energy.
2) J. F. Drake and C. S. Liu, see separate abstract at this meeting.
Observation of Transport in Tokamaks of Arbitrary Shape
and Approximate Numerical Description

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Techniques for accurately measuring local values of heat transport coefficients have been reported previously.¹⁻⁴ They involve the analysis of electron temperature perturbations produced by internal disruptions, made apparent in fluctuations of the soft x-ray Bremstrahlung emission. The advent of elaborate multi-channel x-ray diagnostics looking at the plasma cross section in two orthogonal directions will allow to obtain, by means of the quoted methods, local measurements of heat transport flowing in these two directions. For elliptical cross section plasmas such as those to be produced in ISX-B, values of local transport differing by a large (2 to 4) factor should be observed depending on the direction of observation, because of the influence of geometry. The descriptions of transport in noncylindrical geometry usually imply a surface averaging of transport, effectively effacing local features. The comparison of these descriptions with accurate local measurements does not seem very meaningful. In particular, the influence of transport on the evolution of geometry is not clear. Some preliminary work is presented on approximate computational methods being developed with the purpose of taking into account the effects of local values of transport.

² Visitor from Junta de Energia Nuclear, Madrid, Spain.
EQUILIBRIUM NUMERICAL STUDY OF
THE FORMATION OF THE PLASMA IN TORMAC*

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Results from our two-dimensional, single fluid resistive magnetohydrodynamic simulation of TORMAC are presented. We find that with the existing design, TORMAC suffers from a serious "start-up" problem. Starting with some reasonable initial conditions, the plasma fails to implode towards the desired min-B cusp equilibrium. The reasons for this failure are presented.

We also present some modifications to the external current distribution and timing, which greatly improve the behavior of the plasma during the implosion phase. With these modifications, the plasma is able to implode to a state which resembles the desired cusp equilibrium, and the overall performance, in terms of confinement and temperature, is better than the original TORMAC's.

* Work supported by U.S. Department of Energy under contract EY-76-S-02-2456.
Two-Way Diffusion Equations and Diffuse Reflection of Lower-Hybrid Waves

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We consider the general two-way diffusion equation, $h(\theta)\partial f/\partial x = (\partial/\partial \theta)(D\partial f/\partial \theta)$, for $0 < x < L$, which is well-posed when initial (in x) conditions are given where $h$ is positive and final conditions where $h$ is negative. Here separation of variables does not yield a complete set of eigenfunctions; however, we prove that supplementing that set with a linear (in x) eigenfunction obtains completeness. This eigenfunction expansion has been used in the special case of diffusion through a slab. Another special case of interest is the propagation of lower-hybrid waves through density fluctuations, which can be described by diffusion in perpendicular (to $\vec{B}$) velocity space. The transmitted power falls off only algebraically as the inverse fluctuation thickness, i.e., $-1/L$. We use the eigenfunction expansion to numerically find the transmitted and reflected spectra.

*Work supported by U.S. DoE Contract No. EY-76-C-02-3073.

EBT Neoclassical Ion Transport with Non-Maxwellian $f_0$ and Higher Order Poloidal Expansions* - R. B. Campbell, T. Kammash, The University of Michigan, Ann Arbor, Michigan 48109, and R. J. Kashuba, McDonnell Douglas Corporation, St. Louis, Missouri 63166

There are two basic assumptions inherent in all neoclassical transport calculations for bumpy tori reported on to date (1,2). The first is that the distribution function can be Fourier decomposed in poloidal angle $\Theta$, and only terms up to the first harmonic ($1, \cos \Theta, \sin \Theta$) need be retained. The second states that the $\Theta$ independent distribution is Maxwellian, $f_0 = f_M$. For a simple BGK collision model, relatively high collisionality $\nu\lambda_0 > 1$, and for a $\Theta$ independent poloidal drift frequency $\Omega \approx \Omega(\Theta)$, these assumptions are justified. However, in the lossy regions of velocity space, where $\Omega \approx \Omega_\infty$, the poloidal structure of $\Omega$ becomes very important. Moreover, for a collisionless plasma $\delta^3 < \nu\lambda_0 < \delta$; $\delta / \gamma_\infty$, the zero-order distribution exhibits non-maxwellian features $f_0 = f_{LC}$, specifically a "loss cone" distribution.

Our calculation is based on the bounce-averaged drift kinetic equation for ions, in which we use a simple BGK collision model. The collision frequency is velocity dependent and modeled to crudely take into account the large velocity space derivatives near the $\Omega \approx \Omega_\infty$ loss cone. With this model, and with the restriction $\nu\lambda_0 > \delta^3$ (2), we have calculated four variations of the neoclassical particle diffusion coefficient, $D'$. Two of the coefficients are computed using poloidal terms up to and including the first harmonic ($\cos \Theta, \sin \Theta$), one with $f_0 = f_M$, and the other with $f_0 = f_{LC}$. The other two coefficients are similar to these, but we have retained up to and including the second poloidal harmonic. Sample results are sketched below for typical EBT-S operating conditions. The loss cone results can be interpreted physically by noting that the regions where $\Omega \approx \Omega_\infty$ are depopulated, thus reducing the driving term in the neoclassical fluxes in this region. The plateau result in ref. 2 relies on this lossy area contributing greatly to $D'$, as it should in that case, since the assumption $f_0 = f_M$ was made. This, however, implies that the bulk distribution neither sees nor responds to the loss cone. The introduction of non-maxwellian $f_0$ driving terms seems rather important at low collisionalities since it appears to allow $D'$ to scale like $\nu\lambda_0$, decreasing with decreasing collisionality.

\[ E_\theta \approx 0 ; \quad \frac{e\phi}{kT} = -1 \]

\[ \log_{10}(D') \]

\[ \log_{10}(\nu\lambda_0) \]

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*work supported by DOE
Argonne Beam Propagation and Target Experimental Program for Proposed Heavy Ion Facility

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Abstract

Argonne National Laboratory is proposing to test the feasibility of heavy ion beam drivers for inertial confinement fusion. The first proposed facility would irradiate small targets, and study beam propagation in a background gas with Xe beams. The proposed program structure contains two phases. The Phase I facility could be used to create Xe beams with particle energies of 1.6 to 6.4 GeV and pulse lengths of 2 to 10 nsec. The total beam energy on target would be 1 to 8 kJ. For Phase II the total energy would be increased to 40 to 50 kJ.

We will discuss the direction and goals for the associated programs. Simple unclassified ion beam driver compression targets as well as foil targets will be described, and implosion calculations presented. The important heavy ion background plasma beam instabilities will be reviewed. A discussion of diagnostics for the experimental programs will also be given.
A Finite Element Solution of a Reduced Fokker-Planck Equation

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In this paper we compare several methods for the numerical solution of a reduced Fokker-Planck equation, and conclude that one method, the Rayleigh-Ritz finite element method, has innate advantages over the others. In particular, a minimum principle yields approximations to the flux across the boundary, the curved portion of the domain has a natural boundary condition which needs no special treatment, and a singularity in the solution can be handled easily using nonuniform meshes. We describe an efficient implementation of the Rayleigh-Ritz method using tensor products of one-dimensional piecewise-polynomial basis functions and numerical quadrature rules. We present numerical results comparing the flux estimates obtained using the Rayleigh-Ritz method to previous analytic and numerical flux estimates.

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Alpha-Particle Heating in Tokamaks

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A Monte Carlo alpha-particle heating routine for use in 1-D tokamak transport codes has recently been developed. The algorithm models first-orbit losses, spatial spreading of the plasma heating profile which is due to wide banana orbits, and the time delay in the heating due to the finite slowing-down time. The alpha-particles follow orbits prescribed by the first-order guiding-center drift equations for axisymmetric tokamaks. The average drag caused by Coulomb scattering is used to compute the bulk heating rates for the electrons and ions and self-consistently slow down the sample alpha particles; changes in the orbits caused by the drag are included. We compare our results to previous calculations of the influence of the plasma current and of changes in the alpha-particle orbits on the heating profiles. We then present results from an extensive parameter survey in which we varied the aspect ratio, the current profile, the density profile, the $T_e$ profile, the alpha birth profile, and the position of the wall which acts as the alpha-particle limiter.

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Tearing Modes in a Braided Magnetic Field

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Magnetic braiding, together with large electron mobility parallel to $\mathbf{B}$, has been suggested as an explanation for the anomalously large electron thermal conductivity observed in toroidal confinement experiments. Simple estimates demonstrate that the anomalous electron viscosity $\mu$ that should be an additional consequence of the magnetic braiding can yield tearing mode growth rates much larger than those determined by the (observed) classical resistivity. We calculate the growth rate of the $m=1$ tearing mode as well as that of the (constant-$\psi$) $m \geq 2$ tearing mode. These rates scale as $\mu^{1/5}$ and $\mu^{1/3}$, respectively.

The nonlinear behavior of the constant-$\psi$ mode is calculated in analogy with a previous calculation for the resistive problem. When viscosity dominates, the island width $w$ increases in time as $t^{1/3}$, at a rate much faster than the constant rate determined by resistivity alone. Our estimates indicate that this should typically occur during the initial growth of the island, when its size is a small fraction of the discharge radius; at later times, the usual resistive growth dominates. We also examine the possibility that electron viscosity of this type could play a role in the disruptive instability. Recent calculations suggest that field line stochasticity may onset abruptly as the island width exceeds a critical value. We have modeled this effect by rapidly increasing $\mu(t)$ during a short interval. Immediately upon this increase, $\dot{w}(t)$ accelerates, followed by a return of $\dot{w}(t)$ to the resistive rate. The impulsive increment in $w$ can be a moderate fraction of the discharge radius for plausible values of $\mu$ and could be responsible for triggering disruptions.


Coupling of Lower Hybrid to Acoustic Modes

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We calculate the convective amplification factor and absolute instability thresholds and growth rates for decay of lower hybrid + lower hybrid + acoustic quasimode. Density and temperature gradients establish the thresholds by limiting the spatial extent over which appreciable ion response occurs. Relative pump bandwidths greater than \((m/M)^{1/2}\) can substantially reduce the convective growth factor.

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A POSSIBLE STRANGE ATTRACTOR IN MHD CONVECTIVE INSTABILITIES

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and

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ABSTRACT

We discuss the possibility that in a tokamak for sufficiently large temperature gradients, the convective motion driven by those gradients evolves into a strange attractor. Such dissipative (non-Hamiltonian) motion is intrinsically stochastic, similar to that previously encountered by E. N. Lorenz. This model of MHD turbulence is expected to lead to enhanced heat transport.

We begin by introducing the concept of a strange attractor as it arises in fluid mechanics. Next, motivated by physical considerations, we present a theorem from which the level of turbulent fluctuations may be estimated. Finally, in the context of a simple model, we discuss the possible implications for tokamaks.

References


Cubic Turbulence*

by D. R. Nicholson and D. F. DuBois**

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The nonlinear interaction of a wave with itself is often described by a cubicly nonlinear partial differential equation. Important examples include the nonlinear Schrödinger equation model of Langmuir turbulence, and a model for the nonlinear interactions of drift cyclotron modes which has been applied to mirror plasmas. We develop an approximate statistical theory for such equations. While our development is new, the results are implicit in the elegant formalism of Martin et al. Our theory is analogous to Kraichnan's direct interaction approximation for quadratically nonlinear equations.

We present a progress report of an investigation of the properties of this approximate theory. For example, when applied to the nonlinear Schrödinger equation, the theory yields the oscillating two stream instability as an almost trivial consequence.

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** Address: Los Alamos Scientific Laboratory

The propagation properties of electromagnetic waves in a bounded plasma imbedded in a tokamak type nonuniform magnetic field is studied numerically in the ion cyclotron frequency ($\Omega_i$) range. By solving the coupled wave equations, it is found that, in addition to the global fast cavity mode with $\omega > \Omega_i$, the slow ion cyclotron wave with $\omega < \Omega_i$ can also be a global mode provided that the parallel wavelength is sufficiently short ($k_\parallel c^2/\omega^2 \approx \omega^2_{pi}/\Omega_i (\Omega_i - \omega) > \omega^2_{pi}/(\Omega_i^2 - \omega^2)$). This slow mode has a dominant left-hand polarized electric field and can propagate within the plasma. A unique feature of this mode is that it always has a small poloidal mode number with respect to its center. Since the ion heating is primarily from the left-hand polarized electric field, this slow mode gives a more efficient heating and better energy deposition pattern than that from the fast mode, which has a small and usually ill-placed left-hand polarized component. A recent experimental observation of the increase of loading at $\omega \approx 0.8 \Omega_i$ seems to indicate the excitation of this slow ion cyclotron mode in a tokamak type device.\(^1\) The ramifications of this mode will be discussed in the context of RST and PLT.

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GUIDING CENTER PLASMAS IN THE PRESENCE OF GRAVITATIONAL AND ELECTRIC FORCES

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The most common feature of the two-dimensional guiding center plasma model has been its propensity for the nonlinear formation of large-scale vortices and for exhibiting associated enhanced transport. The large vortices can result from a variety of stimuli: high initial energies, external electric fields, or microscopic instabilities driven by gradients. Here we point out another, essentially universal, mechanism: magnetic field gradients, or more simply, a uniform gravitational field. The essential feature is a guiding-center drift which depends upon the sign of the charge. A slab geometry is considered, with an electric field \( E = -\nabla \phi, \phi=\phi(x,y,t), \) \( B = B_0 \hat{z}, \) \( g = -g \hat{y}, \) \( \partial / \partial z = 0. \) Simulations involving 10,000 particles, with periodic boundary conditions in \( x \) and \( \phi = 0 \) on the \( y \) boundaries, are started from a condition of zero electrostatic energy by randomly loading pairs. Electrostatic energies develop which reach maxima above the random loading value (the threshold value for the onset of the negative temperature regime in the \( g = 0 \) case), and then execute large fluctuations. A maximum in the electrostatic energy as a function of \( g \) is observed, and is unexplained. A most-probable-states analysis leads to the Poisson equation

\[
\nabla^2 \phi = -4\pi e \left( n_0 \exp[-(e\phi + m g y)/\Theta] - n_0 \exp[-(e\phi + m g y)/\Theta]\right)
\]

which has no spatially uniform solution for either sign of \( \Theta. \) Probably the most interesting feature of the analysis is that it appears possible to pass from negative to positive temperatures by changing only the boundary conditions. Clamping the potential between the two faces normal to \( g \) at the same value can enhance the vortex formation in contrast to the situation where a finite potential jump is permitted. This suggests the possibility of suppressing potential island formation in multiplexes by applying appropriate potential differences between the current-carrying rods and the walls.
FIELD REVERSED PLASMA ROTATION AND TRANSPORT

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Field Reversed Plasmas (FRP) have been observed to spin up and subsequently develop a destructive m=2 stability. While the stability limit for spin velocity has been characterized approximately, the cause of rotation remains unclear although several mechanisms have been proposed. We present theoretical evidence for one of these, i.e., the shorted, rotating, open field line plasma causes plasma on unshorted closed lines to spin up by viscous friction. This model seems to explain several observations; the time delay in onset of spin, the rapid spin up rate, and the achievement of rotation rates faster than the diamagnetic drift frequency.

The decay of stable FRPs is necessarily a two-step process; particle diffusion from closed to open field lines followed by convective streaming along field lines. Since the latter process is typically much faster, it will rapidly deplete the open line plasma, leading to steep density gradients near the separatrix; this will accelerate the cross-field diffusion and possibly excite anomalous processes. We describe a model based on a quasi static open field line plasma. The results indicate that the global confinement is a geometric combination of the diffusive and streaming timescales (the geometric mean for the case of classical transport). In addition there is an extremely sensitive dependence on aspect ratio; low aspect ratio plasmas have much faster instantaneous decay rates.

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NONLINEAR SATURATION OF BALLOONING MODES FOR TOKAMAKS

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A CRAY version of our equilibrium and stability code for plasmas in three-dimensional toroidal geometry has been written that runs 30 times faster than the published version for the CDC6600 (cf. F. Bauer, O. Betancourt and P. Garabedian, "A Computational Method in Plasma Physics," Springer Series in Computational Physics, Springer-Verlag, New York, 1978). Fourth order accurate estimates of the energy landscape can be calculated. The improved code has enabled us to study nonlinear saturation of instabilities for screw pinches with realistic distributions of pressure and rotational transform. After difficulties stemming from truncation error in the variational method are overcome, results are obtained that go beyond what has been learned from linear stability theory. An investigation of the saturation of ballooning modes for Tokamaks is in progress.
Free Boundary Equilibria with Multipole Expansion of External Field in Noncircular Tokamaks

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The external magnetic field for noncircular equilibrium is studied in terms of multipole moment coefficients, and these coefficients are expressed in terms of geometrical parameters and the plasma properties. The noncircularity for a given external field configuration is compared with the exact numerical calculation.

The Grad-Shafranov equation is solved for arbitrary noncircular tokamaks. Noncircularity, e.g., b-a/b+a for ellipticity, is assumed to be of the order of the inverse-aspect-ratio; therefore, the elongation ratio of up to about 2 is covered within the framework of the present study. The external poloidal flux is expressed in terms of multipole moment coefficients. By expressing the external field requirement with the moment coefficients, we can directly invert the problem, namely, the geometrical parameters such as a major radius, ellipticity, triangularity, etc., are obtained from the given external fields.

The analytic result is compared with the result of numerical calculation by the Princeton Equilibrium Code, which gives a satisfactory agreement.

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**On leave from Central Research Laboratory, Hitachi Ltd., Tokyo, Japan.
SPECTRUM AND EIGENFUNCTIONS FOR A FIELD EQUATION
WITH STOCHASTIC RAY TRAJECTORIES*

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As a model for linear wave equations arising in plasma physics, with non-
separable geometry, we investigate the two-dimensional Helmholtz equation
$(\nabla^2 + k^2)\psi(x) = 0$. For a racetrack boundary, whose only parameter is its aspect
ratio, the ray trajectories (of geometrical optics) are stochastic, except for
the limiting case of a circular boundary. We examine the eigenvalue spectrum (for
a given parity), and find the eigenvalues well-spaced, with no near degeneracies;
in contrast, for the circle, the eigenvalues are highly clustered, with frequent
accidental near-degeneracies. The eigenfunctions are qualitatively different for
the racetrack and the circle: the racetrack's eigenfunctions have random-looking
nodal curves, which almost never cross. Also, they fill the whole area uniformly,
in contrast to the Bessel eigenfunctions of the circle. The sensitivity of these
features to the aspect ratio will be presented.

*Work supported by the Office of Fusion Energy of the U.S. Department of Energy

MAGNETOHYDRODYNAMICAL INTERCHANGE INSTABILITY IN LOW-β PLASMAS
IN SHEARED SYSTEMS

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ABSTRACT

The stability criterion of a magnetized plasma with respect to interchange in sheared configuration was reexamined. By retaining the finite growth rate, the singularity at the magnetic surface, where the perturbation is constant along the magnetic line, is removed. The resultant wave equation yields the result that the Suydam condition is both a necessary and a sufficient condition for stability at least for the plasma pressure less than \((1 - Δ)β_c\). Here \(β_c\) is the maximum \(β\) for the original Suydam criterion and \(Δ\) is a small positive number (less than 0.03), which presumably can be made arbitrarily small by improving the numerical approximation.

Work supported by U.S. DoE Contract No. EY-76-C-02-3073.
We have made an analytical and numerical analysis of the drift-wave instability in a sheared magnetic field by taking both the current and the finite-β effects into consideration.

Recent analysis has shown the stability of the collisionless drift wave in a sheared magnetic field. The plasma current which produces the magnetic shear can, however, drive an instability. In low-β plasmas where the drift wave can be treated as electrostatic, the stability criterion is obtained numerically as shown in Fig.1. Here u is the electron drift velocity along the magnetic field and the other notation is standard.

For β>m/M, the electron-to-ion mass ratio, the drift wave couples to the Alfvén wave. We found that although this coupling tends to stabilize the short wavelength modes, as shown previously,1) it destabilizes a long wavelength mode. Our method is to first derive a coupled set of equations for the electromagnetic drift wave in the presence of both the current and the finite-β value, and then apply the technique similar to that of Antonsen for the electrostatic mode to the coupled equations. The result indicates the existence of a new-type of instability driven by the combined effect of the current and the coupling to the Alfvén mode. We then obtained numerical solutions for the localized mode of the coupled equations and derived the stability criterion which is shown in Fig.2. In the numerical analysis, we developed a new technique, i.e. the Newton method, in order to obtain the eigenvalue by the shooting method. This technique has considerably improved the convergence of the iterative solution. The result shows that the drift wave, when coupled to the Alfvén mode, can be driven more unstable by a current of substantially smaller electron drift velocity than the electrostatic case.

Two-Dimensional Eigenmode Analysis of the Trapped-Ion Instability*

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An analysis of the two-dimensional eigenmode structure of the trapped-ion instability in axisymmetric toroidal geometry is presented. The approach is based on the drift-kinetic equation in which collisions are modeled by an energy and pitch-angle-dependent Krook operator. The perturbed electrostatic potential $\phi$ is expanded in a Fourier series in $\theta$ to account for the poloidal structure and each harmonic $\phi_m$ is expressed as a truncated Taylor series in the minor radius to account for the radial structure. The governing equations take into account the spatial variations in the equilibrium profiles (e.g., density, temperature, etc.). They also allow for the analysis of eigenfrequencies both less than and of the order of the average ion transit frequency. In addition to the two-dimensional problem, our basic analysis is applied to the familiar radially local problem, in which radial derivatives of the perturbed potential are ignored, and to the one-dimensional radial analysis of Gladd and Ross, in which the mode is assumed to be nearly flute-like. Results corresponding to these two special cases are found to be in reasonable agreement with previous calculations.

The main original contribution of this work, however, is that it is capable of treating the full two-dimensional structure of the instability over the whole plasma cross section. A comparison of a full two-dimensional calculation with its corresponding one-dimensional counterpart shows significant quantitative and qualitative differences in the mode structure and eigenfrequency. Our approach assumes the large aspect ratio limit with circular, concentric magnetic surfaces. It is limited to electrostatic, long wavelength perturbations for which $k_r c_{bi} < 1$ is well-satisfied, where $k_r$ and $c_{bi}$ are the typical mode radial wavelength and the ion banana width.

* Work supported by U.S. Department of Energy Contract #EY-76-C-02-3073.

ANALYSIS OF PLT DISCHARGES WITH HIGH NEUTRAL INJECTION*

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ABSTRACT

We have analyzed a PLT discharge with high neutral injection (P = 2.4 MW) with the 2-D tokamak transport code, G2M, including the previously developed global transport model. 1 The model accounts for the anomalous electron and ion heat and particle transport arising from drift-tearing islands in the outer region, from trapped electron modes in the central region, and from sawtooth modes inside the region, q = 1. Also, the programmed input rate of neutral gas was taken in order to study the density evolution. We find that the build up of the electron and ion temperatures, T_e and T_i, and the density as well as the displacement of the flux surfaces is in close agreement in space and time with experimental results. During the heating process where the ion temperature jumps from 1 keV up to 5.7 keV, the transport improves in the center due to the particle loading increasing the density, though the safety factor remains below 1. As a consequence, the energy lifetime, after a deterioration at the beginning of the turn on of the neutral beam, shows a slow recovery, varying from 37 ms down to 18.5 ms, and then back to 23.2 ms. Various properties of the system during its time evolution are likewise discussed.

* Work supported by the U.S. Dept. of Energy.
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CONDUCTING SHELL STABILIZATION OF FCT EQUILIBRIA*

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The computer code ERATO has been used in an ongoing study of the stability properties of D-shaped flux conserving tokamak (FCT) equilibria with aspect ratio of 4 and elongation of 1.65. As reported previously, equilibria were found with in excess of 10% and stability for values of the toroidal mode number. These equilibria had \( 2.5 \) (about half the aspect ratio) and a q ratio (safety factor at the edge/safety factor at the axis) of 2.0. A conducting shell was assumed, having a radius 20% larger than the plasma radius. Additional studies have now been completed with the conducting shell removed to infinity. The resulting change in stability is about as one might expect on the basis of conducting wall stabilization: the critical value falls from 16% to 10% for \( n=1 \), but varies only slightly for the higher modes whose radial wavelengths are small enough to provide isolation of plasma effects from the shell. To minimize the higher modes and maximize \( \beta \), these equilibria have relatively broad current profiles; they rely on the D-shapedness of the cross section to keep \( q \) above the Mercier criterion limit at the axis and above the empirical limit \( q \) surface \( \geq 2-3 \).

Because plasma stability is determined by the most unstable model, we conclude that wall stabilization is not required to obtain stability at finite \( n \) for \( \beta \) values as high as 10%.


\(^{1}\)D. Berger, R. Gruber and F. Troyon, Second European Conference on Computational Physics, Garching, Germany (1976).

OPTIMIZATION OF TRANSITION COIL DESIGN IN TANDEM MIRROR SYSTEMS FROM THE POINT OF VIEW OF INTERCHANGE STABILITY

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ABSTRACT

Tandem mirror systems are potentially subject to curvature-driven ballooning-interchange modes because of unfavorable field-line curvature in the transition regions between the minimum-B end cells and the axisymmetric central cell. One way to alleviate this problem is to shape the transition coils in a way that optimizes the stability of the system with respect to these modes.

A technique is presented for optimizing the coil design with respect to a particular subclass of curvature-driven instabilities, viz., those with a purely interchange character. The calculation uses a variational procedure to find the transition field configuration that maximizes the value of the quantity $\int d\omega (K_n/B) \partial (p_\perp + p_\parallel) / \partial \psi$, whose positivity is a sufficient condition for stability. $K_n$ is the normal component of curvature, $p_\perp, p_\parallel$ the plasma pressures perpendicular and parallel to the magnetic field, and $\psi$ the magnetic flux.

This approach to improving interchange stability generalizes to finite $\beta$ and anisotropic pressure one based on the low-$\beta$, isotropic pressure stability criterion $\nabla p \cdot \nabla f d\omega / B \geq 0$ used by Riordan et al in the design of the Multiple Mirror Experiment.

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CROSS-FIELD ELECTRON TRANSPORT DUE TO THERMAL ELECTROMAGNETIC FLUCTUATIONS*

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The cross-field electron particle and energy transport for a high $\beta$ thermal plasma is investigated using a magnetostatic plasma model. The diffusion process due to magnetic fluctuations alone is observed to have a Bohm-like scaling, whereas in the presence of both electrostatic convective cells and magnetic fluctuations, the diffusion, because of the strong interaction between these static modes, becomes smaller than the diffusion assuming these modes are independent.

*Work supported by USDOE.
Magnetohydrodynamic Instabilities in a High Shear Helical System

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A rotational transform exceeding unity at the plasma surface, \( \lambda_F(a) > 1 \), and a high shear can be produced in a helical heliotron configuration with \( l = 2 \) short pitch helical coils \(^1\). Current driven kink and tearing modes and pressure driven low \( m \) modes are investigated by applying the stellarator expansion \(^2,3\) to the heliotron configuration.

For the low \( \beta \) current carrying heliotron configuration, the kink and tearing modes become unstable along the line of \( \lambda_{OH}(0) + \lambda_F(0) = n/m \) in the \((\lambda_F(a), \lambda_{OH}(a))\) stability diagram. The \( m = 1 \) modes with \( n > 2 \) give wide unstable regions and large growth rates. Eigenfunctions of kink modes are fairly localized inside the plasma column when the resonant surface exists outside the plasma column. Comparison is made with the results of Heliotron-D experiments \(^4\).

For finite \( \beta \) plasmas, the pressure terms destabilize the low \( m \) modes and give larger growth rates and wider unstable regions than the low \( \beta \) case. The validity of the stellarator expansion is examined by comparing our results with the initial and boundary value problem of linearized MHD equations for the heliotron configuration \(^5\).

For current-less finite \( \beta \) plasmas, \( \beta \) limit due to low \( m \) pressure driven modes is estimated and \( \beta \geq 5\% \) may be expected by tailoring the pressure profile according to the shear parameter.

NONLINEAR KINK INSTABILITIES IN FORCE-FREE FIELDS*

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The theory of Pao\textsuperscript{(1)} for the nonlinear behavior of linearly unstable kink modes in a sharp boundary plasma surrounded by a vacuum is extended to include force-free and distributed plasma currents. Nonlinear stabilization of the external kink (m = 1) instability by force-free fields is enhanced by increasing the current strength in the force-free region up to an optimum value. For fixed force-free currents, nonlinear kink stability is always decreased by increasing the strength of the distributed plasma current. In general, the optimum value of the force-free current increases as the plasma current increases. A condition to determine the nonlinear kink stability is derived for long wave length perturbations.


* Work supported by U.S. Department of Energy under contract EY-76-S-02-2456.
ANOMALOUS DIFFUSION AND PLASMA LEAKAGE
THROUGH OPEN FIELD LINES IN
FIELD REVERSAL CONFIGURATIONS*

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ABSTRACT

Using G1M/HYBRID code, which features self-consistent anomalous resistivities due to various microinstabilities, we have studied anomalous diffusion in reversed field configurations like field reversal experiments (FRX) of LASL. In the FRX, the open field lines at outer regions support the inner closed reversed field configuration. The plasma contained in the closed field region gets diffused out into the open field region due to the lower hybrid drift or drift ion cyclotron instability. The diffused-out plasma leaks out through open field lines, which enables the plasma edge near the open field lines to sustain, in spite of large diffusion, a finite gradient which feeds the microinstabilities. In this manner, the microinstabilities pump out the plasma from the closed field region to the open field region. Our calculation of these leakage rates due to the lower hybrid drift instability and the drift ion cyclotron instability showed a rough agreement, in order of magnitude, with the experimental data of the FRX even though the FRX usually develops the rotational $m = 2$ MHD instability which terminates the containments.

*Work supported by the U.S. Dept. of Energy.
Tensor pressure equilibria are computed for small aspect ratio tokamaks of arbitrary cross section. The perpendicular and parallel pressures are evaluated from a distribution function that models neutral beam injection. The stability of these equilibria to ballooning modes of large toroidal mode number are examined by numerically solving an Euler equation derived from a guiding center fluid energy principle. This second order ordinary differential equation is similar in form to the corresponding ideal MHD equation. The criteria obtained are either necessary or sufficient for the stability of a guiding center plasma depending on whether the double adiabatic contribution to the Euler equation is kept or ignored.

IMPURITY CONTROL BY NEUTRAL BEAM INJECTION

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The use of momentum transfer due to neutral beam injection to drive impurities out of a plasma was apparently first suggested by Ohkawa\(^1\) and has subsequently been examined by several workers.\(^2\)\(^5\)\(^6\) Stacey\(^6\) showed that a general external momentum source could drive radial particle fluxes, in the collisional regime. While the direct effect of external momentum transfer was treated in these earlier works, two important indirect effects were omitted: the modification of the first-order flows in the flux surface and the adjustment of the radial, ambipolar potential gradient in response to beam injection and external drags.

We have used a recently developed generalization of neoclassical theory,\(^7\) which is valid when external momentum sources and drags are present, to make a self-consistent calculation of particle transport in a tokamak plasma in the presence of neutral beam injection and external drags. Our formalism is valid in all collisionality regimes and for arbitrary geometry and beta, although we specialize to the low-$\beta$ limit for clarity. We establish the conditions for which coinjection drives impurities out of a plasma. We estimate that order-unity effects could be observed in PLT and ISX-B, for example, and that beam injection might be a feasible means of impurity control in a reactor-type plasma.

Three important subsidiary results are contained in our work. Since the formalism is developed for a general momentum input, it is generally applicable for the analysis of impurity control by other forms of toroidal momentum input (e.g. rf). As an auxiliary result, we have extended the theory\(^8\)\(^9\) for impurity control by asymmetric particle sources to all collisionality regimes. Finally, we have worked out the general theory for particle transport (subject to the Lorentz form of the friction) in a two-species plasma.

Acknowledgement: This work was sponsored by USDOE.

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STABILITY OF NEUTRAL BEAM HEATED EQUILIBRIA TO BALLOONING MODES

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ABSTRACT

Neutral beam heated PLT equilibria are modeled using the General Atomic 1-1/2-D transport code.* The stability of these equilibria to localized ballooning modes is studied as a function of time during the neutral beam heating. The ballooning stability dependence on the transport generated current profile is illustrated by changing transport coefficients, plasma density, and neutral beam injection parameters. Special attention is paid to current broadening caused by poor beam penetration and its effect on ballooning stability.

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RESONANT SECOND HARMONIC GENERATION OF UPPER HYBRID RADIATION IN A PLASMA

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In the vicinity of upper hybrid resonance (in the higher density regime) an upper hybrid EM wave resonantly excites a second harmonic upper hybrid radiation. The efficiency of harmonic generation is limited by the wave number mismatch created by the density gradient. However, the yield of harmonic conversion for typical laser produced plasmas could be as high as 5% which is in order of magnitude agreement with the experimental observations. This mechanism of harmonic generation is operative in tokamak type plasmas also.

* On leave from Physics Department, Indian Institute of Technology, New Delhi, and supported by Indo-American Exchange Program.

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The perpendicular energy gain, $\Delta W_\perp$, of electrons from the applied extraordinary microwave field in EBT is calculated by means of the stochastical model\(^1\) for the field-plasma cyclotron resonance interactions. In these calculations, the inhomogeneous external bumpy field is chosen to be

$$B(z) = B_0 \left[ 1 - \left( \frac{M - 1}{M + 1} \right) \cos \left( \frac{2\pi}{L} z \right) \right]$$

which simulates the field strength reasonably well for the EBT.\(^2\) Here $M$ is the mirror ratio and $L$ is the distance between the mirror sectors.

The effects of the initial energy of the electrons as well as the mirror ratio on the trapped and untrapped electrons in the bumpy field are discussed. Then the heating rate $\Delta W_\perp/\Delta t$, $\Delta t$ being one reflection time from the mirror, is estimated for the trapped electrons.

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\(^1\)H. Grawe, Plasma Phys. 11, 151 (1969).

FINITE TEMPERATURE EFFECTS ON MICROWAVE PROPAGATION IN EBT

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ABSTRACT

A three dimensional ray tracing code, RAYS, has been developed as a part of the ongoing theoretical study of microwave heating in the Elmo Bumpy Torus device. Recently this code has been improved to include the effects of finite temperature on the ray paths as well as cyclotron damping. Our dispersion relation allows us to study effects at the second and higher harmonic resonances. In particular, we observe conversion of the extraordinary mode to electrostatic Bernstein waves at the upper hybrid resonance and second harmonic resonance. Total absorption rates are given for ordinary and extraordinary mode waves at first and second harmonic resonances for plasma parameters appropriate for EBT-I and projected parameters for EBT-II. We have investigated the influence which the choice of direction for the imaginary part of the refractive index has upon total absorption \( \int_0^s ds \frac{1}{k_1 \cdot \hat{V}_g} \). It was found that when the absorption is weak \(|k_1| \ll |k_2|\) the total absorption is virtually independent of the direction of \(k_1\).

A Simple Annulus Power Balance in EBT-I

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An essential feature of the ELMO Bumpy Torus (EBT) concept is the presence of a relativistic electron annulus in each of the toroidal mirror sections. These high-beta annuli are formed and sustained by microwave heating and are of sufficient density and temperature that diamagnetic currents produce the necessary minimum in the magnetic field required for MHD stability of the toroidal core plasma. Since the electron rings play an important role in the confinement characteristics and performance of EBT, the trade off between the formation of the rings and the power required to sustain them represents an important problem in a fusion reactor. A power balance for the rings which includes drag cooling in addition to the radiation (synchrotron and bremsstrahlung) and annulus electron-electron scattering losses, indicates that drag dominates the annulus energy balance in EBT-I. Drag cooling of the relativistic annulus electrons on the toroidal core plasma appears to provide a reasonable explanation for the decrease in the annulus electron temperature in going from ELMO to EBT-I. Theoretical estimates of the microwave power required to sustain the annulus are found to be within a factor of 2 of the experimentally determined value. Scaling projections are shown for EBT-S and parametric study results for ELMO, EBT-I and EBT-S are presented for various microwave power levels. The Maxwellian distribution function is assumed in these calculations. The results are found to be sensitive to the details of the hot electron distribution function as well as geometric and scaling parameters. Improvements to the model are underway in order to increase its capability and accuracy in assessing the overall power balance.

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RESONANCE WAVE-WAVE COUPLING AND PONDEROMOTIVE EFFECTS IN LOWER-HYBRID HEATING

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ABSTRACT

The electrostatic particle simulation code "EZOHAR" has been applied to study plasma response to high-power, lower-hybrid heating. In general, strong edge heating of electrons and ions, and tail heating of electrons in the interior have been observed. Spatial density modulation by ponderomotive forces does not occur at the edge due to large velocities of particles, but such ponderomotive effects may appear inside the plasma, depending on the density profile. New phenomena, such as resonance excitation of Langmuir waves, have been found where the Langmuir frequency is a harmonic of the external frequency. The electric field associated by these higher harmonics may become comparable with the fundamental modes at the interior.

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Effects of Ion Dynamics on Tearing Modes

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Abstract

Using a simple self-consistent derivation of ion dynamics, we find two terms contributed by ions to the parallel electrical conductivity. One corresponds to the familiar ion acoustic term, and the other, which we call the 'frictional term', is due to the ion-electron collisions. The tearing mode equations are modified by adding these terms to the parallel conductivity, and are then solved to analyze their effects. The stabilizing tendency of the ion acoustic term\(^1\), earlier found on the semi-collisional drift-tearing mode, is found to be true for several other tearing modes. The effect of the ion frictional term is also investigated, and is found to be destabilizing in one case. Quantitative expressions for the change in mode frequency are given. We further show that most known unstable tearing modes are, in fact, the manifestations of the same mode in different regimes of plasma parameters.


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Nonlinear Interactions of Drift-Alfven Waves*

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We present a general gyro-kinetic formalism for the nonlinear interactions of kinetic drift-Alfven waves. The nonlinear equations thus derived include full finite ion-Larmor radius (FILR) effects and are valid in the strong-turbulence regime. Applying this formalism to the parametric decays of kinetic drift-Alfven waves, it is found that the nonlinear decay processes are modified, both qualitatively and quantitatively, by the FILR and diamagnetic-drift effects.

* Work supported by U.S. Department of Energy Contract #EY-76-C-02-3073.
BURN CONTROL VIA REGULATED RIPPLE
APPLIED TO REACTOR-GRAGE PLASMAS

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Control of a reactor-grade plasma may be lost if a sizable thermal excursion occurs after ignition is achieved. Because of its strong temperature dependence and its finite value in the critical center region of the plasma, the magnetic ripple produced by the toroidal field (TF) coils is a promising means of inhibiting such a thermal runaway.\(^1\) However, achieving this ripple via a conventional TF-coil configuration provides little flexibility for subsequent modification of startup operation. Furthermore, the degradation of confinement associated with ripple transport renders heating to ignition more difficult. Although there is a range over which fixed ripple values lead to a satisfactory burn, i.e., a stable, ignited plasma within prescribed beta limits, the size of this "window" suggests a somewhat restrictive operating mode.

These difficulties can be relieved by a TF-coil network characterized by a small number of large superconducting TF-coils supplemented by copper pull-back coils,\(^2\) a TF-array of the type employed in the recent NUMAK design.\(^3\) The dynamical scenario proposed is to activate the "correction" coils during startup to minimize the impact of ripple on both plasma confinement and auxiliary heating demands, and to reduce the current in these coils when ignition is achieved, thus enhancing ripple losses to a level sufficient for burn control. In this way, the ignition requirement is considerably relaxed, resulting in a much larger effective operating "window". Furthermore, the small number of superconducting coils provides a spatial distribution of ripple more suitable for burn control. This approach may provide the flexibility needed to track the plasma and to control the reactor power level.

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\(^*\) Work supported by Department of Energy, Contract EY-76-C-03-0167, Project Agreement No. 38.
\(^3\) R. W. Conn, et al., Trans. of Third Topical Meeting on the Technology of Controlled Nuclear Fusion, Santa Fe, New Mexico, 351 (May 1978).
Electron Landau Damping of Instabilities in Short, Fat, Field-Reversed Ion Rings

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The integral equation for the most dangerous \( \omega = \frac{v_A}{L} \) normal modes of a high energy, low density field-reversed ion ring immersed in a cool, high density background plasma is simplified to a form allowing practical numerical solution, for an arbitrary (axisymmetric) geometry. One qualitatively different feature of the short, fat ring, as opposed to the long layer or bicycle tire, is that electron Landau damping is much less effective, because there are far fewer resonant electrons. In a long layer or bicycle tire, the magnetic field is uniform along a given field line, and resonance requires \( \omega = k \parallel v \parallel \), i.e., \( v \parallel = v_A \). In a short, fat ring, the magnetic field varies along a field line, and electrons with \( v \parallel \leq v_\perp \) are trapped; for trapped electrons the resonance condition is \( \omega = \omega_b \) (where \( \omega_b = v/L \)), i.e., \( v = v_A \). For moderately large \( \beta \), \( v_e \gg v_A \), there are far fewer electrons with \( v = v_A \) than with \( v \parallel = v_A \). This result does not apply to ion damping since \( v_i \leq v_A \).

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Kink Instabilities of a Field Reversed Ion Ring with a Toroidal Magnetic Field

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The low frequency MHD stability of an axisymmetric field reversed ion ring in a current-carrying background plasma with a toroidal magnetic field is studied. A generalization of the energy principle\(^1\) which treats the background plasma by fluid equations and the beam by kinetic theory is employed. The major effects upon stability are the MHD response of the plasma and beam, a collective reaction of the beam, and betatron resonances.\(^2\) It is found that, if the beam and plasma currents are roughly equal, and if the exterior region contains a cold but highly conducting plasma, there is a window of stability for kinks with safety factor \(q\) as low as \(1/2\). The results may also explain the anomalous trapped current losses in the RECE-Christa electron ring experiment.\(^3\)

\(^*\)Work supported under U.S. Department of Energy Contract EY-76-S-02-3170.

With the use of an energy principle of W. Newcomb (LLL Report UCID 17182 (1976)) the stability of an axisymmetric mirror machine with anistropic pressure is studied. The analysis is particularly simple in the low beta limit. For a mirror machine the low beta limit may be taken in several ways. Unpublished results of W. Newcomb on a wide class of unstable systems are recovered. Other unstable configurations are also described. In the appropriate limit a stable configuration is given for a plasma with a very weak singularity in the magnetic field, $B$ is smooth but $\nabla B$ tends to infinity at a point. If such a singularity is smoothed out, then the plasma will be unstable only on the outer edge, where line tying may stabilize the system. Some numerical examples will be given and extensions to high beta systems will be considered.
The energy spectrum of drift wave turbulence with $|n_k/n_0| > \omega_k/\omega \ci$ is shown to be divided into two regions. One is the region with wavenumbers larger than a critical wave number $k_c$, in which the spectrum obeys the two-dimensional hydrodynamic dual-cascade law where the energy cascades into smaller wavenumbers. Here, the $\omega$ spectrum is broad and the unidirectional $k$ spectrum obeys $k^{-8/3}$ or $k^{-4}$ inertia range spectrum.

The other region has wavenumbers smaller than the critical wavenumber $k_c$ and the spectrum obeys the resonant wave interaction law of the weak turbulence theory. Here the $\omega$ spectrum is narrow and peaked near the frequencies at which the resonant condition, $\omega_k = \omega_k' + \omega_k''$ is satisfied for $\kappa = \kappa' + \kappa''$ and the spectrum cascades to lower frequencies. The critical wavenumber is decided roughly by the condition $k_c \rho_s = (\kappa/k)^{3/2} \left( e^{\phi_k} / T_e \right)^{-3/2}$ where $\rho_s = \sigma_s / \omega \ci$, $\kappa = |d \ln n_0 / dx|$.

The situation is analogous to Rossby wave turbulence.

REFERENCES

THERMAL FLUCTUATION LEVELS
AND CONVECTIVE AMPLIFICATION

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ABSTRACT

It is well known that the thermal level of fluctuations in a stable nonuniform plasma can greatly exceed the uniform plasma level due to local regions of instability. Using the method of Kent and Taylor,\(^1\) the thermal fluctuation spectrum for electrostatic drift waves in a sheared magnetic field is calculated (the extension to magnetic perturbations is straightforward). We find that the thermal level spectral function, over a wide range of parameters, is far below the experimentally measured level.

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Simulations of DCLC Modes Near Linear Marginal Stability
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ABSTRACT

A particle-fluid hybrid simulation code has been used to study nonlinear properties of the drift-cyclotron-loss-cone (DCLC) instability near marginal stability. We are using a simple local model focusing on ion nonlinearities. The simulation allows only electrostatic perturbations and adopts the usual one-dimensional slab configuration for drift wave propagation. The electron response is taken to be the low frequency (ω << ωpe, ωce), cold-fluid, linear response with E x B and polarization drift effects retained.

We have investigated Maxwellian, subtracted-Maxwellian and delta function f_i(v) for nonuniform plasmas near linear marginal stability, i.e., at low densities 2 ≤ ωp_i/ω_c_i ≤ 10. Drift-cyclotron and DCLC instabilities were observed which exhibited frequencies |Re ω| ∼ δ(ω_c_i) and growth rates 0 ≤ Im ω/ω_c_i ≤ 0.2 in good agreement with linear theory over the range 0.2 ≤ a_i/L_n ≤ 0.4. The most unstable wavelengths are characterized by ka_i ∼ (m_e/m_i + ω_c_i^2/ω_p_i^2)^{-1/2}. These grew from an initial small level to a large amplitude in all cases. Nonlinear saturation was accomplished by ion trapping as evidenced by particle orbits of selected particles, vortex formation in phase space, and amplitude oscillations of the electrostatic potential at the trapping frequency ω_t = δ(1)k|eϕ/m_i|^1/2 ∼ ka_i|eϕ/T_i|^1/2ω_c_i. Trapping was accompanied by spreading and filling in of the velocity distribution function, and by slowing of the average v_i. The implied relaxation of the density gradient increased dramatically with the onset of trapping but remained insignificant, < 1 or 2%. The saturation of a single dominant DCLC mode was in excellent qualitative agreement and in good quantitative agreement with the trapping theory of Aamodt and Bodner (1969), |eϕ/T_i| ∼ (ω_p_i/ω_c_i)/k^2a_i^2 at saturation. A good fit to the data for subtracted Maxwellian f_i(v) was also given by |eϕ/T_i, hole| ≤ 1 at saturation. Our simulation results near marginal stability exhibited a general insensitivity of the saturation levels with respect to a_i/L_n, in disagreement with some recent theoretical calculations. This, as well as the feedback influence of a self-consistent density profile, L_n(1) + L_n(t), will be described in further detail.

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Stability Analysis of
Runaway Distribution Function

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Abstract

The steady state electron runaway distribution function has been computed numerically using a spline collocation scheme for velocities \( u = v/v_e \leq 10 \). The tail of the distribution function (5 \( \leq u \leq 10 \)) has been fit by a simple analytical formula which is parameterized in terms of \( Z_{\text{eff}} \) and \( E/E_D \). The analytical formula is compared to previously derived theoretical formulas for the distribution in the runaway regime. The stability against the high frequency electrostatic mode due to the \( \lambda = -1 \) cyclotron resonance is then investigated using the analytical formula to extrapolate the distribution function to high velocities where numerical computation is inefficient. The stability boundary is calculated in terms of parameters \( Z_{\text{eff}}', E/E_D', \omega_p/\omega_c, k_\parallel \) and \( k_\perp \).

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THE NONLINEAR EVOLUTION OF THE ION MIRROR INSTABILITY*

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Collisionless shocks propagating normal to an ambient magnetic field heat a plasma primarily in the directions perpendicular to the field. The resulting anisotropic distribution function is unstable to modes which would reduce this anisotropy. In many fast \( \theta \) pinch experiments, the shock propagation time is much greater than classical electron self collision time, but much less than the classical ion self collision time. A study of these ion anisotropy reducing modes is thus necessary to the understanding of particle endloss from such pinches, since the endloss rate becomes large on timescales of the order of the implosion time.

The ion mirror mode, having unstable \( m = 0 \) waves, may be addressed by a two-dimensional (r and z) model. In order to include finite ion Larmor radius effects, this mode is studied with a hybrid (Vlasov ions, fluid electrons) simulation code. The early growth of the wave will be compared with linear theory and the nonlinear evolution of the instability will be presented. The saturation mechanism will be discussed.

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Ion-Temperature-Gradient Instability in Toroidal Plasmas

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The stability of the ion-temperature-gradient mode in a toroidal plasma has been investigated. Using the newly developed ballooning mode formalism, we have derived an ordinary difference-differential equation which includes full kinetic effects. The equation is examined in various limits where it reduces to an ordinary differential equation. Analytic and computational studies show that for \( \eta_i = \frac{1}{d \ln T_i / d \ln n} > 1 \) toroidal effects further destabilize the mode and hence the corresponding growth rates far exceed those obtained from the slab calculations. However, it is also found that toroidal effects give rise to higher \( \eta_i \) threshold compared to the slab case. Extensive numerical calculations over a wide range of parameters have been carried out to delineate the regions of instability. Unlike the universal drift waves, which require weak shear (\( \hat{s} = r q' / q < \frac{1}{2} \)) for the nullification of shear damping by toroidal effects, the present \( \eta_i \) instability persists even for \( \hat{s} = 1 \) and hence is of relevance to present-day beam-heated tokamaks.

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HIGH BETA STELLARATOR STABILITY THEORY*

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An analytic study of the stability of a diffuse high beta stellarator with arbitrary wall corrugation is presented. It is found that a solvability condition for the equilibrium of such a plasma is intimately related to a sufficient condition for stability. The results of the calculation suggests that if the equilibrium model is valid, then all high beta stellarators are unstable.

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Plasma Diffusion in the Presence of Strong Turbulence*

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Plasma diffusion in the presence of electrostatic turbulence has been studied analytically and numerically. First, the two-dimensional convective cell turbulence is studied in the guiding-center limit and keeping the finite ion inertia. It is shown that the mode-coupling equations for both models are essentially the same which indicate large mode-coupling coefficients for long-wavelength fluctuations and, hence, the presence of strong turbulence even for modest level of fluctuation. Numerical simulations reveal the spreading of localized plasma density through vortex formation and at the same time the spreading of electrostatic energy toward long-wavelength modes (inverse cascades).

Drift wave turbulence and the associated particle diffusion are studied in a steady state using a quasineutral simulation model in which the electrons follow Boltzmann distribution. Numerical simulation reveals the diffusion in this case is much smaller than the previous case and the electrostatic fluctuations do not easily cascade toward long-wavelength fluctuations \( (k_p L < 1) \). Both observations are interpreted in terms of small mode-coupling coefficients for long-wavelength fluctuations in this model.

Finally, a coupled set of equations are derived for drift wave and convective cells using fluid theory. For drift turbulence, the coupling to convective cells is much more important than the drift wave nonlinearity. Numerical solutions of the mode-coupling equations will be presented.

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ON THE CYLINDRICAL LIMIT OF VARIOUS MHD PHENOMENA

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In dealing with toroidal systems, cylindrical coordinates are used either with $z$ along the straightened toroidal direction, as in most stability and wave problems, or with $z$ along the vertical symmetry axis of the system, as customary in equilibrium studies.

In this paper, assuming the appropriate axis orientation, the following essentially one-dimensional topics are discussed: 1. The nature of singularities and cutoffs of the Hain-Lüst equation. 2. The existence of toroidal surface MHD-modes. 3. The simplest possible form of a ballooning-like instability. 4. The qualitative equilibrium-pressure profile versus major radius in finite-beta toruses.

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Magnetohydrodynamic Stability Analysis Using Approximate Codes

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Linearized magnetohydrodynamic (MHD) stability of tokamak equilibria of arbitrary cross section presently may be examined by large numerical codes such as PEST\textsuperscript{1} and ERATO.\textsuperscript{2} These codes are time consuming and may be impractical for comparison or optimization studies involving many equilibria. However, such studies may be done with the aid of smaller, faster, special-purpose, or approximate codes. All of the codes discussed here are based upon the Lagrangian formulation of the linearized MHD equations of motion. Approximate forms of the Lagrangian may be derived from geometrical considerations such as aspect ratio and vertical elongation or from the limit of high toroidal mode number.

An example of such a code is that developed from the Mercier criterion.\textsuperscript{3} The Mercier code is used to study interchange modes, radially localized to magnetic flux surfaces within the plasma. Another such code is that developed for the high toroidal mode number ballooning mode.\textsuperscript{4} This code examines the stability of modes that are radially extended and predominate toward the outside of the plasma toroidal cross section. Strictly speaking, these analyses are not independent, but must be treated so in practice. Stability information gained from these two economical codes is practically sufficient for the study of "pressure-driven" modes in the interior of the plasma.

Long wavelength (small toroidal mode number) "current-driven" modes, such as kinks and axially-symmetric displacements, also may be studied by an approximate code for equilibria with vertically elongated cross section.\textsuperscript{5} A greatly simplified Lagrangian is obtained using a double expansion in inverse aspect ratio and elongation. A new analysis has been carried out to second-order in elongation to properly describe the vacuum region surrounding the plasma.

These three codes have been used to examine the stability of equilibria with simply connected flux surfaces and comparison of results has been made with ERATO. These approximate codes proved to be economical and may be used to study stability of equilibria with internal separatrices, such as doublets.

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\textsuperscript{3} C. Mercier, \textit{Nucl. Fusion} \textbf{1}, 47 (1960).


We have developed a self-consistent nonlinear resonance broadening theory for electrons in a drift-wave turbulent, sheared magnetic field. The phase space islands overlap at very low fluctuation levels resulting in stochastic electron orbits. With shear, radial diffusion combines with rapid parallel motion to induce random poloidal motion and electrons decorrelate at a rate $\omega_c = [(v \cdot k_j)^2 D]^{1/3}$. In tokamaks, this exceeds the decorrelation rate $k_\parallel^2 D$ for ions in a uniform magnetic field. It is shown that linearly stable drift waves can be destabilized for $\omega_c > \omega$, which occurs at very low levels of turbulence. Nonlinear stabilization at modest saturation levels occurs when the broadened inverse electron Landau resonance balances the turbulently enhanced shear damping. A turbulent diffusion coefficient, $D_e \sim 15 \Delta_{PB}^3 \rho_s \omega_s/L_s$, where $\Delta_{PB} = (L_s/L_n)^3 (m_e/m_i)$, is required to saturate electrostatic drift modes for which $\beta (L_s/L_n)^3 < 1$.

PARTICLE SIMULATION OF DRIFT-CYCLOTRON INSTABILITY

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The drift-cyclotron instability is a collisionless instability of a Maxwellian magnetized plasma driven by the free energy associated with a spatial density gradient normal to B, requiring \( k_\perp \) and \( v_\perp \) only. To study the linear and nonlinear behavior of this instability we use an electrostatic particle code with no restrictions on the dynamics of electrons as well as ions; namely, both species are magnetized and treated fully nonlinearly. During the linear stage our simulations show exponential growth in time over a few ion cyclotron periods, with the growth rates in quantitative agreement with the predictions of a linear nonlocal theory, while the real frequencies match only qualitatively with the predictions of a linear theory. The latter discrepancy might be related to the generation of a zero-frequency collisionless vortex mode. At saturation, the total electrostatic field energy reaches a few percent of the initial ion kinetic energy for most runs, with mass ratios ranging from 25 to 200. These simulation saturation levels are compared with some nonlinear theories; namely, simulations produce the scaling of the nonlinear frequency shift theory, \( \zeta \approx \kappa \delta^2 \) [where \( \zeta = (e\phi/T_i) \) saturation, \( \kappa = (\alpha_i/n)(dn/dx) \), \( \delta = m_e/m_i + \omega_e/\omega_i \)] and that of the trapping, \( \zeta \approx \kappa^{-3/4} \delta^{1/4} \), in various parameter regimes. By the saturation time, the phase space pictures of electrons and ions reveal vortex-like structure and appreciable density-profile modification in some cases. The details of the comparison with linear and nonlinear theories will be presented.

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Within the framework of the expansion a three-dimensional MHD equilibrium around its magnetic axis (see, e.g. [1 , 2]) a class of stable stellarators without ohmic heating is considered which may be characterized by five independent parameters. The magnetic axes are a set of closed curves described by two parameters, the number of periods and the helical amplitude (l = 1 field). The elliptical plasma cross-section (l = 2 field) turns at the constant rate - π/L_p (L_p period of the axis) with respect to the normal of the axis. For a suitable range pressure gradients, given by the parameter dp/dV on the magnetic axis (V volume inside flux surfaces) these configurations have magnetic surfaces which are nearly centered [3]. In addition two stabilizing triangularity parameters (l = 3 fields) are used, one of which serves to satisfy a stability criterion. \( \beta \)-values of about 10 % together with an aspect ratio of about 20 have been found taking into account the necessary stability criterion.


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Anomalous Reconnection in
Disruptive Processes in Tokamak Like Plasmas

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Abstract

Interpreting the results of numerical simulations a picture is given of the mechanisms that lead to rapid expansion of magnetic flux during disruptive processes in tokamaks. The competing role of resistivity and electron viscosity in the rapid growth of a \((m,n) = (3,2)\) tearing mode in the presence of large \((2,1)\) magnetic islands is studied quantitatively. When viscosity is dominating, the growth rate scales as \(\gamma \propto \mu^{1/6}\), the process being very insensitive to the actual value of \(\mu\). Finally certain shortcomings of the \((3,2)-(2,1)\) mode interaction theory in explaining major tokamak disruptions are outlined. While interaction of any low \(m\)-number modes of different helicity may lead to an explosive transport of flux and electron energy across a certain region, a model of the major disruption seems to require a coupling to the \((1,1)\) mode.

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Similarity Solutions of Partial Differential Equations
Using MACSYMA*

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The use of the MACSYMA algebraic computing system to aid in the construction of exact (nonlinear) similarity solutions of systems of second-order, quasi-linear partial differential equations is discussed. Specifically, MACSYMA is used to calculate systematically the generators of the infinitesimal group under which the considered equations are invariant. Once the group is known, its invariants and consequently the similarity form of the solutions of the partial differential equations can be obtained. Finally, the (hopefully nontrivial) subgroup which leaves invariant the boundary curves and boundary conditions of the problem is found. The use of MACSYMA in obtaining similarity solutions is illustrated by an example from fluid mechanics.

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Axial Collisional Heating of
Linear Magnetic Fusion Systems

by

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Long plasma columns can be heated economically by low frequency
compressive pumping with an axial wavelength, $\lambda_p$, that is of the
order of or long compared to the mean free path, $\lambda$. Heating rates
are calculated as a function of $\lambda/\lambda_p$, $\beta$, and pumping frequency, $\omega_p$, by
a second order perturbation method and by numerical simulation,
using collisional viscosity and heat flow co-efficients. A maximum
in the heating rate is found near the resonance between the pumping
phase velocity, $\lambda_p/\omega_p$, and the axial magneto acoustic velocity (cusp
velocity). It is shown by numerical simulation that in spite of
the detuning of the wave velocities that occurs as the plasma
temperature changes, increases of plasma temperature by a factor
of ten are quite plausible.
Electron Stability Analysis of the Inhomogeneous Beam Plasma System--

Application to the Electrostatic Double Layer*

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ABSTRACT

The electrostatic double layer has been proposed as a mechanism for particle acceleration. Such a B.G.K. type structure (related to the collisionless shock) has been observed in double and triple plasma devices, and has been the subject of several computer simulations. We model this structure in terms of two interpenetrating cold beams (one electron and one ion), together with background populations. The stability analysis, facilitated by employing the localizing approximation, is performed. A classical perturbation expansion using the wave amplitude and the ratio of the wavelength to the characteristic scale length of the medium as small parameters, yields the energy flux conserving approximation. Such an expansion breaks down at transition points. In order to understand the nature of these transition points we reformulate the problem in terms of a system of five first order differential equations. This system is then subjected to a reductive perturbation procedure which reduces it to tractable independent systems of lower order. We observe that in addition to the four traditional modes observed in the homogenous theory there exists an additional singular mode. The behavior of this additional mode is discussed.

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"Pinch-Tormac"- A New Fusion Device. T. Hatori and A.K. Sen*, Institute of Plasma Physics, Nagoya U., Japan. We propose a new figure 8 fusion device whose linear segments are Theta pinches and whose curved ends are tormac like sectors. To derive the sheath thickness of the transition layer between the closed and open field lines, we use the Braginskii transport equations with friction and thermal forces and momentum terms corresponding to particle losses from the sheath. We find that the transit time of ions in the linear segment is a new parameter which is critical in the determination of the sheath confinement time and thickness. However, the sheath thickness is between one and two gyroradius and the sheath confinement time is typically $\sim 10^{-4}$ s. For high fusion type parameters and length of the linear segment between 100 to 150 m one can obtain $n_{\tau} \sim 10^{15}$.

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TWO DIMENSIONAL STRUCTURE AND VARIATIONAL PRINCIPLES
FOR TOROIDAL BALLOONING MODES
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We employ a direct two dimensional representation of ballooning modes and evaluate their stability, in axysimmetric toroidal devices, for the following equilibrium configurations: a low-$\beta$ equilibrium, where the flux surfaces can be described by concentric circles and a finite-$\beta$ equilibrium in which the flux surfaces are still circular, though with shifted centers. We obtain marginal stability curves as well as a picture of the resulting trial function for the perturbing displacement. A comparison with the marginal stable eigenvalues and eigenfunctions, obtained by a well-known infinite series representation\textsuperscript{1}, yields good agreement. The advantage of our direct representation is that the topological and physical properties of the considered modes are immediately evident.


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The Trapped-Untrapped Electron Boundary Layer in Tokamak Geometry

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Boundary layer effects are found to be important when solving the drift kinetic equation in toroidal geometry. A Lorentz operator is used for collisions for all of velocity space in a radially local analysis, applying no special boundary conditions at the trapped-untrapped electron boundary. Electron Landau resonances and trapped particle effects are thereby consistently connected in the transition region. This improves upon analyses using a Krook model for collisions which solve in the resonance and trapped particle regions separately, then seek a plausible connection criterion.

The drift kinetic equation is here numerically solved, and distribution functions showing boundary contributions will be exhibited. The results are used, with simple assumptions for the ion physics, to examine the trapped electron mode, which is found to be damped for most parameter regions studied. Evidence indicating that this may be due to a collisionally broadened Landau resonance at the boundary layer will be presented.
STABILITY OF FIELD REVERSED THETA PINCHES*

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In field reversed theta pinch experiments at Los Alamos and elsewhere, stationary plasma configurations which are stable for many Alfvén transit times have been produced. Three-dimensional, initial value calculations have been performed with the MALICE code to study the stability of such a configuration to ideal MHD modes. In these simulations, toroidal mode numbers of n = 0, 1, 2, and 3 are resolved with little effect from finite grid resolution. Higher n modes are attenuated or totally suppressed.

Configurations which are completely stable to low n modes are found by allowing plasma pressure on open field lines outside the separatrix. The stability of n = 1 modes is shown to depend on the details of the open field pressure profile.

Rotationally driven modes are also examined using the simulation. Depending on the equilibrium and the rotational velocity the n = 2 or the n = 3 mode may be most unstable. Comparison of these results with the experimental observations and with other theoretical work indicates qualitative agreement.

*Work performed under the auspices of U. S. Department of Energy.
Solid Material End Plugging of Linear Magnetic Fusion Systems

by

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Studies of solid material end plugging of linear open ended confinement systems have shown that plugs with Z (atomic number) > 1 can eliminate plasma end loss and considerably reduce the plasma column length needed to limit the energy end loss rate to a given value. These studies include line and continuum radiation energy loss rates and have been done with a time dependent, numerical, MHD model which includes Monte Carlo alpha particle transport, and with a quasi-stationary analytic model. Calculations done with reactor parameters indicate system lengths less than ten kilometers. Improvements in performance are obtained from use of multi material layered end plugs.
CHARACTERISTICS OF IGNITED, HIGH-WALL-LOADING CATALYZED DEUTERIUM TOKAMAK PLASMAS*

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ABSTRACT

Scoping studies for ignited catalyzed-deuterium tokamak plasmas are carried out with emphasis on attaining medium to high power loading. Neoclassical scaling for the ion energy confinement time and various empirical scalings for electron energy confinement time are used. The critical beta limit is determined by MHD ballooning mode theory. Because no tritium breeding is required, the plasma can be surrounded by a thick conducting wall which raises the limiting beta.

Both analytical and numerical solutions are found for equilibrium ignition, taking into account radial profiles of plasma parameters. In the numerical solutions, the relative magnitudes of $T_e$ and $T_i$ are calculated self-consistently. Cyclotron radiation is found to be a minor loss mechanism at the high densities ($n_e > 3 \times 10^{14} \text{ cm}^{-3}$) and medium temperatures $<T> \sim 25$ KeV used here. $<T_e>$ is always at least 0.80 $<T_i>$. The results indicate that to achieve a total wall power loading of $\sim 3$ MW/m², the reactor dimensions can be kept reasonable ($R=8.4$ m, $a=3$ m), if $B_{\text{max}}$ (at the conductor) is 15 to 16 T and $<\beta> = 0.15$. If $\tau_e \propto T_e^{1/4}$ the required $B_{\text{max}}$ is reduced to about 13.5 T. In the extreme case of $\beta(0) = 1.0$ with appropriate radial profiles, $B_{\text{max}}$ as small as 12 T is sufficient.

The thermal stability of practical operating regimes is under investigation.

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It is generally considered that the alpha particles generated by D-T fusions in a fusion reactor must be pumped away as neutral helium at the plasma edge. A corollary is that divertors are necessary to pump the charged alpha particles into a remote chamber where hopefully a means can be found to exhaust helium sufficiently and thus prevent quenching of the burn by an accumulation of alpha particles in the plasma. However, as has been pointed out previously, a possible difficulty with this scenario is that in toroidal plasmas with any reasonable amount of magnetic field ripple most of the alpha particles should pitch angle scatter into the ripple loss region, become ripple trapped, and drift vertically out of the machine by $B \times VB$ drifting along $|B|$ contours. Thus, the problem of handling alpha particles is probably not primarily one of handling a diffusing thermal component, but instead becomes one of providing "channels" at the top or bottom of the tokamak through which the energetic ($\sim 0.3-3$ MeV) alpha particles can drift out of and be removed from the plasma chamber.

In this work we estimate the fraction of the alpha particles that are lost through the ripple trapping process, and the fraction of their energy that is deposited in the plasma before they are lost. Since D-T fusion alpha particles are produced with a monoenergetic (3.5 MeV), but isotropic pitch angle distribution, a small fraction of them ($v_\alpha/v_d < \sqrt{\delta}$, where $\delta$ is the ripple depth) are born in the ripple loss region and are lost immediately without depositing any of their energy in the plasma. The remaining alpha particles slow down by collisional drag on the background plasma in a manner similar to neutral beam injected fast ions. During the slowing down process they pitch angle scatter, with the scattering becoming most significant for alpha particle energies at or below the critical energy ($E_c \sim 33T_e$ for alpha particles in a 50:50 D-T plasma) at which energy is transferred equally to plasma ions and electrons. Thus, the alpha particles should deposit a large fraction of their energy in the plasma before being scattered into the ripple loss region and drifting out of the machine with a remaining energy of $E_c \sim 300-600$ keV. That is, the ripple effects may provide the primary "pump" for removing alpha particles from the plasma. More detailed estimates of these processes and of other ramifications of magnetic field ripple effects will be presented.

References


FCT HEATING OF FREE BOUNDARY EQUILIBRIA

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We have used an accurate, efficient 1-1/2 D transport code to study the evolution of a free boundary tokamak with FCT heating. We find that by reasonable adjustment of the vertical field and the current in the primary windings the $\beta$ of the equilibrium can be raised from one percent to twenty three percent without separatrix formation at the plasma edge, appearance of reversed current regions or undesirable shape changes. However, unless the primary current is adjusted, unexpected plasma compression or surface current may result.

The equilibrium module of the code employs a combination of a semi-fixed boundary Buneman solver with an efficient surface Green's function for the plasma current to reduce the computation time. Feedback of the vertical field and primary flux is used to position the plasma against a fixed limiter and to adjust the plasma volume to avoid skin current formation.

In agreement with previous theory,$^1$ we find that $\beta_j$ saturates with increasing $\beta$ and scales as $\beta^{1/2}$ for high $\beta$; $I_p$ rises linearly at low $\beta$ but begins to saturate at higher $\beta$; the required vertical field also begins to saturate at high $\beta$ and the primary flux must be decreased. Evolution of the plasma shape depends upon the current profile. With broad current, the plasma edge becomes elliptical at high $\beta$ in a uniform vertical field, whereas, for peaked current, the plasma edge is almost circular even at $\beta \approx 20\%$. Interestingly, the density at the magnetic axis decreases with increasing $\beta$ because of increasing volume near the axis due to the outward diamagnetic shift.

These results have implications for rapid heating of a tokamak because they show that nonintuitive changes in the poloidal field coil currents may be required to preserve the plasma shape and volume and to avoid destabilizing skin currents. These same methods will be used to adopt the diffusive 1-1/2 D transport code to study free boundary plasmas.

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$^1$Research sponsored by the Office of Fusion Energy (ETM), U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

**Visitor from JAERI, Tokai, Japan.

In order to study drift wave instabilities, we have developed a numerical model - TEDI - to study the time evolution of drift waves. The first calculations testing the model are shown here. These include 1) the linear evolution of a drift mode in a cylinder, and 2) a reproduction of results obtained by a well-known eigenmode solver. Kinetic equations are used for both ions and electrons, the ion equation being a gyro-kinetic equation, and the electron equation being a drift-kinetic equation. These are solved on a grid in velocity space and "radius." A slab geometry is used for these early tests. Landau damping is shown to be correctly included.

Computer simulations of dissipative trapped-electron modes in toroidal plasmas, including curvature and gradient drift effects, are presented. These simulations are based on the linearized electron drift-kinetic equation, Fourier transformed with respect to the poloidal and toroidal angles. No a priori distinction is made between trapped and circulating particles, and collisions are represented by a Lorentz model giving pitch-angle diffusion, which does not necessitate the introduction of the often-used "effective" collision frequency, based on an assumed trapped-electron distribution.

A series of computations shows the dependence of the growth rates on $\rho_e/r$ and on $\nu/\omega_*$ ($\rho_e$: electron gyroradius, $r$: flux surface minor radius, $\nu$: collision frequency, $\omega_*$: drift wave frequency). In the weak collision regime, $\nu/\omega_* \ll 0.1$, curvature drift resonance effects are strong but are destabilizing only for $\rho_e/r$ below a critical value. These growth rates decrease rapidly for collision frequencies $\nu \gg 0.1 \omega_*$, and for $\nu \approx \omega_*$ the dissipative trapped-electron instability occurs. In this regime, Landau damping due to resonant circulating electrons reduces the growth rates, and modifies significantly the mode structure of the dissipative trapped-electron instability.

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MATHEMATICAL PROBLEMS ARISING IN
ADIABATIC COMPRESSION OF PLASMA

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ABSTRACT

We present some results on the "Generalized Differential Equations" (GDE) of adiabatically evolving plasma equilibria. These are non-linear differential-functional equations of the form $\Delta \psi = F(V, \psi, \psi', \psi'')$, where $V = V(\psi)$ is the volume (area) inside the level sets $\psi(\mathbf{r}) = \psi$, a constant, and the derivatives on the right hand side are with respect to the dependent variable $V$.

We describe so-called microcanonical averages and their derivatives and simple cases for the nonlinear problem. An existence and uniqueness theorem is given for the associated linearized problem, which is also a functional-differential equation. Finally we will mention an isoperimetric problem related to the geometry of GDE's, and this variational formulation will be used to study examples of bifurcation, exchange of stability and transfer into more complicated geometries.

† Work supported by U.S. DOE contract No. EY-76-C-02-3077.
THE HAMILTONIAN FOR A CHARGED PARTICLE
IN AN ELECTROMAGNETIC FIELD*

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It is advantageous to have a Hamiltonian formulation of the guiding center equations so that the equations are internally self consistent (i.e., they conserve energy exactly to whatever order desired). We obtained the Hamiltonian for a charged particle in toroidal geometry through the use of Poisson brackets. This procedure is simpler than the generating function approach, especially for obtaining higher order terms in $1/\Omega$ ($\Omega$ being the gyrofrequency). Since this was not a perturbative problem, Lie transforms were inapplicable.

The Hamiltonian obtained has no explicit dependence on the gyro angle, and hence possesses an invariant momentum corresponding to the magnetic moment. In addition, coordinates corresponding to the poloidal and toroidal angles were obtained together with their conjugate momenta. Thus, the coordinates are directly related to the geometry as opposed to systems which require knowledge of the length along a field line. For axisymmetric configurations, the momentum conjugate to the toroidal angle is conserved.

We have studied the evolution of tearing modes using a reduced set of low $\beta$, three dimensional, resistive MHD equations which includes the effects of resistivity and toroidicity. These equations are the generalization to toroidal geometry of the equations employed in Ref. 1. A three-dimensional code, Lobeto, is used to numerically advance this set of equations.\textsuperscript{2}

A detailed analysis in the linear regime has been performed to investigate the toroidal effects on the linear growth rate and eigenfunctions of the tearing modes. We have considered several safety factor profiles and different values for the aspect ratio. These results show that semianalytic calculations based on the coupling of only two modes can give a reasonable understanding of the toroidal effects in the large aspect ratio limit, if the modes are wisely chosen. A scheme for performing such calculations has been developed.

The toroidal coupling between the 1/1 and 2/1 tearing modes during the nonlinear phase has also been studied. We have found that the 2/1 tearing mode can be destabilized by the 1/1 mode through the toroidal coupling.

\textsuperscript{*}Research sponsored by the office of Fusion Energy (ETM), U.S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

\textsuperscript{**}Visitor from J.E.N., Madrid, Spain.


\textsuperscript{2}H. R. Hicks, E. Carreras, and S. J. Lynch, abstract submitted to this conference.
FREE AND FORCED $m = 0$ OSCILLATIONS OF A SHARP-BOUNDARY VLASOV-FLUID SCREW PINCH

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A dispersion differential equation has been derived for the study of finite ion gyroradius effects in free and forced oscillations of a high-$\beta$, sharp-boundary screw pinch. The dispersion differential equation is derived from the equations of the Vlasov-fluid model. An approximate solution of the linearized ion Vlasov equation for the sharp-boundary pinch is obtained analytically via assumption of fast gyration and small gyroradius. This approximate solution together with the Ampere equation generates the dispersion equation, which explicitly exhibits two length scales: a MHD length scale (the pinch radius) and a microscopic scale (the ion gyroradius). In the limit of vanishingly small ion gyroradii, the equation and two of the boundary conditions reduce to the ones derived from the guiding center plasma model; with finite ion gyroradii, boundary-layer phenomena are obtained.

The dispersion differential equation has been applied to study free and forced axisymmetric oscillations of a sharp-boundary screw pinch. In the case of free oscillations, we examine the effects of finite ion gyroradii on the eigenfrequencies and eigenfunctions of two types of modes: magnetoacoustic modes and mirror modes. As the ion gyroradius increases, the eigenfrequencies are modified; however, the eigenfunctions become considerably distorted from the guiding center plasma solution, this probably signals the breakdown of the approximation.

In the case of forced oscillations, we compute the coil impedance as a function of real frequency. Two separate resonances are noted: (relatively high frequency) magnetoacoustic resonances and a (relatively low frequency) sloshing resonance. A magnetoacoustic resonance involves primarily radial plasma motion; whereas the sloshing resonance in the guiding center plasma description exhibits substantial flow along the magnetic field lines between the high- and low-pressure regions. As the axial wavelength of the excitation increases, the magnetoacoustic resonance frequency approaches a low-frequency cutoff, while the sloshing resonance frequency scales with the axial wavenumber. The important implications for rf heating in high-$\beta$ systems are as follows:

1. The use of the magnetoacoustic resonance requires high frequencies.
2. Arbitrarily low frequencies may be achieved for the sloshing resonances by using arbitrarily long wavelength excitations.
3. Absorption is due to ion transit-time damping: it depends critically upon the value of $\omega/kv_{th}$, the ratio of the resonance frequency to the thermal ion transit frequency.
4. Comparable resistances can be achieved with each type of resonance.
5. Finite ion gyroradius effects modify the solutions in ways similar to those described above for free oscillations.

*Work performed under the auspices of the U. S. Department of Energy.
PARTICLE ORBITS IN FIELD-REVERSING ION RINGS: ERGODIC OR NOT?

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An analytic and numerical study has been made of the single particle orbits in self-consistent ion ring equilibria. In most of the cases studied the numerically computed orbits are non-ergodic indicating the existence of a constant of the motion in addition to the Hamiltonian and the canonical angular momentum. In one compressed ring equilibrium limited stochastic behavior was found with about 10% of the particles in the ring being ergodic. An analysis of possible effects of a third constant of the motion has been begun.

An understanding of the ergodic to non-ergodic transition is obtained from an analysis of the orbital stability of the class of particles which have orbits near the mid-plane of the ring. An example of a stable mid-plane orbit, which is the typical case, is shown below. The dotted line is the poloidal projection of the orbit, and the solid line is a constant curve of the effective potential.
Numerical Approaches to a Time-dependent Non-linear
Fokker-Planck Equation in Two Velocity Coordinates

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Some numerical approaches to a Fokker-Planck equation for a single species of particle trapped in a joint magnetic electrostatic square-well potential are discussed. The distribution function is assumed to be independent of gyration phase. The Poisson equations for the Rosenbluth potentials are solved in the perpendicular and parallel velocity coordinates using a fast Poisson solver and then these potentials are differenced for the Fokker-Planck coefficients. The Fokker-Planck equation itself is discretized using tensor product B-splines in the Galerkin form of the equation. A comparison with a finite difference discretization is made. The resulting ordinary differential equations in time are solved using a stiff ODE package. A fully implicit method (fixed step size backward Euler) and a time centered scheme (fixed step size trapezoidal rule) are also described.

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Renormalized Dispersion Tensor for Electromagnetic Vlasov Turbulence

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The nonlinear dispersion tensor for electromagnetic fluctuations in a turbulent Vlasov plasma is derived. The calculation extends the electrostatic results given recently by Krommes and Kleva. The formalism employs straightforward but powerful functional techniques to express the dispersion function entirely in terms of observable quantities such as the electric field fluctuation spectrum. Explicit formulas are given in the Direct Interaction Approximation; the relation to weak turbulence theory is then demonstrated. The application of the results to the turbulence theory of finite-β drift waves is discussed.

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WAVE PARTICLE TRANSPORT FROM ELECTROSTATIC INSTABILITIES: AN OVERVIEW*

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Wave-particle transport from short wavelength electrostatic instabilities driven by currents both across and parallel to a unidirectional magnetic field in a Vlasov plasma is reviewed. Both electrons and ions are taken as magnetized, and propagation is in the plane defined by the drift velocities and the magnetic field. A consistent procedure is used to evaluate and compare wave-particle exchange frequencies of momentum and energy for the lower hybrid drift, ion cyclotron electron drift, universal drift, ion acoustic current and ion cyclotron current instabilities. In this model, resistivities and heating frequencies of the universal drift instability are substantially greater than those due to the other drift modes. And wave-particle transport due to the ion cyclotron electron drift instability is larger than that of the lower hybrid drift instability at $T_e > T_i$. The results are applied to the problem of thermal flux inhibition vs. enhanced radial diffusion in a linear theta pinch.

*Work performed under the auspices of the U. S. Department of Energy.
STUDIES OF CURRENT DUE TO RF INDUCED RUNAWAY IN THE DIIA LOWER HYBRID EXPERIMENT*

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In a tokamak, a slow wave antenna of finite length will radiate waves within a broad range of phase velocities parallel to the toroidal magnetic field. For a range of plasma parameters corresponding to the Doublet-IIA experiment, a quasilinear-collisional theory of lower hybrid wave absorption is used to examine penetration of the full spectrum excited by the antenna. It is found that radiation at parallel phase velocities in excess of the Dreicer velocity $v_D$ can penetrate to the plasma center. The quasilinear diffusion of this component of the wave spectrum dominates collisional diffusion, even though the nominal phase velocities excited by the antenna are of the order $v_D/2$.

As a result, it is predicted that antennas in Doublet-IIA that radiate waves nominally characterized by $n_\parallel = 11$ and $n_\parallel = 14$ may induce an observable runaway current (but no substantial heating in these cases). The most pronounced effects occur in accordance with the experiment for the lower density discharges with $n_\parallel = 11$. Simple numerical estimates give rise to runaway currents consistent with experimental observation. Hence the experiment provides evidence of quasilinear diffusion in the tail of the electron distribution.

For completeness, a more comprehensive analysis of runaway current has also been undertaken using a 2-D velocity space Fokker-Planck code including terms describing an applied dc electric field, QL electron diffusion due to rf, and braided magnetic field induced transport.

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Convective Drift Wave Instability In A Sheared Magnetic Field* 

by 
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The universal drift instability as an initial value problem is studied in the slab geometry. The linearized drift kinetic equation is integrated numerically in the two-dimensional phase space (x, v_∥), where x is the inhomogeneous coordinate, and v_∥ is the component of the velocity parallel to the magnetic field. k_x ρ_i is not assumed to be small, and we have found no absolute instabilities associated with finite ion gyroradius effects. In the weak shear limit perturbations that are local in both space and time are found to excite the "marginally stable" drift wave eigenmodes after substantial convective growth of the perturbation. Energy amplification factors of O(10^5) have been obtained. The contribution of these modes to the equilibrium fluctuation spectrum will be discussed.


*This work was supported by US Department of Energy Contract No. EY-76-C-02-3073.
Transient Amplification of Shear Alfven Waves*

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It is shown that a current-carrying plasma, such as that in a tokamak or in a pinch, could be subject to transient amplification of magnetic field fluctuations of shear Alfven waves. This conclusion was based on a preliminary study\(^1\) of a slab model of a plasma described by the ideal MHD equations. For parameters typical of tokamak geometry, the amplitude of a shearing wavelet may gain by a factor of 50-100 in a time scale of order 5-10 µsec before it eventually decays. It is speculated that these magnetic fluctuations, while "ever-present", may enhance the energy loss in a plasma, and in the worst case, may even trigger other instabilities if these fluctuations attain a sufficiently high level.


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Numerical Simulation of Plasma Confinement and Heating by Field-Reversed Ion Rings

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The RINGA code$^\dagger$ has been used to study confinement and heating of a finite-$\beta$ plasma on closed field lines produced by a field-reversed ion ring.

Plasma is described by a Grad-Shafranov term in the field equation which is in addition to the term representing the current contributed by the ion ring. The plasma pressure $p(\psi)$ is a given function which is nonzero only on closed field lines.$^2$ As we increase the plasma pressure, the field reversal factor $\xi$, the total magnetic field energy $E_B$, and the ring halfwidths $\Delta r$ and $\Delta z$ increase, while the total particle energy $E_p$ decreases. Most importantly, the innermost flux surfaces of the field-reversed region become stable to the interchange mode, as determined by $\int d\alpha/|B|$.

We have also included the fast ion-electron drag term to describe the slowing down of the energetic ions by means of the equation

$$\dot{p}_\theta = -v_{ie} m v_r v_\theta,$$

where $p_\theta$ is the ion canonical angular momentum. The loss of particle energy is balanced by increases in the plasma energy and pressure. Recent results from these studies will be presented.

$^\dagger$Work supported under U.S. Department of Energy Contract EY-76-S-02-3170.


PLASMA TURBULENCE NEAR A MAGNETIC FIELD REVERSAL POINT

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An important question regarding the stability of plasmas with reversal magnetic fields is whether microscopic processes exist in the vicinity of the reversal point; and if so, what are their effects. Particle simulation is a convenient method of studying the development of instabilities in such a configuration, since complicated particle orbits make analytic theory intractable. The simulations are done in two dimensions perpendicular to the magnetic field, so that the resulting turbulence is due to a cross-field instability, instead of the collisionless tearing mode. Away from the null point where strong density gradients exist, the lower hybrid drift instability is excited, giving rise to electrostatic and electromagnetic fluctuations of comparable size. At the null point the fluctuations are larger and primarily electromagnetic. The fluctuations are not simply an extension of the lower hybrid drift instability, since nonlocal linear theory shows that this mode is stabilized at the null point by finite beta effects; rather, they are parametrically driven by the lower hybrid drift waves. The turbulence at the field reversal point produces strong electron heating. Both linear and non-linear behavior will be described.


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NUMERICAL SIMULATION OF IMPURITY TRANSPORT AND
PLASMA DECONTAMINATION BY IMPURITY DRIVEN MODES
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The effects of impurity driven modes are analyzed with a one-dimensional impurity transport model which includes both neoclassical and anomalous transport. The expressions of the anomalous fluxes contain the quasi-linear effects of both collisional and collisionless impurity driven modes. The evaluation of the neoclassical fluxes takes into account the different collisionality regimes of the main ions. A single impurity species is included in the model, and it is assumed that the magnetic field is constant in time and the electron and ion temperatures are proportional. We then study the time evolution of the main ion density, the total impurity density and the ion temperature. Furthermore, since the results of ref. (1) are derived for a plasma contaminated by an impurity ion in a single ionization state, most of the computations are done for this simple case. However, the effects on the results of the $\partial n_i / \partial r$ terms, in the neoclassical fluxes, are examined by using the coronal equilibrium model.

Parameters typical of the Alcator device are used, and the value of the relative ion temperature gradient ($\eta_i = d\ln T_i / d\ln n_i$) is varied. We find that when $\eta_i < \eta_c$ (with $\eta_c \approx 1$), no anomalous transport occurs and collisions cause an accumulation of impurities at the center. However, when $\eta_i > \eta_c$, the impurity driven modes produce an outward flow of impurity ions until the impurity density profile is peaked at the edge of the plasma. In this case the neoclassical terms do not affect the results considerably, because the anomalous fluxes exceed the neoclassical ones. Furthermore, the peak impurity density in steady state is typically $2 \div 5$ times larger than the value at the center.

SELF-HEALING OF BALLOONING MODES
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The growth rates and the stability limits of ideal M.H.D. ballooning modes have been obtained for models which include the poloidal angle dependence of the poloidal field and the rate of magnetic shear.\(^1,2\) A first threshold for instability is reached when there is a sufficiently large pressure gradient acting against the curvature of the magnetic field lines. However, further increase of the pressure gradient has a stabilizing effect because of the "stiffening" of the poloidal field lines on the outer side of the torus and the poloidal angle dependence of the shear. This occurs because the general governing equation exhibits non-linear dependence on the pressure gradient. As a result, a second stability region appears.\(^3,4,5\)

In the vicinity of the magnetic axis, where the magnetic surfaces can be described accurately by shifted circles, all the equilibrium parameters of the model are related in a simple way to the rate of magnetic shear and to the pressure gradient.\(^4\) In this limit, we can determine the critical values of magnetic shear and pressure gradient which define the two boundaries of the instability domain.

In order to confirm the predictions of the model with more realistic finite-beta configurations, we have tested the stability of a sequence of flux-conserving Tokamak equilibria generated numerically. The solutions of the general ballooning mode equation based on this exact equilibria again demonstrates the existence of a second stability region.\(^6\)

THE COUPLING OF THE RESISTIVE-$\alpha$ AND ION TEMPERATURE

GRADIENT INSTABILITIES IN A SHEARED MAGNETIC FIELD

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ABSTRACT

It is shown that the shear in the magnetic field couples together the resistive-$\alpha$ and ion temperature gradient instabilities to form a single strongly growing mode. This result is first obtained from a consideration of the structure of the turning points of the two modes and then confirmed by a numerical solution of the full radial eigenvalue problem. The dependence of the growth rate of this instability on shear, curvature and collisionality etc will be given. Comparisons will be made between the characteristics of this instability and the low frequency fluctuations observed on the Culham Levitron.
RESISTIVE INSTABILITIES IN THE REVERSE FIELD PINCH

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The work described here concerns investigations of resistive instabilities in the reverse field pinch configuration. We have developed a new numerical code which solves the full set of linearized resistive instability equations for one dimensional equilibria with $B_\theta(r)$, $B_z(r)$ and $\beta$ arbitrary.

The novel feature of our approach is that the equations are solved as an eigenvalue problem. Those codes in existence, treating similar problems, utilize an initial value approach. Two advantages of the eigenvalue approach are as follows. (1) The numerical procedure is inherently very fast. A converged eigenvalue (to three significant figures) using a grid with 500 points is obtained in about 10-15 sec. on a PDP-10. (2) the eigenvalue approach allows us to examine the behavior, not only of the fastest growing mode, but of any mode. This feature is important in understanding the complete spectrum of instabilities.

In the first version of the code we treated the case of constant resistivity and incompressible displacements. Even though there is no rigorous proof, substantial theoretical and numerical work indicates that under these assumptions, unstable modes are purely growing and not overstable. This feature, which is sometimes not true, was also incorporated in the code. None of these assumptions, however, is at all critical to the numerical method.

We have run the code for RFP like profiles for a wide range of parameters. The results for the $m=1$ mode are as follows:

1. In general, for profiles satisfying the Suydam criterion, there are two unstable modes for any given values of $k$ and $S$ (Reynolds number). One of these is called the fast mode, the other one, the slow mode.

2. For fixed $S$, the fast mode smoothly transforms from the ideal MHD mode, to a tearing mode, to a resistive interchange mode as $k$ is varied.

3. Curves of growth rate $\gamma$ vs $S$ (for $S \leq 10^5$) with $k$ as a parameter show a smooth transition from one mode to another; that is, for these values of Reynolds number, we do not see clearly distinct regions where say $\gamma \sim S^{-3/5}$ as predicted by analytic theory.

4. For the whole range of unstable $k$, the growth rate of the slow mode scales inversely with Reynolds number, $\gamma \sim 1/S$. We interpret this, not as instability, but as a resistive diffusion motion away from our only approximate initial equilibrium.

5. If the fast and slow growth rates are plotted simultaneously vs $k$ for fixed $S$, these curves intersect at two different $k$ values. For values of $k$ outside this range, it is extremely likely that unstable modes exist, but with complex eigenvalues.

6. Finally, we point out that the space between the intersecting $k$ values is a function of $S$. Typically, for $S=150-200$ these values coalesce indicating a strong interaction between diffusion and resistive instabilities. This result raises questions about two and three dimensional MHD simulations which are often forced to operate at relatively low Reynolds numbers because of computer limitations.
Numerical Calculations of Necessary and Sufficient Conditions for MHD Stability of a Stationary Field Reversed Mirror Plasma*

by

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Stationary plasma configurations have been observed in the field reversed theta pinches (FRX) at Los Alamos¹ and elsewhere which are observed to persist for many MHD Alven transit periods. Recent 3D time dependent computer simulations of these profiles with the MALICE code have also failed to show MHD instabilities. It is not clear that these results really indicate MHD stability because of other effects present in the experiment or in the code. In the experiment finite Larmor radius stabilization may reduce or eliminate the instabilities. The code, on the other hand, can only evolve long wavelength modes without dissipation while shorter wavelengths suffer numerical dissipation and those on the sub-grid scale are absent altogether. Hence stability of shorter wavelength modes is not assessed. We have turned to the MHD energy principle to get a better understanding of the field reversed plasma stability. Given equilibrium configurations calculated from a 2D r,z code (CYLEQ) for a scalar pressure profile \( P = P(\psi) \) we use our stability code (STABCRIT) to evaluate the MHD energy principle. A generalized Sturm–Liouville problem is solved on each field line to evaluate the necessary and sufficient conditions for stability. Criteria for interchange and co-interchange (ballooning) displacements are found. Methods, first proposed by Johnson², for calculating marginally stable pressure profiles are also being investigated.

References:


* Work performed under the auspices of the U. S. E. D. E. by Lawrence Livermore Laboratory and Los Alamos Scientific Laboratory under contract numbers W-7405-ENG-48 and W-7405-ENG-36 respectively.
Recent experimental measurements of density fluctuations in the neutral-beam-heated PLT by means of microwave scattering have suggested the possible presence of low-frequency drift-type microinstabilities. In this paper it is pointed out that a number of physical characteristics predicted by the linear theory of such modes are in apparent qualitative agreement with experimental results. Taking into account conditions appropriate to the experiment, a comprehensive calculation of electrostatic drift waves in a tokamak geometry has been carried out. It is found that instead of a single type of instability (such as the trapped-ion modes), the dominant drift modes are actually hybrids of the trapped-electron mode, trapped-ion mode, and the ion-temperature-gradient-driven drift instability. These linear eigenmode calculations employ one- and two-dimensional codes embodying all features known to be important to the stability of toroidal drift waves. The necessary equilibrium profiles are obtained from transport code calculations which model well the experimental results. Among the particular points of qualitative agreement between linear theory predictions and the microwave scattering results are: (1) the strong enhancement of fluctuations at the outside of the torus corresponds to the ballooning character of the calculated eigenfunctions; (2) the linear instability thresholds on the ion temperature and the ion temperature gradient fall roughly within factors of two of those observed experimentally for a large increase in the fluctuation level; and (3) the two maxima in the computed linear growth rate curve (as a function of either poloidal or toroidal mode number) are found to be close to the observed peaks in the k-spectrum of the density fluctuations.

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The Trapped Ion Mode in the Presence of Drift Wave Fluctuations

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Abstract

The influence of a spectrum of drift wave fluctuations on the trapped ion mode is investigated. It is shown that a broad spectrum of drift wave fluctuations with a monotonically decreasing $k_\parallel$-spectrum leads to a stochastic damping of the trapped ion modes. For the drift wave spectrum given by a previous theoretical model the nonlinear turbulent damping is sufficient to inhibit the onset of the usual trapped ion mode. In contrast, the analysis predicts that a peaked $k_\parallel$-spectrum of drift modes results in the stimulation of trapped ion modes.

Two theoretical approaches are used to derive the trapped ion mode dispersion relation renormalized by the presence of drift wave fluctuations. In one case general mode coupling equations are derived and reduced with the properties of the trapped ion-drift wave interactions. In the second approach the separation of the space-time scales is used at the outset to write a wave-kinetic equation for the drift modes propagating in the presence of the slowly varying trapped ion mode.

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Ion Temperature Drift Instabilities in a Sheared Magnetic Field*

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Results from the first particle code simulations of ion-temperature-gradient-driven drift instabilities in a sheared magnetic field are reported. This type of instability\(^1\) has received renewed interest recently because the beam-heated PLT experiment exhibits characteristically large ion temperature gradients. The purpose of this investigation is to verify the linear theory of the instability and to study its nonlinear consequences. The simulation has been carried out using a 2 1/2-D code in a sheared slab geometry, where exact dynamics are kept for the ions, while the electron response is assumed to be adiabatic, i.e., \(\dot{n}_e/n_o \approx e\phi/T_e\). The latter approximation is in accordance with the usual theoretical model and has the advantage of suppressing the unnecessary discrete particle noise associated with the electron motion. In the linear stage of the instability, the simulation results agree very well with the WKB and shooting code calculations of the mode frequency, the growth rate, and the spatial structure. In the nonlinear stage, a large amount of ion energy transport has been observed. For the present simulations, the dominant nonlinear saturation mechanisms are found to be the quasilinear diffusion of the ion temperature profile and the Doppler frequency shift resulting from the build-up of the ambipolar potential. Details will be reported along with preliminary results using 3-D models.

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SUPPRESSION OF CURRENT DRIVEN ION CYCLOTRON WAVES
BY A LOWER HYBRID PUMP IN A Q MACHINE*

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In this paper we have explained the experimental results of Lashinsky et al on the suppression of current driven ion cyclotron waves by a lower hybrid pump in a Q machine. The lower hybrid pump (of finite wave number) interacts with the cyclotron wave to produce high frequency sidebands, which in turn couple (through the dominant $E \times B$ electron nonlinearity) to the pump to produce a low frequency ponderomotive force. When the ponderomotive potential is out of phase with the wave potential (as is the case with Q machine parameters), the wave frequency suffers a downward shift and the ion cyclotron damping is greatly enhanced, thus stabilizing the instability. This effect is shown to arise when the electron oscillatory velocity exceeds the acoustic speed.

*This work supported by the Department of Energy.
ECRF Absorption Related to EBT. J. F. Pipkins and R. L. Hickok, Rensselaer Polytechnic Institute. -- A single particle model has been used to study ECRF absorption in a non-uniform magnetic field such as occurs in EBT. As a first step we use a non-relativistic calculation in slab geometry and solve the equation \( m \frac{dv}{dt} = q \vec{E} + q (v \times \vec{B}) \) where \( \vec{E} = E_0 (\vec{e}_x \cos \omega t + \vec{e}_y \sin \omega t) \) and \( \vec{B} = -\vec{e}_z B_0 (1 + \epsilon_x + \delta x^2) \). The solution of this non-linear equation shows that the magnetic field gradient sets an upper limit on the runaway energy of the resonant electrons, but the energy fluctuates between zero and this maximum level. The resonant layer also generates a positive potential barrier and a suppression of the magnetic field, but they also fluctuate with the runaway energy. The response curve for this resonance resembles that of a "soft" oscillator -- i.e. the restoring force decreases with amplitude. If the resonance zone is limited in the field direction (as it is in EBT) there will be a return flux that must be included in the self-consistent field. For appropriate length rings the response curve will change to resemble a "hard" oscillator. Experimentally it is observed that EBT operates at a drive frequency which corresponds to \( 2 \omega_{ce} \) at the ring location. At the operating density the upper hybrid resonance also occurs at the ring location and may be responsible for the energy absorption. If this is true then varying the density corresponds to walking along the response curve. For a "hard" oscillator characteristics, decreasing the density will trap the rings -- i.e. the energy of the resonant particles will no longer fluctuate between zero and the maximum, but will be restricted to small fluctuations about the maximum. If typical parameters for EBT are substituted into the model, the results are in qualitative agreement with experimental measurements.

\[\text{Supported by DOE under Contract EY-76-S-02-2229.}\]
Two time scales enter in magnetic field diffusion problems where the diffusion is through a shell of radius \( a \) and thickness \( \Delta \), as for example in the poloidal field control of a tokamak plasma. The time scale of the diffusion equation, \( \Delta^2 \dot{B} = \mu_0 (\partial \dot{B}/\partial t) \), is \( \tau \approx \mu_0 \Delta^2 \), and the eddy current rise time is \( \tau \approx L/R \approx \mu_0 a \Delta \), which is much longer if the shell is thin.

This paper presents numerical calculations of the field diffusion through a cylindrical shell, which could be magnetic, as well as conducting, in order to represent the use of ferritic materials in wall and blanket/shield design. The appearance of the two time scales is examined.

A model of a plasma in such a shell is also presented. If the plasma were to experience a perturbation, an increase in its pressure, for example, the conducting wall would tend to keep the corrected external field from penetrating and restoring the desired equilibrium. On the other hand, as with copper shell tokamaks, the conducting shell also tends to retain the plasma in its original equilibrium. The net result of these competing effects is examined.

* Work supported by the U. S. Department of Energy.
Variational Principle for Magnetohydrodynamic Equilibrium States

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A variational principle is given for constructing magnetohydrodynamic equilibria, with arbitrary pressure, subject to a generalized set of constraints. These constraints are global, may be shown to be complete for axisymmetric systems and are generalized versions of the conventional class which conserve only the number of particles, entropy and magnetic helicity. This generalization is achieved by the inclusion in the integral constraints of a complete set of weight functions of the totoidal flux $\Psi$. The Euler-Lagrange equations for minima of the energy are derived and are direct generalizations of the results obtained by Taylor.† By a suitable choice of the basic functions, realistic pressure and density profiles of interest in tokamaks and reversed field pinches may be generated. By considering the second variation of the generalized thermodynamic energy criteria for the stability of the equilibrium states to ideal MHD and a class of dissipative perturbations are obtained.

*Work supported by U. S. DoE Contract No. EY-76-C-02-3073.
PONDEROMOTIVE EFFECTS OF AN ELECTROMAGNETIC WAVE IN A NONUNIFORM MAGNETIC FIELD

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The ponderomotive Hamiltonian is derived for an electromagnetic wave of arbitrary polarization, and with spatial variation of wavevector and amplitude. The magnetostatic field is nonuniform and has arbitrary geometry. The perturbation vector potential is represented as \( \tilde{A}(x) \exp i(\psi(x) - \omega t) + \text{c.c.} \), where \( k(x) = \nabla \psi(x) \). In terms of the gyromomentum (or generalized magnetic moment) \( \mu \), guiding-center position \( X \), and parallel momentum \( P_\parallel \), the result for the ponderomotive Hamiltonian is (\( m = c = e = 1 \))

\[
\begin{align*}
K_2(X;P_\parallel;\mu) &= \frac{|\tilde{E}(X)|^2}{2} + \sum_{l = -\infty}^{+\infty} \left( \frac{\partial}{\partial \mu} + k_{\parallel}(X) \frac{\partial}{\partial P_\parallel} \right) \omega - \lambda \Omega(X) - k_{\parallel}(X) P_\parallel \\
&= \frac{\left| H_\perp(X;P_\parallel;\mu) \right|^2}{2}
\end{align*}
\]

where \( H_\perp(X;P_\parallel;\mu) \) is the Fourier component of the perturbation:

\[
|H_\perp(X;P_\parallel;\mu)| = \left| \frac{\Omega(X) \tilde{E}_\perp(X)}{\omega k_{\perp}(X)} J_{\perp} + \frac{P_\parallel \tilde{E}_\perp(X)}{\omega} J_{\parallel} + \frac{2i \Omega(X) \mu B_\parallel(X)}{k_{\perp}^2(X)} \frac{\partial J_{\perp}}{\partial \mu} \right|;
\]

\[
\tilde{E}(X) = i k_{\perp}(X) \times \tilde{A}(X), \quad \tilde{E}_\perp(X) = i \omega \tilde{A}(X), \quad \tilde{E}_\parallel(X) = k_{\parallel}(X) \cdot \tilde{E}(X);
\]

\( J_{\perp} \) is the Bessel function of argument \( k_{\perp}(X) \sqrt{2 \mu / \Omega(X)} \), and \( \Omega(X) \) is the unperturbed gyro-frequency. The equations of motion are derived, and the interpretation of the physical meaning of each term is presented. The ponderomotive effects on the containment of particles in a mirror field are analyzed. The expressions for the displacement of the turning point and the shifts of the gyro-, bounce- and drift-frequencies are obtained.


3. R.G. Littlejohn, paper at this meeting.
A GUIDING CENTER HAMILTONIAN USING PHYSICAL VARIABLES*  

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A guiding center Hamiltonian is rigorously derived within the framework of a systematic ordering scheme. The result to lowest two orders is

$$H(X,U) = B(X)\mu + \frac{1}{2}U^2 + O(\varepsilon^2)$$

where $X$ and $U$ are the position and parallel velocity of the guiding center and

where $\varepsilon$ is the ratio of gyroradius to scale length. The use of magnetic vector potentials or Euler potentials is avoided, the Hamiltonian and all associated variables being expressed directly in terms of locally measurable quantities such as velocities and magnetic fields. Effects beyond lowest order in gyroradius, such as second order drifts, are relatively easy to study. Perturbations such as small amplitude electromagnetic waves\(^1\) can be treated within a Hamiltonian framework.

A mathematical novelty of the method is the use of noncanonical coordinates in phase space.\(^2\) Thus

$$[X,X] = \frac{\varepsilon}{B} b \times + O(\varepsilon^2)$$

$$[X,U] = \hat{b} + \frac{\varepsilon U}{B} b \times (\hat{b} \cdot \nabla \hat{b}) + O(\varepsilon^2)$$

where all field quantities are evaluated at the guiding center position. Hence

$$\dot{U} = [U,H] = [U,X] \cdot \frac{\partial H}{\partial X} = -\mu \hat{b} \cdot \nabla B + O(\varepsilon);$$

$$\dot{X} = [X,H] = [X,X] \cdot \frac{\partial H}{\partial X} + [X,U] \frac{\partial H}{\partial U} = \hat{b} U + \frac{\varepsilon}{B} b \times [\mu \nabla B + U^2 \hat{b} \cdot \nabla \hat{b}] + O(\varepsilon^2).$$

1. C. Grebogi, A.N. Kaufman, and R.G. Littlejohn, paper at this meeting.  
Multipole Equilibria with Beta Equal to One

R.L. SPENCER, University of Wisconsin-Madison—Hemholtz's free boundary conformal mapping technique is used to find \( \beta=1 \) sharp boundary equilibria for linear multipoles with conducting walls. It seems to be always possible to find such equilibria, indicating that there is no equilibrium beta limit in multipoles. Octupole equilibria are studied for all values of the fluid pressure. At low pressures, cusp equilibria are obtained. As the pressure is increased, the fluid closes on itself in the bridge region. At the instant of closure, the vacuum is split into separate regions, and for yet higher pressures, more parameters are needed to uniquely determine an equilibrium.

Thus, there is a kind of bifurcation when the pressure exceeds the value for closure in the bridge. Although they are unstable, these sharp boundary \( \beta=1 \) equilibria are useful because they provide an opposite extreme from the zero beta equilibria obtained from vacuum flux plots. They also provide information on the parameter ranges for which high beta equilibria exist, and on the number of parameters required to determine equilibrium uniquely.

*Work supported by USDOE.
LH-Quasimode Parametric Excitation at the Edge of a Tokamak Plasma.† E. VILLALON,‡‡ MIT—Parametric excitations via quasimode decay of a lower-hybrid pump wave are shown to be strong near the edge of the plasma. Calculations in the shadow of the limiters for Alcator A heating experiment will be presented. This region is characterized by a big drop in the electron temperature which makes pump and sideband become strongly coupled. The linear theory predicts that the rf-power is mainly distributed between the wave numbers $n_z = 1$ to $3$ (where $n^+_z = c k / \omega$). The excitation of fields with high values of $n_z$ (e.g. $n_z \sim 7$ or $8$) is significant, and may lead to a shift of the initial power spectrum toward higher $n_z$'s. These waves may transfer energy to the electrons, through linear Landau damping, as they get inside the plasma. A nonlinear analysis of the steady-state evolution has also been carried out, showing that the most powerful sideband fields are created in this region of the plasma.

† Work supported by U. S. Department of Energy Contract (ET78-S-02-4682).
‡‡ Supported by Grant PFPI (MEC, Spain).
A numerical simulation of adiabatic compression in various reversed field (RF) plasma configurations has been carried out using the "1-1/2 D" methods originally proposed by Grad. The RF configuration is produced numerically by tying the field lines at the two ends of the device. Specifically, RF plasma with and without toroidal field has been investigated in two kinds of current experiments, theta-pinch and liners, which require the imposition of different boundary conditions at the wall.

For both physical cases the radial compression of the plasma is accompanied by a strong axial contraction, the axial contraction being strongest for the theta-pinch case where the plasma tends to move to a bicycle tire shape whereas in liners the initial elongation is increased during compression. Numerical results show evidence of similarity like solutions. In both cases the plasma beta is increased during compression; this contradicts previous predictions based on "1D" simulations. Simple arguments show that under adiabatic evolution interchange stable plasmas remain stable; conversely interchange unstable plasmas remain unstable. Numerical results for typical experimental plasmas will be shown which illustrate the interesting features of plasma compression.


Work supported by U.S. DOE Contract No. EY-76-C-02-3077.
ELECTRON HEATING BY LOWER HYBRID WAVES
IN THE PRESENCE OF ANOMALOUS TRANSPORT

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ABSTRACT

We consider the effect of anomalous transport on electron heating by lower hybrid waves in a low-to-medium density plasma. For definiteness, we assume stochastic magnetic fluctuations as the mechanism for anomalous transport. The modification in the quasistationary electron distribution is examined both in the weak RF and strong RF limits. Physical pictures are presented to explain the distinction in the two cases. In the weak RF regime, anomalous transport can eventually reduce the RF damping rate. For high power heating experiments, anomalous transport can significantly enhance the electron heating rate with a concomitant increase in anomalous loss rate. This can reduce the efficiency of RF heating in two ways: (1) heating is shifted toward the plasma periphery thus increasing heat losses for example by direct heat convection; and (2) the modification of the electron distribution increases anomalous losses resulting in less power available for heating. The relevance of present study to current lower hybrid heating experiments will be discussed.

Work supported by Department of Energy, Contract No. EY-76-C-03-0167, Project Agreement No. 38.

POSTER SESSION
The effects of trapped electrons on drift-Alfvén waves has been studied by a number of authors. Previous models have failed to recognize that the trapped electrons because of their bounce motion are unable to respond to the perturbed parallel magnetic vector potential \( A_n \). As a result of this oversight, the coupled radial eigenvalue equations for \( A_n \) and the electrostatic potential \( \phi \) and/or the perturbed distribution function for the electrons exhibit a non-physical singularity at the mode rational surface.

In order to remove the preceding difficulty, a more careful derivation of the perturbed trapped electron distribution function \( f_t \) is performed by separating \( f_t \) into portions even and odd in the parallel velocity. The resulting radial eigenvalue equations for \( \phi \) and \( A_n \) are well behaved at the rational surface. In the limit in which the boundary layer between the trapped and untrapped electrons can be ignored, this set of radial differential equations is solved numerically to determine the eigenvalue. Preliminary results show that the electrostatic drift branch of the trapped electron mode is weakly affected by finite \( \beta \), which in most cases is a destabilizing influence. In addition, the trapped electron tearing mode (\( \phi \) odd, \( A_n \) even about the rational surface) is found to have a growth rate smaller than the twisting mode (\( \phi \) even, \( A_n \) odd about the rational surface).

For the smaller collisionalities a velocity space boundary layer exists between the trapped and untrapped electron distribution functions so that a Krook model is inappropriate. A model will be presented which employs a pitch angle scattering collision operator to treat this boundary layer.

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Stability of High Beta Tokamaks to Ballooning Modes*


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Ballooning modes are found to possess a second stable regime for high beta. The range of unstable beta values depends on the details of the equilibrium, and in particular, on shear, which can be strongly stabilizing.

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*Work supported by U. S. DoE Contract No. EY-76-C-02-3073.
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A Nonlinear Mode Below the Electron Plasma Frequency

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Abstract.

We find the exact Vlasov distribution function for a one dimensional boundary value problem. A large amplitude, high frequency, spatially modulated wave $E(x)\cos(\omega t - kx)$ launched by an external source changes significantly the plasma equilibrium. By assuming nonresonant wave-particle interaction we find the nonlinear dispersion relation to all orders in the electric field amplitude and second order in $k\lambda_{De}$. Above a certain critical $E_c$ and $(k\lambda_{De})_c$ an undamped nonlinear mode, other than the ordinary plasma wave, exists. Its range of frequencies is $(1/2, 1/3)$ of the frequency of the Langmuir wave.

The existence of the nonlinear mode implies that the velocity dependent ponderomotive potential will lead to anomalous propagation in plasmas. The effect of the ions has been neglected and the ponderomotive and ambipolar potentials are balanced to produce charge neutrality. The normal modes we considered are orders of magnitude above the ion acoustic wave.

*Work supported by National Science Foundation (Grant ENG77-00340).
Equilibrium and Thermal Stability Properties of Ignited Plasmas with Advanced Fuel Cycles

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Ignition requirements are determined self-consistently for plasmas with D-\textsuperscript{3}He and catalyzed-D cycles. Present confinement studies indicate that these plasmas may be characterized by \( \tau_e/\tau_i \ll 1 \) which facilitates ion-electron decoupling. This decoupling would be enhanced by anomalous slowing down of the fusion products; in this case all of the energy of the fusion products is transferred to the plasma ions. The possibility of anomalous slowing down has been recently suggested by Molvig.\textsuperscript{1} The anomalous slowing down results in a factor of two increase in the fusion power density relative to the case with classical slowing down. Similarly the minor radius of the ignited plasma can be reduced significantly (~30%) in the case of anomalous slowing down. Thermal stability properties are studied using a simplified Fokker-Planck model of the fusion products. It is found that the ratio between \( \tau_{\text{runaway}} \) and \( \tau_{\text{global}} \) is

\[
\frac{\tau_{\text{runaway}}}{\tau_{\text{global}}} < 0.5
\]

at the temperature that results in the minimum size or in the maximum power density.

\textsuperscript{†} Work supported by U.S.D.O.E. Contract No. EG-77-S-02-4183.A002
\textsuperscript{*} Westinghouse Co., Pittsburg Penn.
\textsuperscript{1} Kim Molvig, Ignition Experiment Design Meeting, M.I.T. Cambridge (Jan 1979)
Features of Ignited Operation\footnote{Supported by U.S. D.O.E. Contract No. EG-77-S-02-4183.A002}
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Regimes of ignited operation in D-T plasmas are explored in terms of a general requirement on $n_T$, $n_i$, $T_e$ and $T_i$. The conditions under which $T_i > T_e$ and $T_i < T_e$ are found. The amount of electron-ion decoupling is calculated as a function of the ion temperature and the ratio of $\tau_e/\tau_i$. Thermal stability characteristics are determined in the context of the four dimensional ignition requirement. An empirical scaling for $\tau_e$ and neoclassical for $\tau_i$ are used to project the features of ignited operation in recent next step tokamak reactor designs. Devices with similar values of $\tau_e/\tau_i$ have similar equilibrium and stability properties. Operation at $T_i > T_e$ and at high ion temperature can significantly reduce the value of $n_T$ at ignition\footnote{J.F. Clarke, Ignition Experiment Design Meeting, MIT Cambridge (Jan 1979)} and therefore leads to a reduction in the beam energy required in full size full density startup\footnote{L. Bromberg, D.R. Cohn and J. Fisher, MIT Plasma Fusion Center Report RR-79-3 (March 1979)}. Thermal runaway times are very short ($\sim \tau_e$) until ion temperatures approaching 50 keV are reached\footnote{L. Bromberg, D.R. Cohn and J. Fisher, MIT Plasma Fusion Center Report RR-79-3 (March 1979)}. The effectiveness of gas control and compression-decompression as means of plasma control are discussed.
ELECTRON TRANSPORT IN RANDOM MAGNETIC FIELDS

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ABSTRACT

Electron transport in a random magnetic field has been studied taking into account the effect of the perpendicular wavelength $d$ of the perturbing fields. If the perturbing fields have low amplitudes and the typical particle dissociates itself from the field line before it diffuses a distance $d$, the diffusion coefficient is given by the Rechester-Rosenbluth$^1$ formula. Whereas in large amplitude stochastic fields, the typical particle can dissociate itself after it diffuses a distance $d$, the diffusion law given by Kadomtsev and Pogutse$^2$ is obtained. In a high-$\beta$ plasma, such that the random magnetic field results from excitation of magnetostatic modes,$^3$ the relationship of the diffusion law to Ohkawa's formula$^4$ (which fits Alcator scaling) is discussed.

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COUPLING OF DRIFT MODES IN A TORUS

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ABSTRACT

The coupling of electrostatic drift modes due to ion, magnetic curvature drift in a torus is examined analytically and numerically. With the perturbed electrostatic potential written as

$$\phi = \sum_{j=-\infty}^{\infty} \phi_j(x) e^{i(m+j)\theta - il \phi - i\omega t}$$

we reduce the general system of coupled differential equations for the poloidal harmonics $$\phi_j(x)$$ to a single equation by considering a class of solutions

$$\phi_j(x) = e^{ik_0 j_0(x - j\Delta)} 0 \leq \kappa < 2\pi.$$ 

$$x$$ is the radial distance from the $$q = \frac{m}{\lambda}$$ rational surface and $$\Delta$$ is the spacing of the surfaces. Solutions with $$\kappa = 0(\pi)$$ correspond to outward (inward) ballooning modes. In contrast to previous work we emphasize that only the inward ballooning mode (which is least stable at outer radii with strong shear: $$s = rad\frac{dq}{dr} > 1/2$$) is physically realizable within the constraints of the theory.

Numerical and perturbative analyses with adiabatic electrons show that about half of the shear damping is nullified. However, when nonadiabatic electrons are considered, numerical solutions show that, discounting trapped electrons, mode coupling is insufficient to destabilize electron drift modes within the practical ranges of shear, temperature gradients, and current drive for a tokamak.

Work supported by Department of Energy, Contract No. EY-76-C-03-0167, Project Agreement No. 38.
Recently, ballooning instabilities, as obtained from the ballooning instability equation [1], have gained much interest and it has been shown [2] that the ballooning stability properties of self-consistent axisymmetric equilibria are characterized by a ballooning unstable band in the plane poloidal $\beta$ vs. shear. Defining $\beta_p = 1 - \left[ \frac{\partial \phi}{\partial \chi} \right] (0)$, $\zeta = 2 \pi^2 R^3 \left[ \dot{q}/q \right] (0)$, the boundaries of the unstable band are approximately given (to within 10% accuracy) by $\beta_p \approx 0.45 \sqrt{\zeta}$, $\beta_p \approx \sqrt{\zeta}$, in the range $5 < \zeta < 40$. We have now written a code which evaluates the ballooning instability criterion over the whole plasma cross-section of any given axisymmetric equilibrium. In particular, this code will be applied to nonlinear equilibria with circular cross-section to determine, for given pressure profiles, ballooning marginal profiles of the toroidal current $J$.

[1] Connor, J., Hastie, R., Taylor, J.B.,


"This work was performed under the terms of the agreement on association between the Max-Planck-Institut für Plasmaphysik and EURATOM."
We have developed a one-dimensional MHD simulation code including both plasma transport and impurity effects that follows the time evolution of a Reversed Field Pinch (RFP) through a series of hydrostatic equilibria. These effects may be separated since impurities radiate energy out of the plasma causing an adiabatic change to a new equilibrium but do not contribute to the motion of plasma across magnetic flux surfaces. The full equations are thus split into two sets, one which contains plasma transport and another radiation. Two codes were developed and linked together to solve the full problem. The transport code has been described earlier. The radiation code is essentially a set of ideal MHD equations, with each impurity charge state treated as a separate fluid, that contain energy loss terms due to radiation. When written in a Lagrangian coordinate system based on the poloidal flux, these become a simple set of ordinary differential equations.

Results are presented for several RFP operating parameters, including those of ZT-S and ZT-40 at Los Alamos. These results show that the electron temperature in the ZT-S experiment is radiation limited due to oxygen impurities at present operating densities and impurity levels. The strong dependence of radiation loss on density for a fixed relative impurity concentration is also shown. Using classical transport coefficients for ZT-40, we find that large electron temperature gradients can be created when the plasma burns through a radiation barrier in only one region. Classical electron thermal conductivity is not large enough to strongly couple neighboring plasma radii and reduce these gradients. In addition, we find that the plasma is usually Suydam unstable in the outer third of the discharge.


First and second harmonic ion cyclotron resonance heating is being studied as a possible alternative to neutral beam heating of the end plugs of a tandem mirror. As a simple model we consider midplane and off-midplane heating in a parabolic well. The energy gain per pass is calculated from the single particle equations of motion for an RF wave travelling obliquely to the local magnetic field. This analysis differs from previous theoretical treatments in that arbitrary harmonics and large doppler shifts are allowed.

The resulting $\Delta v_\perp$ is given in terms of a generalized Airy function, which is evaluated asymptotically in cases of interest. Various applications to the Phaedrus experiment will be discussed.
The appearance of a continuous spectrum in the ideal MHD equations is related to the non-ellipticity of the time-independent equations. Every family of characteristic surfaces gives rise to part of the continuous spectrum. The familiar Alfvén and Cusp continua arise from the presence of pressure surfaces which are characteristic surfaces even when the equilibrium state involves mass flow. Ballooning modes are related to the existence of a second family of magnetic flux surfaces. This approach enables one to treat ballooning modes not through minimization of $\delta W$ and the use of eikonal forms but by direct derivation from the differential equations, in a way similar to the traditional treatment of the Alfvén continuum.

The equations determining ballooning modes will be derived for closed field line systems. It will be demonstrated that in a mirror configuration, the outcome is equivalent to the modes obtained in Ref. 1 in the limit $m \rightarrow \infty$. We anticipate that an approach based on these ideas will help resolve questions concerning boundary conditions for ballooning modes in sheared systems.


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Drift-Wave Eigenmodes in Toroidal Plasmas*

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Effects of toroidal couplings on the shear damping of drift-wave eigenmodes are studied using the ballooning-mode formalism.\textsuperscript{1-3} WKB analyses are then carried out for the drift-ballooning eigenmode equation\textsuperscript{4,5} in the cold-ion limit. It is found that two types of eigenmodes exist. One is slab-like and the other is toroidicity-induced. Depending on the parameters and the type of the eigenmodes, toroidicity can either enhance or reduce the shear-damping rates. Both analytical and numerical results will be presented.

\* Work supported by U.S. DoE Contract No. EY-76-C-02-3073.
1 A. Glasser, et al. (to be published).
CHERENKOV RESONANCE AS AN FLR EFFECT ON THE ALFVEN-ION-CYCLotron MODE

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From hybrid-kinetic theory (Vlasov ions and guiding-center electrons) an eigenvalue equation for electro-magnetic perturbations with $\omega \leq \omega_{ci}$ in collisionless $\theta$-pinches with anisotropic ion energy was recently derived. This equation is presently reduced to two coupled, ordinary second order linear differential equations by an expansion in thermal ion gyroradius. These equations are supplemented by appropriate boundary conditions for the case when the plasma is surrounded by a cylindrical, perfectly conducting wall. For weak inhomogeneities a local dispersion equation is obtained that can be solved using standard numerical procedures.

Both within the context of global and local analysis, the leading order correction terms contain Cherenkov resonances absent in the homogeneous case. This resonance is due to the existence of a radial ion pressure gradient: the unperturbed electric field associated with this gradient induces an electric field component parallel to the instantaneous magnetic field. This electric field is annihilated by the rapid electron motions parallel to the magnetic field, thereby inducing an electric field disturbance parallel to the direction of wave propagation. For high-$\beta$ the phase-velocity of the wave is comparable to the thermal ion velocity. Conclusively, Cherenkov resonance as an FLR effect influences stability of the Alfvén-Ion-Cyclotron wave and might change the amount of anomalous transport ($T_{ii}$) associated with it.

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THE MICROWAVE SPHEROMAK*

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A recent design proposed to construct a "spherical" tokamak by inducing a ring current perpendicular to a uniform d.c. magnetic field, producing a separatrix along the axis of the configuration and closed toroidal magnetic surfaces, i.e., the Spheromak. It is the purpose of this paper to propose a scheme to generate this configuration with the use of electromagnetic current drive.

The device is set up in a cylindrical microwave cavity, along the axis of which a uniform d.c. magnetic field is imposed. The required toroidal current is presumed to be driven by r.f. fields that are oriented in the azimuthal direction. This can be done by driving the cavity in a mode of the form TE_{2mn}, where \( \ell, m, n \neq 0 \). The electric fields of such a mode are of the form:

\[
E_r = -\frac{\alpha J_\ell(k_1 r)}{k_1 r} \sin \phi \sin k_3 z
\]

\[
E_\theta = -J_\ell'(k_1 r) \cos \phi \sin k_3 z
\]

\[
E_z = 0
\]

Note that if \( \ell \neq 0 \), the azimuthal variation of the azimuthal component of E is a standing wave which can be broken into two travelling waves. By suitably tuning the cavity, a single travelling wave can be excited. If \( \ell \gg 1 \), then conditions for r.f. current drive can be satisfied. A similar configuration may be set up in a spherical cavity.

Plasma rings carrying such currents have previously been excited,\(^1\)


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INITIAL RESULTS OF TANDEM MIRROR TRANSPORT CALCULATIONS

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ABSTRACT

A code previously used for mirror radial buildup studies¹ has been modified to study tandem-mirror radial transport. The revised code includes modified endloss terms, extra (non-ambipolar) transport coefficients to describe enhanced diffusion caused by the quadrupole field, a procedure to solve for the radial potential profiles in the solenoid and plugs from requirements of charge neutrality, and bounce-averaged (over plugs and solenoid) equations for electrons. Atomic physics and finite gyroradius effects were included in the original code and have been retained. Initial runs have been made with one central cell ion species, deuterium, a constant (in radius and time) ion source, and a deuterium plug with fixed density and temperature profiles. In these runs, quadrupole field effects were treated by using order-of-magnitude approximations² to resonant ion³ and neoclassical electron transport coefficients.

Trial runs have been made with no transport and various combinations of classical and quadrupole enhanced electron and ion particle and energy transport, using TMX parameters. The results obtained indicate that resonant transport depresses the mid-solenoid density and raises the ion temperature, by a few percent, when compared with the results obtained using solely classical transport. A substantially higher (20%) mid-solenoid density and lower (40%) ion temperature are obtained in runs with all transport coefficients as compared to runs with no transport. In runs with particle transport but no energy transport, the equilibrium density profiles are lower than those obtained with no transport, and the depression is due mostly to ion resonant transport.

¹Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contracts W-7405-ENG-48 and ET-78-5-02-4636.

A radial transport code for tandem mirror devices has been developed.

An arbitrary number of central cell ion species described by density profiles \( n_a(r,t) \) and temperature profiles \( T_a(r,t) \), plug ions of density \( n_p(r,t) \) and energy \( E_p(r,t) \), and electrons of density \( n_e(r,t) \) and temperature \( T_e(r,t) \) are considered. The quantity \( r \) is the radius of a magnetic flux surface at the midplane of the central solenoid, and the above densities and energies are azimuthal averages.

Particle diffusion, heat conduction, heat convection, energy exchange, charge exchange, ionization, end-loss and acceleration due to the radial electric field are modeled. Empirical, classical, neoclassical and resonant transport models are included. Axial loss rates are computed using a general Pastukhov formula. Radial ambipolar potential profiles in the central solenoid and plugs consistent with charge neutrality are determined.

The transport equations are time-advanced using an implicit, iterative finite difference algorithm. Spatial gradients are centered, with the exception of the convection term, which uses upwind differencing. Particle and energy conservation up to roundoff error is maintained.

Preliminary results of applications to the Tandem Mirror Experiment (TMX) are presented.

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*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.*
TRANSPORT EQUATIONS FOR TANDEM MIRROR MACHINES

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ABSTRACT

We have derived a set of one-dimensional transport equations for tandem mirror machines. This set of equations includes endloss and classical as well as quadrupole-field enhanced radial transport, and correctly describes transients due to time-varying magnetic and electric fields. The equations can be derived from either the drift-kinetic equation or the Boltzmann equation.

We have developed an analytic approximation to the resonant plateau diffusion coefficients of Ryutov and Stupakov\(^1\). It is derived by neglecting the azimuthal \(\nabla B\) drift compared to the azimuthal \(E \times B\) drift, and by adopting a semi-empirical model for the pitch-angle dependence of the radial displacement per bounce (obtained by fitting numerical drift calculations). The resulting expression is used to obtain numerical results\(^2\) and analytic estimates for resonant transport in TMX, a scaled-up tandem mirror experiment (MFTF-B), and tandem mirror reactors.

\*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.

2. A. A. Mirin, R. H. Cohen, M. E. Rensink and J. Killeen, Paper at this meeting.
PARTICLE MOTION IN A CYCLOTRON RESONANT FIELD

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ABSTRACT

The time averaged equations for a particle's motion in a mirror field with electric field r.f. present near the central cyclotron frequency (or its harmonic) is derived. The averaging method is an extension of the technique used by Aamodt and Bodner (1) who studied such particle motion in a uniform magnetic field. We obtain four coupled nonlinear equations for the axial position, axial velocity, perpendicular velocity and the particle's relative gyrophase with respect to the wave phase. When the rate of change of relative gyrophase, \( \omega - \omega_c(s) \), is larger than the bounce period, \( \omega_B \), we derive a ponderomotive force for axial motion. In the opposite limit, \( \omega - \omega_c(s) < \omega_B \), we obtain an analytic description of superadiabatic motion. In both limits we show that there exists low energy particles that are trapped at the center of the mirror even in the presence of a repelling ambipolar well. This may have a stabilizing effect on the amplitude of loss cone modes. The intermediate limit, which is more difficult to analyze analytically, is the regime of stochastic motion. Numerical solutions will be compared to the analytic theory.

1Aamodt and Bodner, Phys. Fluids 12, 1971 (1969)

"Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48."
The effect of an externally applied large amplitude, spatially uniform electric field at the lower hybrid frequency on the drift-cyclotron loss-cone mirror instability is investigated. It is found that the lower hybrid field has a stabilizing effect on the drift-cyclotron loss-cone mode if
\[ \omega_{LH} < \omega < \omega^* \]
where \( \omega_{LH} \) is the lower hybrid frequency, \( \omega \) is the applied wave frequency, and
\[ \omega^* = \left( c + \sqrt{c^2 + 4\omega_{LH}^2} \right)/2 \]
\[ c = \frac{\omega_{LH}^2}{\omega_{ci}^2} \]
on-otherwise, the lower-hybrid field has a destabilizing effect on the drift-cyclotron loss-cone mode.
Three Dimensional Fluid Simulations of Drift Waves

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Abstract

The ion pressure gradient driven drift instability has recently been invoked as a possible explanation for the rather high frequency, large amplitude density fluctuations observed in strongly beam heated PLT plasmas. A 3D code is developed to investigate the nonlinear behavior of this instability. A simple set of fluid equations is used for the electrostatic potential, the parallel ion velocity and the ion pressure, neglecting quasilinear relaxation of the density. Besides giving saturation levels of this type of drift instability, these model computations are of more general value to understand the basic nonlinear dynamics of electrostatic drift waves, in particular the coupling of parallel phase-velocities generating convective cells.

This work is supported by the U.S. Department of Energy Contract DE-AC05-79ET53036.
A significant improvement of the particle MHD code is achieved by implementing the Lax-Wendroff method for advancing the magnetic field in a way analogous to Makino et al.Sharper mode spectra than are obtained by the Lax method have been observed with the present code, as the numerical diffusion of the magnetic fields is reduced to the order of $(k\Delta)^4$. Thanks to the low magnetic diffusion with this algorithm along with the particle nature of the code, we are able to simulate problems with sharp plasma boundaries and large density ratio. The Adam-Bashforth method (extrapolated leapfrog method), accurate also up to $(k\Delta)^4$, has been tried and compared to the Lax-Wendroff code: we find that the Adam-Bashforth code is more susceptible to numerical difficulties in the case of handling sharp boundaries. Applications of the 2-1/2 D code have been started with studies of flute and ballooning instabilities and the area wave propagation in a high $\beta$ plasma column. For a sharp boundary plasma in a gravitational field with finite $\beta$, we see a critical ratio of $k_n/k_\perp$ $(k_n$ parallel to $B_0)$ below which the ballooning mode is unstable.


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†Supported by a D.G.E.S. Fellowship.
The application of lower hybrid range of frequencies (LHRF) to tandem mirror geometry for direct heating of electrons, is of interest because it allows a significant relaxation of the ion energy required for end plugging of a fusion plasma. We have considered the wave parameters required for this application. The condition for efficient absorption of wave energy by electron Landau damping is consistent with both the accessibility condition and the avoidance of the mode conversion layer. By proper choice of wave frequency, electrons at the loss boundary can be selectively heated and driven out. This process by which the potential barrier is amplified can therefore be very energy efficient. For parameters typical of tandem mirror reactor, the window in $\omega$-$k$ space for effective application of LHRF waves is technologically much more attainable than either ICRH supplementary heating of ions in the end plugs or the application of ECRH that are presently being considered.
Simulation of Multi Impurity Species Transport in Tokamaks

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We continue to upgrade our numerical simulation of transport of multiple species of impurities in tokamaks. Our present emphasis is to develop a calculational module treating impurity transport and atomic physics that can be utilized generally in tokamak transport simulation codes. As part of our review of neoclassical and classical ion diffusion coefficients we have developed some approximations for Pfirsch-Schluter regime coefficients that significantly reduce the complexity of the expressions while maintaining high accuracy. We present some comparisons, both of individual coefficients and of complete simulations, in which exact and approximate forms have been used.

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REAL-TIME MHD COMPUTATIONS FOR NONCIRCULAR TOKAMAKS ON A HIGH-SPEED ARRAY PROCESSOR

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ABSTRACT

One of the most important tasks in high-\(\beta\) noncircular tokamak experiments, such as Doublet III, is to shape and maintain a desirable plasma cross section throughout a discharge. Sequences of time-resolved MHD equilibrium analyses, produced by fitting experimentally-measured magnetic data, greatly enhance prospects for the successful operation and diagnosis of these noncircular plasma experiments. It would be especially useful if the experimental data could be processed on a real-time basis and the analyzed results returned within the 5-10 minute interval between shots.

The computational processing involved is significant: the General Atomic MHD equilibrium code takes approximately 0.45 sec/step on the MFECC A-7600 machine and typically, 100 steps are needed to produce one set of analyzed data corresponding to a single instant within a particular plasma shot. To satisfy the stringent requirement on the computation time, we have interfaced a high-speed array processor AP-190L capable of performing several million floating-point operations per second with the existing data acquisition system at General Atomic based on the USC DEC System-10 computer. The GA free boundary MHD equilibrium code has also been converted to run on the DEC-10-AP-190L system.

Detailed structures of both the computer system and the MHD code will be presented. Initial timing comparison between A-7600, CRAY-I, and DEC-10-AP-190L will be presented as well.

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EFFECTS OF SHEAR ON DRIFT-CYCLotron INSTABILITY

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We have studied the effects of magnetic shear on the Drift-Cyclotron (DC) Instability by including the exact particle orbits in a sheared magnetic field. The main effect of including the exact particle orbits is to introduce the Shear Kinematic Drift (SKD)\(^1\) which essentially modifies the potential in the Weber equation.\(^2\)

The growth rates and the critical shear needed to trigger the stabilising process were calculated and compared with the conventional theory which uses uniform field orbits. We find that the growth rate (\(\gamma\)) for the very short wavelength, the \(k\rho_e \gg 1\) modes, now depends on the parameter \(\alpha' = \alpha / (k\rho_i)^{1/2}\), where \(\alpha = \omega_s / \bar{\omega}\) (\(\omega_s = \rho_i S \delta / \omega_{ci}\)) is the characteristic SKD frequency; \(\rho_i\) is the Larmor radius; \(\omega_{ci}\) is the ion-cyclotron frequency; \(S\) is the inverse shear length, and \(\bar{\omega} = \omega - \omega_{ci}\), the resonance factor.) When \(\alpha'\) is greater than a critical value \(\alpha'_c\), we find that \(\gamma_{SKD}\) is significantly less than \(\gamma_{\text{conventional}}\). When \(\alpha'\) is less than \(\alpha'_c\), \(\gamma_{SKD}\) is slightly greater than \(\gamma_{\text{conventional}}\). Detailed calculations and the results of the numerical study on the more general second order differential equation with the full potential will also be presented.

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A Monte Carlo Model of Particle Motion in Field-Reversed Mirrors — MCFRM

by

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Fusion products (fps) are found to have a significant effect on both the steady-state particle and energy balances in Field-Reversed Mirror (FRM) plasmas in spite of the small size of the plasma (radius equal to only a few fuel ion gyroradii). In fact, over 40% of the fp energy is retained in the closed-field region of a D-He FRM with S (the ratio of the plasma radius to the fuel ion gyroradius) equal to 5. This results in an energy multiplication factor (Q) of 12 as compared to the Q of 2.0 associated with the same system without fp heating. Unfortunately in a steady-state system, the desirable contribution of fp heating is necessarily accompanied by an increase in the fp ash buildup which reduces the actual Q value of the above system to 4.5. An accurate calculation of fp ash deposition is therefore seen to be an important consideration in evaluating steady-state FRM design concepts such as SAFFIRE.

To facilitate this, a Monte Carlo particle code, MCFRM, has been developed. It couples the Hill's spherical vortex representation of a field-reversed equilibrium, with a Monte Carlo treatment of Coulomb scattering; thus providing a complete picture of fp thermalization in the FRM, even at lower energies where pitch angle scattering becomes important. The basic algorithm will be discussed, along with results from several test cases which were run to establish the validity of the model. Additional results will also be presented which summarize the affect of fp heating and ash deposition on the SAFFIRE reactor, and illustrate several possible means of ash control.


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*This work supported by Department of Energy Contract EY-76-S-02-2218.
A Numerical Investigation of the Evolution of the Electron Distribution Function in Tokamaks*

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Recent papers on high frequency instabilities in tokamak discharges relied on simple models of the anisotropic electron distribution function. In order to improve the calculations, the shape of the full electron distribution function has been determined numerically as it evolves in time in a tokamak discharge. The numerical code which describes the particle dynamics includes an inhomogeneous magnetic field (trapped and untrapped particles), the applied electric field (Ohmic heating) and Lorentz collisions.

The electron dynamics are described by the drift-kinetic equation expressed in energy and magnetic moment variables. The resulting three equations: 1) trapped electrons, 2) co-streaming passing electrons, and 3) counterstreaming passing electrons, are then transformed to a compact domain where the actual numerical procedure is employed. The differential equations are written in conservation form.

The electron distribution function has been determined for a range of electric field strengths and varying degrees of magnetic field inhomogeneity. Also in this region of parameter space the plasma resistivity and production of runaway electrons have been calculated. This information should aid in determining whether the changes in the tokamak discharge parameter alters the electron distribution function which in turn determines the runaway electron instability seen in tokamaks or whether the electron distribution function remains unchanged and some other physics is responsible.

* Work supported by U. S. Department of Energy.

A code has been written at Wisconsin for numerical simulation of alpha particle motion in vacuum fields of stellarator-type geometry. This code implements the Lorentz force equation for the magnetostatic case, avoiding problems encountered in guiding center methods which rely on the existence of adiabatic invariants of motion. This permits the code to be used in the study of the longitudinal invariant of motion for localized particles, which is of importance in the theory of superbanana diffusion.

Several orbit types have been studied in a comparison between torsatron and stellarator reactor configurations (R major = 30 M; R minor = 4 M; B = 5 Tesla; 20 field periods). These simulations involve 3.5 MeV alpha particles launched under identical initial conditions in the reference stellarator and torsatron of the \( \alpha = 3 \) type. The reference machines are designed to match flux surfaces, ripple profiles, mod-B surfaces and transform profiles within the regions enclosed by their separatrices.

The orbits obtained have been compared in terms of their precession rates, action, magnetic moment, and turning points. The absence of a strong longitudinal invariant of motion has been observed. Conservation of the action occurs on a piece-wise basis in general, with discontinuities due to particle detrapping at helical mirror boundaries. The computed stellarator orbits are qualitatively similar to earlier simulations for lower energy particles. However, for the torsatron, significant differences in the localized orbit types are seen. In particular, the conditions for superbanana orbits, and their apparent absence in the torsatron geometry, are found to be related to the topological differences between the stellarator and torsatron designs used for these calculations.


2A. Gibson and J. B. Taylor, Physics of Fluids 10, 2653 (1967).

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Computational and Analytic Study of Ballooning Modes in Highly Elongated Tokamaks*

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Computational results for high n ballooning modes in highly elongated elliptic plasma is presented. The effects of elongation, toroidicity, shear, and pressure profile are studied computationally using the ORNL BALOON code and these results are compared with and clearly understood through the analytic calculation of high n ballooning modes in highly elongated elliptic plasmas. The marginal ß's are plotted as functions of ßp for different elongations, shears, pressure profiles, and aspect ratios. The analytic results predict and the computational results verify that high elongation, low aspect ratio, and broad pressure profile enhance the marginal beta value for ßp less than unity but severely reduce ß for ßp larger than unity with pressure p(ψ) = AψM and safety factor q(ψ) = B - CψN. Stability sensitively depends on shear, q(ψ), as a function of poloidal Ψ, even when q(ψ) at the magnetic axis and at the plasma edge are fixed, while equilibrium is insensitive to q(ψ) in this case. Detailed comparisons between computational results and analytic results are given for a variety of cases.

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Nonlinear Magnetohydrodynamics in Three Dimensions,
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We describe a simple numerical method for nonlinear magnetohydrodynamics in three dimensions that is designed to be efficient in any problem characterized by long thin geometries and low speed flows, and can be incorporated easily into existing initial boundary value codes.

The method is similar to that of Jardin et al [1] in that the terms corresponding to the fastest time scale are systematically identified. However, it is different in that the terms are not then formally isolated. Rather, only those terms corresponding to the fastest time scale are made fully implicit. This selectively or semi-implicit formulation is demonstrably twice as fast as an earlier implicit code [2], and potentially even faster. Further, because the magnetohydrodynamic equations rather than derived equations are differenced, it is conservative in the low speed flow limit, and incorporates resistive transport.

The analysis, the formulation, and the results of the calculation of an initial shear flow discontinuity in one dimension, and helical equilibria in three dimensions will be presented.

The nature of the anomalous transport of electron thermal energy in existing experiments on magnetically confined toroidal plasmas is discussed and a new form of the relevant electron thermal conductivity, that is consistent with the observed temperature profiles, is presented. In particular, scalings of the energy replacement time and the applied loop voltages that are consistent with the experiments are obtained. In the presence of ohmic heating alone a simple analytical form of the relevant electron temperature profile can be derived. The appropriate diffusion coefficient can be written as

$$D = \frac{e^2}{4\pi} \left( \frac{E}{\nu_{le}} \frac{\omega_{pe}}{\nu_{le}} \right)^{2/\nu} \times \frac{E}{\nu_{le}},$$

(1)

where

$$\nu_{le} = \frac{e^2}{4\pi} \left( \frac{E}{\nu_{le}} \frac{\omega_{pe}}{\nu_{le}} \right)^{2/\nu}$$

is the resistive diffusion coefficient,

$$E_{pol} = \frac{1}{r}$$

is the local poloidal field,

and the other quantities have well known definitions. This diffusion coefficient has been incorporated in the transport model and code that are described in Ref. 2 and have been utilized to simulate a variety of plasma discharges. The set of experiments for which Eq. (1) appear to be most appropriate have been performed on the Frascati FT device in which it has been possible to vary the plasma current by a significant factor, up to 600 kA.


2. B. Coppi and A. Taroni, Report PRR-79/7, R.L.E., Massachusetts Institute of Technology, (Cambridge, Ma., 1979)
ABSTRACT

Experiments under construction at LLL propose to make magnetized plasma rings by gun injection into a guide field leading to a quadrupole mirror trap. These rings are then to be heated from their initial kev to temperatures around 20 kev by neutral injection. A number of theoretical models have been developed to describe plasma rings, with or without embedded toroidal fields and with poloidal and toroidal plasma flows. Single particle studies help to connect simple multifluid models to kinetic theory of these various equilibria. Numerical solutions of the equations enable the initial and final plasma states to be calculated and the accessibility of the transitions to be assessed.

As an example of the more general theory, we consider the pressure balance equations for a rotating fluid plasma species. This can be reduced to a Bernoulli type equation on each magnetic surface, relating pressure, electric field $E$, and rotation speed, and a pressure balance equation across the surfaces, relating centrifugal forces and pressure gradients. Entering the solutions into Amperes law gives a modified Grad Shafranov equation for a multifluid equilibrium:

$$\Delta^* \psi_0 = -\frac{4r^2}{c^2} \sum_j \sum_j n_{0j} c_j^2 \frac{\partial H_j}{\partial \psi_j} \exp(H_j - \frac{e_j}{c_j^2} (\phi + \frac{r^2}{2} \frac{\partial H_j}{\partial \psi_j}))$$

where $H_j(\psi_j)$ is an arbitrary profile function for each species, depending on the total flux $\psi_j = \psi_0 + e_j^{-1} c_m n_j r^2 \frac{\partial H_j}{\partial \psi_j}$. The potential is determined by the quasineutrality equation:

$$\sum_j n_{0j} c_j^2 \frac{\partial H_j}{\partial \psi_j} \exp(H_j - \frac{e_j}{c_j^2} (\phi + \frac{r^2}{2} \frac{\partial H_j}{\partial \psi_j})) = 0$$

These fluid equations do not describe the contribution of large gyroradius ions to the fields. Orbit studies in axisymmetric reversed fields, such as Hill's vortex, show that a third invariant exists which is easily destroyed by collisions or quadrupole fields. High energy ions are therefore included in the studies via distribution functions, $f$, of energy $\varepsilon$ and canonical angular momentum $p_\theta$.

A transport theory is required to determine $H_j$ and $f(\varepsilon, p_\theta)$ self consistently, so present studies use model profiles.
GENERALIZED WKB METHOD IN ONE DIMENSION

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ABSTRACT

A generalized WKB method is developed for calculating eigenvalues and eigenfunctions of electromagnetic waves governed by integral equations or, equivalently, differential equations of arbitrary order in a medium that is inhomogeneous in one dimension. The method extends previous work with one component of polarization (e.g., electrostatic waves) to the three components of polarization of an arbitrary electromagnetic wave. The wave amplitudes are expressed in terms of a superposition of eikonal solutions. We find that the amplitude variation in space has a compact explicit solution when the kernel is symmetric with respect to position in the inhomogeneous direction x, and in the components of polarization. With a suitable choice of components, this symmetry applies to a wide class of problems and includes Landau damping.

The local wave number, \( k(x, \omega) \), is determined by setting the "local dispersion relation" to zero. The rules for obtaining the appropriate local dispersion relation are given and they contain corrections from what one usually expects. A specific example will be given.

The WKB solutions fail near turning points, where two solutions, \( k(x, \omega) \), merge in the complex \( x \)-plane. In the vicinity of the turning point, the merging waves satisfy an Airy equation, which allows for the determination of reflected amplitudes and phases away from the turning point. By mapping the wave trajectories and demanding single valuedness of the solution, a general phase integral dispersion relation can be determined.


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STABILITY AND FORCE-FREE FIELDS IN AN ELLIPTICAL CYLINDER

George Vahala (William and Mary)

Force-free fields in an elliptic cylinder are generated by finding the eigenfunctions and point spectrum of the curl operator $\nabla \times \vec{B} = \lambda \vec{B}$ with $\vec{B} \cdot \hat{n} = 0$ at the wall. It is found that the point system is completely determined by the geometric boundary condition and consists of discrete eigenvalues on the real line. The minimum (non-zero) eigenvalue is the first zero of the radial Mathieu function.

Thus the circular cylinder is a singular limit much like zero shear is a singular limit\(^1\). In the limit of zero eccentricity, the point spectrum remains completely determined and discrete, but for the circular cross section itself, the point spectrum consists of a continuous eigenvalue (extending to the origin) together with discrete eigenvalues bounded away from the origin.

The implications of this singular limit on Taylor's reversed field theory\(^2\), on linear MHD stability of the Lundquist solution\(^3\) and the discrepancy with nonlinear stability results\(^4\) will be considered.

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Using Antonsen's technique, we first show that the collisionless drift and drift-Alfvén eigenmodes are stable in a sheared slab magnetic field. Noting that Antonsen's technique is valid only for cold ions, we have developed a more powerful theory for analyzing the stability of drift-wave eigenmodes. The theory employs the S-matrix technique in the complex plane and is based on the observation that the physical system, after a suitable complex transformation, exhibits wave-flux conservation. Applying this technique, we prove that, with full kinetic-ion effects, the collisionless drift-wave eigenmode is stable. We further demonstrate that this theory can also be applied to the case with arbitrary radial wavenumbers. Here, due to the finite-ion-Larmor-radius effects, the usual differential equation is replaced by an integral eigenmode equation. The universal drift-wave eigenmodes is found to remain absolutely stable.

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VORTICES IN 2-D GUIDING ENTER PLASMA WITH GRAVITY*

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To study the convection cells in multipoles geometry, the equilibrium state of a two-dimensional guiding center plasma under gravity which simulates curvature effect is considered. The most probable state of this system can be described by a Poisson equation with Boltzman density distribution for both electrons and ions. We found a conformal mapping which can transform away the gravity and reduce the equation to the nonlinear sinh-Poisson equation. Exact solutions are found and the effect of the gravity is to introduce a natural period independent of boundary condition in the direction perpendicular to the gravity. The guiding center plasma is thus quantized in this direction. The simplest solution shows a somewhat stochastic distribution of vortices of various sizes with the smaller ones at the bottom and the larger ones forming coherent structure near the top. The implications of these highly complicated vortex structures to the convective-cell transport in multipoles devices will be discussed.

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SHAPE OPTIMIZATION OF TOKAMAK PLASMAS TO LOCALIZED MHD MODES

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ABSTRACT

We employ a numerical technique to optimize the shape of tokamak plasmas to achieve maximum stable volume average beta, $\beta$, with respect to localized interchanges and localized ballooning modes. A free boundary equilibrium is calculated numerically and its stability to internal modes is assessed. Optimization is then accomplished by automatically varying the equilibrium boundary conditions in the direction of increasing maximum stable $\beta$. For plasmas without internal separatrices, we find the optimal shape to be a strongly modified dee with a large indentation on the inside edge of the plasma. The maximum stable beta exceeds 14% for a moderately-peaked current profile with beta poloidal = 1, aspect ratio = 2.76 and b/a, the height-to-width ratio of the rectangular limiter, = 3.0. Optimal doublet shapes are also presented. MHD stability to external modes is evaluated for the indented dees using ERATO.$^2$

Work supported by the Department of Energy, Contract No. EY-76-C-03-0167, Project Agreement No. 38.


MODELLING OF STAGED LASER HEATING*

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The laser solenoid is a linear magnetic fusion concept which employs heating by an axially propagated laser beam and adiabatic magnetic compression. A number of experiments on this concept have been carried out including a large on-going experiment at Mathematical Sciences Northwest, Inc., (MSNW). Laser heating and adiabatic compression overlap in time and occur on comparable timescales. The objective of this work is to characterize the plasma conditions which may be achieved by this method.

Staged laser heating is treated using a dual approach which includes an analytical model to give approximate scaling and a one-dimensional magnetohydrodynamic code for precise solutions. Previous analytical models have assumed the laser heating time to be much less than the compression time or else have held the electron density constant. Our analytical model accounts for staged heating on comparable timescales as well as including two temperatures and crude radial structure. Results will be presented characterizing the plasma temperature and plasma radius as a function of laser energy and filling pressure for conditions relevant to the current MSNW experiment and hypothetical reactors. Results of the 1D MHD code DYNASOR will also be presented and compared with experimental data. The code includes additional effects of importance such as radial dynamics, classical radial thermal and field diffusion, ionization, and impurity radiation loss.
The Effects of Low Frequency Electromagnetic Turbulence on Toroidal Plasmas

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Abstract

We have constructed a quasilinear theory for the effects of low frequency electromagnetic turbulence in a toroidal plasma, ignoring terms $O(E^2_{\parallel} r/R)$. The qualitative behavior of the diffusion tensor and the special role played by $E_{\parallel}$ will be discussed. In addition we shall point out the similarities of this theory to our earlier slab model theory and indicate the future applications which are planned.


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ON MODE CONVERSION OF LOWER HYBRID WAVES

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ABSTRACT

The problem of mode conversion of lower hybrid waves in the absence of a reflection layer is considered. It is found that partial mode conversion takes place for some ranges of plasma parameters. Nonetheless, the RF power can penetrate to the plasma center. The quasilinear behavior of the electron Landau damping sets a power dependence to mode conversion. For small incident lower hybrid powers, Landau damping may be too large for mode conversion to be observed. In such cases, mode conversion can become possible if the RF power exceeds some critical level because of quasilinear flattening of the electron distribution function with the resultant weakening of electron Landau damping.

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Stabilization of Trapped-Electron Shear-Alfven Instabilities by Temperature Gradient

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Abstract

Localized shear-Alfven modes with large m-numbers\textsuperscript{1,2} are shown, numerically, to be strongly damped by the collisionless electron response in the presence of a temperature gradient. The trapped-electron drift-tearing instability\textsuperscript{1} is stabilized by this effect in a tokamak unless the local inverse aspect ratio, r/R, exceeds a critical value, typically between 0.1 and 0.2. Analytical models demonstrate the scaling of these results with plasma parameters.


This work is supported by the U.S. Department of Energy Contract DE-AC05-79ET53036.
Stable Spheromak Current Profiles*

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The original spheromak concept was the small aspect ratio limit of a toroidal magnetic confinement system, with no hole in the middle and no external toroidal field coils. When that was found to be Mercier unstable at very low $\beta$ unless a small hole was put in the middle, it became clear that the essential feature of a spheromak is the absence of external toroidal field coils rather than the spherical shape.\footnote{M. N. Bussac, et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 7th International Conference, Innsbruck, 1978) IAEA-CN-37-X-1} It is therefore worth while to consider the large aspect ratio limit of the spheromak, modeled by cylindrical fields $B_\theta(r)$ and $B_z(r)$, with $B_z = 0$ for $x \equiv r/a > 1$. We have studied a variety of smooth current profiles of the form $J = \sigma \hat{B}$, with $\sigma = \sigma_0 (1 - x^{p_1})^{p_2}$ for $x < 1$ and $\sigma = 0$ for $x > 1$, and found stability to all kink and tearing modes for $p_1 = 15$, $p_2 = 2$, and with a wall at $x = 1.0475$. The smooth decrease of the current towards the edge of the plasma is more realistic than previous step-function models. Stability with the wall removed from the edge of the plasma is important for practical reasons such as impurity control and thermal isolation.

*Work supported by the United States Department of Energy Contract No. EY-76-C-02-3073.
LOWER HYBRID HEATING AND CURRENT GENERATION
IN VERSATOR II*

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We have used a one-dimensional transport code to model the time dependent plasma heating and current generation that we expect to result from the lower hybrid RF heating experiment being prepared for Versator II. Some of the features of the code are described elsewhere.\(^1\) RF energy deposition into the bulk plasma is included through appropriate quasilinear equations which describe parallel electron and perpendicular ion Landau absorption. A modified version of Fisch's theory\(^2\) has been used to obtain the quasilinear corrections to the linear damping and the generation of RF current via plateau formation in the tail of the electron distribution function. Provisions for inductive effects have been included in the code.

We have followed the time evolution of electron temperature and RF current both during and after an RF pulse for various combinations of waveguide array configuration and initial (pre-heating) plasma state, assuming the generation of a Brambilla power spectrum. We have attempted in this manner to estimate optimum operating parameters for the Versator II RF experiment.


* Work supported to USDOE
The linearized, 3-D hybrid code RINGHYBRID [1] was developed at Cornell University for the purpose of studying the low-frequency stability of field-reversed ion rings in a background plasma. With minor modification the program is capable of examining the stability of axisymmetric field-reversed mirror equilibria.

We have examined the dispersion properties of waves in the background plasma (in the absence of any ring) in greater detail than previously presented, and find finite cell-size effects to be of the sign and approximate magnitude expected.

A preliminary study of infinite-layer stability as a function of layer strength, background density, and external field gradient has been carried out. We observe stability of the MHD precessional mode when the external magnetic field increases strongly with radius, and instability in the opposite limit; near zero field gradient, however, results are inconclusive. We observe a regime of decreasing growth rate as the layer strength increases, as suggested by theory [2,3]. We have also examined the effects of various boundary conditions applied to the field equations at the outer wall and on axis.

The current plasma model requires the cold fluid background ion component, in addition to the electron and hot-ion components, because the present field solver requires a small Δt for convergence when regions of small total ion density exist. In order to model inhomogeneous hot mirror plasmas more efficiently, modifications of the fieldsolving algorithm to remove this limitation have been proposed. We have also begun to consider models which match the usual equations in the plasma region to another set of equations assumed to hold in a surrounding vacuum region.

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THE DISTRIBUTION OF AND CLASSICAL TRANSPORT BY ALPHA PARTICLES
IN A THERMONUCLEAR PLASMA*

by

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The velocity distribution of alpha particles produced at a constant rate by thermonuclear reactions in a Maxwellian plasma is obtained analytically from the Fokker-Planck equation. The time-asymptotic distribution can be divided into three regions: a thermalized region with a nearly Maxwellian distribution, a slowing-down region with a power law distribution, and a high-energy region with a rapidly decreasing exponential distribution. A more detailed treatment, including the time evolution, loss term and a weak parallel electric field is given for the slowing-down region, which contains the majority of the alpha particles, and for the high-energy tail. The time evolution of the density, momentum, kinetic energy and heat flux in calculated. The electron and background ion contributions are given separately to show the effects of each species. In particular it is found that the electrons are more rapidly heated by the alpha particles than are the background ions.

* Research supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Financiadora de Estudos e Projetos (FINEP).
Parametric Decay Heating with an Electron Cyclotron Wave

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The standard ECRH schemes for heating tokamaks require $\omega_0^2 > \omega_{pe}^2$ at the region to be heated; however, development of gyrotrons at the required frequencies for high-density plasmas has been slow. As long as $\omega_0 < \Omega_e$, a normally incident extra-ordinary wave ($\omega_0$) propagating from outside the torus encounters its first cut off at $\omega_{pe}^2 = \omega_0^2 + \omega \Omega_e$. Thus, the wave can reach densities such that $\omega_{pe}^2 > \omega_0^2$. If the wave is focused to a sufficient intensity in a region near this cut off, parametric decay of the wave can transfer its energy to the plasma. The dispersion relationship for such a decay into an ion-acoustic wave and a Langmuir wave is derived. It is found that plasmas with a wide range of densities can be heated with a fixed frequency source. A threshold value of the wave intensity for decay to occur is found. Estimates of the power required for an useful coupling of energy to the decay waves are made assuming a diffraction limited focus. Frequencies as low as 20 GHz could be used to heat a typical PLT discharge.

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PARTICLE SIMULATION OF X-POINT DYNAMICS*

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A simulation of the magnetic x-point dynamics has been performed on a 2-1/2 dimensional magnetostatic particle code. The full dynamics of electrons and ions in a doubly periodic system is represented save the displacement current. The x-point is created by two temporally rising parallel rod currents in the z-direction with the cross section of the rod oblate in the x-direction. Proper care is taken to accommodate the $k = 0$ component of the current which should show up in the Ampere-Maxwell equation in spite of the neglected displacement current. When the current rise time is approximately equal to the wave traveling time from the rod to the x-point, we observe shock fronts converging to the x-point. Jetting from the x-point and into the o-points is apparent from flow vector plots. The plasma dynamics is dominated by the induced $E_y \times B_z$ drift current near the rods. The currents in the x-y plane show a number of vortices which tend to disappear with a slight tilt of the rods in the x-z plane. In this collisionless regime, the x-point seems stable so far. Attempts at x-point heating by ringing the plasma at the appropriate wave traveling time will be reported.

*Work supported by DOE and NSF.
SIMULATION STUDY OF THERMAL VERSUS PARTICLE DIFFUSION*

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Multi-species simulations were run using the 2½-dimensional electrostatic particle code with fixed magnetic field as implemented on the CHI computer at UCLA. Ions of 25 to 100 electron masses were found to have a diffusion rate as low as 25% of the electron rate. This is attributed to partial cancellation of ExB drift velocity when averaged over the large ion Larmor orbits, whereas the smaller orbit electrons can move with the full local ExB velocity.

*Work supported by USDOE.
Ballooning instabilities are believed to be a major limitation on the value of the plasma beta that can be achieved in a tokamak. The non-linear evolution of these instabilities is being investigated by using a 3-D MHD code in toroidal geometry, which integrates MHD equations in time by an explicit, leap-frog finite-difference scheme.

For a particular class of equilibria, which are diamagnetic, with a broad current profile and an elongated cross section, our preliminary results indicate that the ballooning instabilities do not saturate at small amplitudes. The results also show filamentation of the toroidal current and its self-reversal at some parts of the cross section.

*Work supported by USDOE.

COALESCENCE OF MAGNETIC ISLANDS*

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Tearing instabilities, and the accompanying processes of magnetic field reconnection and island formation, are believed to play an essential role in areas such as magnetic oscillations in tokamak discharges and stability of reversed-field theta pinches. Recently, Finn and Kaw\(^1\) studied the tendency of magnetic islands to coalesce into larger units by investigating the stability of an exact hydromagnetic equilibrium consisting of an infinite chain of magnetic islands in slab geometry. We present the results of an extensive numerical analysis of this configuration which includes nonlinear and finite-resistivity effects.

We treat the coalescence process as an initial-value problem and solve the incompressible MHD equations using a semi-implicit method.\(^2\) Our results confirm the existence of the coalescence instability in the ideal MHD limit, but we find no evidence for a threshold in island width. The linear growth rates are found to be large compared to those for purely resistive processes such as the tearing mode. The linear mode structure has only a weak dependence on resistivity. The resistive contribution to the growth rate has an \(S\) dependence similar to the inverse fractional power dependence of the tearing mode. In the nonlinear regime, saturation of the mode in the ideal case is observed due to flux piling up at the X point, while in the nonideal case the merging process is observed to proceed to completion.

*Work supported by USDOE and NSF.

Stability of Drift Waves in a Field Reversed Configuration*

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In some field reversed plasma configurations, e.g., \( \theta \)-pinches, ion rings, etc. toroidal field is absent. The drift waves in such geometry are therefore not stabilized by magnetic shear. However, the short connection length of the poloidal field is an important stabilizing influence. We have modeled such a field reversed configuration by a cylindrical Bennett pinch in the limit of large aspect ratio. We take account of both radial density and magnetic field gradients and derive the radial eigenmode equation for the perturbation from kinetic theory. Using the method of quadratic forms\(^1\) we show that in the low \( \beta \) limit the electrostatic universal mode is stable. The short connection lengths of the field lines lead to ion Landau damping, which accounts for the stability. In the finite-\( \beta \) case the drift shear Alfvén mode becomes important and the modes are now described by two coupled equation in \( \phi \) and \( A_{||} \). Again using the quadratic form method this mode is shown to be stable under quite general conditions.

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NEOCLASSICAL TRANSPORT IN EBT*

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ABSTRACT

We employ a suitably bounce averaged drift kinetic equation to identify three specific neoclassical transport regimes in EBT: a collisional or Kovrizhnikh\(^1\) regime in which the collision frequency, \(\nu\), is much greater than the poloidal drift frequency, \(\Omega\); an intermediate or "plateau" regime in which \(\nu\) is less than \(\Omega\) but still large enough to smooth the particle distribution; a collisionless or "banana" regime in which the guiding centers of slowly precessing particles trace out banana shaped orbits before the particles suffer a collision. We employ realistic bounce averaged drift velocities and calculate transport coefficients, correct to lowest order in \(\nu/\Omega\), for the plateau regime, the operating regime of the present and planned devices. We find that this transport is independent of collision frequency. Results of calculations from a one-dimensional transport code incorporating the plateau transport coefficients are presented and compared to experimental data.


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We describe particle simulation results for the axis encircling orbit model recently discussed by Aamodt, Catto and Rosenbluth. The model employed is identical to the analytic one: perfectly concentric ion orbits, cold electrons, electrostatic model, electron polarization drift neglected. Results from the code have confirmed the analytic results and have helped to elucidate the behavior of the unstable mode when various analytic approximations are of uncertain validity. The code produces details of the eigenmodes $\phi(r)$ along with accurate real frequencies and growth rates as a function of the system parameters $\omega_{pi}/\omega_{ci}$, $R_{wall}/R_p$, and azimuthal mode number $\ell$. All specific cases observed agree with the general predictions of the analytic model and in addition there are the following points of detailed agreement: $\ell = 1$ stable; a purely growing mode exists at high density and high $\ell$; nearly identical predictions for $\phi(r)$, $\gamma$, $\omega$; for a specific case marginal stability results when $R_{wall}$ is decreased below a critical value. The good agreement between code and analysis illustrates the utility of particle simulation techniques for determining the linear stability of spatially inhomogeneous equilibria. The present equilibria model is of course almost the simplest example; we are now extending the code to allow more general ion orbits.

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Electron cyclotron resonance heating of tokamaks at the fundamental harmonic was shown to have great potential. If the electron density of future tokamaks is so high that the fundamental harmonic is not accessible, we may have to use the second harmonic of the electron cyclotron resonance. Under reactor conditions \( n_e > 10^{14} \text{ cm}^{-3}, T_e \approx \text{a few KeV} \), the second harmonic of the ordinary mode and the extraordinary mode could be absorbed efficiently at oblique incidence. Results of the numerical and the analytic calculations will be presented.

* Work supported by DOE.

1. "Electron Cyclotron Resonance Heating of Tokamaks at \( \omega = \omega_{ce} \)". E. Ott, B. Hui and K. R. Chu, to be published.
A NEW TRAPPED-ION INSTABILITY WITH LARGE FREQUENCY
AND LARGE RADIAL WAVE NUMBER

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The need for theoretical previsions concerning anomalous transport in large Tokomaks, as well as the recent results of PLT, ask the question of the process responsible for non-linear saturation of trapped-ion instabilities. This in turn necessitates the knowledge of the linear behaviour of these waves at large frequencies and large radial wavenumbers.

We study the linear dispersion relation of these modes, in the radially local approximation, but including a term due to a new physical effect, combining finite banana-width and bounce resonances. Limiting ourselves presently to the first harmonic expansion of the bounce motion of trapped ions, we show that the effect of finite banana-width on the usual trapped-ion mode is complex and quite different from what is generally expected.

In addition we show, analytically and numerically, the appearance of a new branch of this instability. Essentially due to this new effect, it involves large frequencies (ω ∼ ω_b) and is destabilized by large radial wavelengths (k_x A ∼ 1, where A is the typical banana-width). We discuss the nature of this new mode and its potential relevance of the experiments.

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Because of increasing interest in Tokamak plasma with small aspect ratio and high pressure we investigate in this paper the higher order effects of the inverse aspect ratio expansion on the toroidal equilibrium and stability. The conventional high pressure ordering, $\mathcal{O}(1)$ and $\mathcal{O}(\varepsilon^2)$, where $\alpha$ and $\gamma$ are free parameters of the flux function and pressure respectively, is reasonable when the aspect ratio is large. We introduce in this calculation a different ordering which is more suitable for small aspect ratio tokamaks, namely $\mathcal{O}(1)$ and $\mathcal{O}(\varepsilon^2)$, and use it to analyze the equilibrium of isotropic and anisotropic pressure that includes plasma mass flow which cannot be ignored in such toroidal devices as the Two Component Tokamak. We find that substantial corrections in the displacement of the magnetic axis as well as in the critical equilibrium $\beta_\parallel$-value occur when second order effects of small aspect ratio are included in a toroidal plasma with isotropic pressure. Preliminary results on the stability of high ($\mathcal{N}$) mode number of such equilibria will be presented and discussed.


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ANALYTIC THEORY OF THE TRAPPED ELECTRON MODE
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ABSTRACT

The 2-D problem of the electrostatic trapped electron mode in the limit $k_r \rho < 1$ is analytically investigated. Both the curvature-drift resonance of the trapped electrons and the Landau resonance of the transit electrons are included in the analysis. It is first shown that the Pearlstein-Berk type approach of retaining only the ion sound term in the ion response, which we also adopt here, is justified when $L_L / L_s$ is large, in which case $\omega$ is near the value given by the usual local approximation. The resultant system of radial differential equations coupling the Fourier harmonics in the poloidal angle can be reduced to a single differential-difference equation because of a certain symmetry possessed by the system. This latter is recast into the form of a standard matrix eigenvalue problem after expanding the solution in a complete set of parabolic cylinder functions. The relevant matrix elements are evaluated with the exact orbit of the trapped electrons rather than the harmonic oscillator approximation. The matrix eigenvalue problem can be solved and analytic dispersion relations obtained in the limits $\Delta / x_T << 1$ and $\Delta / x_T >> 1$ where $\Delta$ is the distance between neighboring mode rational surfaces and $x_T$ the width of the parabolic cylinder functions. The dispersion relations are solved numerically to determine the growth rates and the unstable region for values of parameters representative of tokamak operation. The 2-D mode structure will also be discussed.

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GATO

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ABSTRACT

GATO evaluates stability of a tokamak equilibrium with respect to a linearized ideal MHD displacement and can treat equilibria with one of two magnetic axes in a general axisymmetric toroidal configuration. A variational approach to the problem is used; the displacement vector is expanded in terms of a set of basic functions and substituted into the Lagrangian of the system. GATO uses an orthogonal coordinate system and finite hybrid elements. Since the MHD spectrum is ill conditioned and the matrices are large, attention has been given to the eigensolver. The problem is solved using an improved version of the direct method (inverse iteration plus Choleski decomposition). The improvement was obtained by taking advantage of the sparseness of the matrices. Matrix reordering is being investigated and may further improve the method. Initial results obtained using GATO will be presented.

Work supported by Department of Energy, Contract No. EX-76-C-03-0167, Project Agreement No. 38.

1F. J. Helton and R. W. Moore, 8th Conf. on Numerical Simulation of Plasmas, Paper OD-1.
TWO TRANSPORT MODELS FOR NON-CIRCULAR AXISYMMETRIC DEVICES*

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Two programs are described which solve the differential equations of 1-D plasma transport in an axisymmetric toroidal plasma of arbitrary cross section. Both programs assume the existence of an arbitrary number of Maxwellian ion species which have a common temperature profile. The electrons, whose density is determined through quasi-neutrality, have a separate temperature profile.

The first program, TOAD, writes the transport equations in terms of adiabatic invariants—mass, entropy $P(V')^{5/3}$, and magnetic flux, and advances these quantities implicitly, which is to say that the equations are independent of the time rate of change of the volume between flux surfaces. The 2-D equilibrium equation is solved by a variational method in which the time advanced adiabatic variables are the input parameters. Subsequently, one solves for density and temperature.

The second program, FPTE, does not advance the adiabatic variables implicitly, but splits them into two parts (e.g. $P$ and $(V')^{5/3}$). The time derivative $\frac{d}{dt}V'$ is determined through the equilibrium calculation, and this value is used during the repetition of the transport cycle. This process is repeated to convergence.

The code FPTE also takes into account the presence of energetic non-Maxwellian ion species arising from beam injection. The full nonlinear 2-D Fokker-Planck operator is solved using a recently developed CRAY optimized Fokker-Planck package. In addition, the equilibrium calculation is designed to accommodate anisotropic pressures.

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FINITE-LENGTH THEORY OF COLLECTIVE FREE-ELECTRON LASERS*

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Free-electron lasers represent promising sources of tunable coherent radiation; potential applications of fusion interest include plasma heating (ECRH) and driving systems for pellets. The small-signal gain of such a device operated in the stimulated Compton mode is derived here without limitation on the density of the relativistic electron beam employed. Expansion of the exact result in powers of the linear susceptibility $\chi$ reproduces the vacuum gain formula\(^1\), and shows that the leading plasma correction causes a slight enhancement (not reduction!) of the optimum vacuum gain. The theory is founded on the oscillation-center approach\(^2\), and generalizes past work further by including a static guide magnetic field and an arbitrary distribution of beam momenta; the principal assumption is small gain in the available length (i.e., a recycled system).

For denser beams ($|\chi| > 1$), a new regime of operation is discovered. It is shown that stimulated Compton scattering persists in a finite-length system, and that the Compton gain can easily rival the finite-length Raman gain which is derived for comparison. The possibility of a short-wavelength laser (visible, ultraviolet, even x-ray) operated in this new regime (plasma-modified Compton effect) is discussed.

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High-$B$ Tokamak Transport Modelling Studies

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The attainment and maintenance of a high-$B$ ($B > 4\%$) plasma by neutral beam injection is an important goal of present tokamak experiments. We discuss work in progress on a number of modelling problems suggested by such experiments: optimization of the configuration by beam deposition and current programming, the role of neutral beam induced currents in determining the shear profile, possible use of Fisch-Bers lower hybrid current drive for tailoring profiles [1], and properties of the final stage of FCT evolution.

As part of this study we have modified our 1 1/2 D transport code to:

- calculate lower hybrid power dissipation and current generation using a module developed by D. Ehs (ANL) [1].
- calculate magnetic reconnection of non-monotone current profiles and magnetic island effects due to saturated tearing instabilities, using criteria developed by Waddell, Carreras and Hicks.

As before, beam-deposition is calculated with the Fowler-Rome noncircular version of the Freya module developed by D. Post (PPPL). We find:

- self-consistent transport-generated 'stable' D-shaped plasmas with $B > 10\%$ ('stable' = linearly unstable over less than 20% of the plasma volume), and that the TOSCA results do not violate the ballooning criterion.
- that benefits for ballooning stability from force-free currents (from external voltage programming or lower hybrid current drive) may be counter balanced by tearing mode-induced enhanced transport.
- that the Zakharov-Snafranov catastrophe [3] (loss of equilibrium on the post-FCT timescale) should not be accessible in the near future.
- that neutral beam induced currents completely determine the shear profile at low density for co-injected plasmas, and thus remove some of the profile tailoring capability. Perpendicular injection, restores it, but a study of anisotropic-pressure equilibrium and stability is needed for this case [cf. A. Cooper, this meeting].

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Shear Damping of Drift Waves in Toroidal Geometry

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Abstract

In toroidal geometry the magnetic field strength, shear and curvature vary over a magnetic surface. This non-uniformity induces a coupling between drift-modes centred on different rational surfaces, which can inhibit convection of energy away from the mode centre and so reduce shear damping.

For short wavelength modes ($k_y a_i \gg 1$) the only important coupling effect is due to trapped electrons, and earlier work has shown that this is ineffective in reducing shear damping. For long wavelength modes ion magnetic drift terms provide an important coupling term, and this is shown to be effective in removing shear damping when $r_n/R \gtrsim k_y^2 C_s^2 (r_{q''}/q)$.

A two dimensional eigenvalue equation modelling long wavelength drift modes in a large aspect ratio, circular cross section Tokamak (with $\beta \sim O(\varepsilon^4)$) is reduced, using the techniques developed for mhd ballooning modes, to a one dimensional eigenvalue equation from which the shear damping can be calculated. Numerical solution of this equation shows that the analytic 'strong coupling' theory developed in reference (1) and employed extensively since, is accurate over much of the range of possible parameters, but that it fails to predict an important class of undamped modes.

Application to a typical Tokamak suggests that, for modes of a given wavelength, shear damping may be absent for modes in an inner core region of the cross section, while in the outer regions the normal, slab value of shear damping may prevail, though the break between these two types of behaviour is not necessarily at $r_{q''}/q = 1/2$, predicted by strong coupling theory.

The relationship of the eigenvalue obtained from the reduced one dimensional equation, to the global eigenvalue, and to the determination of the radial structure of the eigenmode is investigated in higher order of an expansion in $(a_i L_s/r_n^2)$.

References


Ionic Cross Section Relevant to Plasmas

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The diagnostic and the radiative power loss from hot thin plasmas require cross sections and rate coefficients of the ions present in the plasma. The electron impact collisional excitation cross sections appear to be very important. This paper will report on the present status of the cross sections and the analytic forms available for accurate and practical calculations.
A TRANSPORT ESTIMATE FOR EBT IN THE BANANA REGIME

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In a bumpy torus, the destabilizing vertical drift caused by the toroidal component of the magnetic field is suppressed by the introduction of a poloidal drift due to magnetic field inhomogeneities. As a result of the poloidal drift, the electrostatic potential $\phi$ is able to play a sensitive role in the transport since the poloidal component of the $E \times B$ drift will always tend to cancel the poloidal magnetic drift of one species for some velocity range.

This cancellation has been identified as the dominant transport mechanism in the ion plateau regime, $\epsilon^{3/2} < v_i/\Omega_p < 1$, where $v_i$ and $\Omega_p$ are the typical ion-ion collision and ion poloidal drift frequencies, and $\epsilon$ is the inverse aspect ratio. In the low collisionality banana regime $v_i/\Omega_p < \epsilon^{3/2}$, the ions having canceling $E \times B$ and magnetic drifts not only continue to dominate the transport, but are also able to trace out banana-shaped drift surfaces (when their bounce average motion is projected onto a cross section of the torus). Because of the resulting boundary layer structure in velocity space previous approaches become inadequate or inappropriate.

In the simplified model presented here the ion and electron diffusion coefficients are evaluated in the banana regime from the full Fokker-Planck collision operator by assuming that the gradient $B$ drift dominates over the curvature drift, and that the bumpiness of the magnetic field may be treated as small in the collision operator. The model estimates the diffusion coefficients from the entropy production, and predicts an ambipolar containment time proportional to $1/v_i$ and independent of aspect ratio, rather than the more favorable scaling of $1/\epsilon^{2}v_i$ normally assumed.

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It is common practice in RF heating calculations to evaluate the antenna loading solely due to the interaction of the plasma with the far field of the antenna. However, in actual experiments a substantial amount of plasma (poorly confined) always surrounds the antenna structures. The present study considers various kinetic interactions (transit-time damping, boundary absorption, nonlinear generation of ion acoustic waves) that arise in this environment and which can lead to the anomalous heating of the plasma surface. The prototype problem in this class of phenomena (the Landau half-space) is examined in detail and a numerical calculation of a RF heated plasma confined by two reflecting walls (of relevance to multipacting breakdown) is found to attain a universal equilibrium in which the asymptotic kinetic energy $KE$ satisfies $<KE> = aN\epsilon\phi$, where $N$ is the number of particles, $\epsilon$ the charge, $\phi$ the peak RF potential, and $a$ is a constant which is independent of initial conditions and RF parameters.

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It is well known that deformation in tokamak flux surfaces result from magnetic perturbations that are helically resonant with the equilibrium magnetic field $B_0$. We discuss the broadening of this $k \cdot B_0 = k'' B_0 = 0$ resonance due to stochastic magnetic perturbations. A diffusion equation for the field lines results whose diffusion coefficient has a broadened $k''$ resonance. The diffusion coefficient is similar to that obtained recently by Hirshman and Molvig. We show an analogy between this stochastic magnetic field model and the velocity scattering of particles by electrostatic waves. The implication that a magnetic island is analogous to a BGK equilibrium is discussed. The model gives the usual expression for island width, as well as the criterion for onset of stochasticity, i.e. island overlap. Finally, we suggest the application of this $k''$ broadening model to nonlinear MHD problems.


TURBULENT MODEL OF MAGNETIC BRAIDING II: PRESSURE CORRELATION FUNCTION AND SELF-CONSISTENCY, P.H. Diamond, D.J. Tetreault and T.H. Dupree, M.I.T., -- We have studied the theory of pressure fluctuations in a cylindrical tokamak in the presence of stochastic magnetic fields. Using methods from the clump theory of Dupree, we have calculated the pressure fluctuation correlation function, \( \langle \delta p(1) \delta p(2) \rangle \), starting from the equation \( \mathbf{B} \cdot \nabla p = 0 \). As pressure is constant along field lines, the structure of the correlation function gives information about field line correlation. It is shown that two lines are correlated when they are within an island (or resonance) width of each other radially and are separated in arc by less than a poloidal wave length. It is shown that large poloidal mode number is necessary to destroy field line correlation. The field line exponentiation length is calculated. The pressure fluctuation correlation function is given by \( \langle \delta p(1) \delta p(2) \rangle = 2 Z_c D \langle \delta p / \delta r \rangle^2 \), where \( Z_c \) is the correlation length for a "clump" of field lines and \( D \) is the magnetic diffusion coefficient. The conversion of average pressure (or poloidal field energy) gradients into fine scale fluctuations is seen to be caused by stochastic magnetic diffusion. Similarities to 1D Vlasov turbulence are indicated.

Using momentum balance and Ampere's Law to obtain \( \nabla \times \mathbf{B} \) in terms of \( \langle \delta p \delta p \rangle \), the possibility of a self-consistent, "stochastic equilibrium" is investigated. The role of marginally stable ideal M.H.D. modes in such a steady state and the effect of stochasticity near the \( \mathbf{K} \cdot \mathbf{B} = 0 \) resonance layer are discussed.
The sheath at the interface between a plasma and a wall is a common feature of Langmuir probes, plasma interaction with the limiter or divertor collector plate in tokamaks, and axial electron heat flow in mirror machines. This problem was originally studied by Tonks and Langmuir$^1$ for the cold ion case; a lengthy series of papers has followed on their classic work. We consider here the formulation of the plasma-sheath equation for a collisionless plasma with arbitrary ion temperature in plane geometry. Outside the sheath this equation is replaced by the plasma equation (quasi-neutral approximation, zero Debye length limit), for which an analytic solution for the electrostatic potential is obtained. In addition, the ion distribution function, the wall potential, and the energy and particle fluxes into the sheath are explicitly calculated. The plasma-sheath equation is also solved numerically with no approximation of the Debye length. The numerical results compare well with the analytical results when the Debye length is small.

Research sponsored by the Office of Fusion Energy, U.S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corp.

* University of Wisconsin
+ Georgia Institute of Technology
Orbits and Transport in Three-Dimensional Geometries

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In nonsymmetric systems the evaluation of drift orbits and transport is difficult by traditional methods. We find the particle drift orbits can be rapidly evaluated using a magnetic coordinate system $\mathbf{B} = \mathbf{V}_a \times \mathbf{v}_e = \mathbf{V}_x + \mathbf{V}_y + \mathbf{V}_z$ with $\alpha, \psi, \chi$ the coordinates. Transport can be simply evaluated by a Monte Carlo technique using Lorentz scattering. If $\mathbf{V} \times \mathbf{B} = 0$, then $\beta = \gamma = 0$ and the drift equations become with $\rho = v/(eB/mc)$

$$\frac{d\alpha}{dt} = -c \frac{3\phi}{3\psi} - \left(\frac{c}{e} \mu + \frac{eB}{mc} \rho \right) \frac{2B}{3B}, \quad \frac{d\psi}{dt} = c \frac{3\phi}{3\alpha} + \left(\frac{c}{e} \mu + \frac{eB}{mc} \rho \right) \frac{2B}{3B}$$

$$\frac{d\chi}{dt} = \frac{eB}{mc} \rho, \quad \frac{d\rho}{dt} = -c \frac{3\phi}{3\chi} - \left(\frac{c}{e} \mu + \frac{eB}{mc} \rho \right) \frac{2B}{3B}$$

with $\phi(\alpha, \psi, \chi)$ the electric potential and $B(\alpha, \psi, \chi)$ the magnetic field strength.

To evaluate transport across the pressure or $\psi$ surfaces, the particle pitch $\lambda = v_\parallel /\nu$ is changed after each time step of length $t$ from $\lambda_0$ to $\lambda_N$

$$\lambda_N = \lambda_0 (1 - 2\nu t) \pm [(1 - \lambda^2_0)(2\nu t)]^{1/2}$$

with $\pm$ a random sign and $\nu$ the collision frequency. Let $D(E, \psi) = (\psi^2 - \overline{\psi^2})/t$ with the bar a time average of the orbit, then due to the averaging effect of the Lorentz operator and the drifts, the kinetic equation can be written

$$\frac{3f}{d\psi} = \frac{1}{s} \frac{3f}{d\psi} \left(sD \frac{3f}{d\psi} \right)$$

with $f(E, \psi)$ a local Maxwellian and $s(\psi)d\psi$ the volume element. Moments of this equation give the transport.

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Would like adjacent poster - Boozer and Kuo-Petravic.
Evaluation of Orbits and Transport in Three-Dimensional Geometries

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The evaluation of α and beam ion drift orbits as well as the heat transport is essential for the assessment of any magnetic fusion concept. In nonsymmetric geometries like stellarators, EBT, and the tandem mirror, these calculations are subtle and difficult by usual methods. Even ideally symmetric devices like the tokamak and toroidal Z pinch have significant symmetry breaking ripple effects. In this paper, we report computational evaluations of orbits and transport obtained by integrating the drift equations in α,ψ,χ coordinates as described in a companion poster by Boozer and Kuo-Petravic. An eighth-order multi-step Runge-Kutta method was used with a variable timestep to allow for very different time scales when the particle is in the ripple trapped and untrapped regions. In this way, we were able to follow single particles for $5 \times 10^7$ ion cyclotron periods with energy conservation good to 1 in $10^7$.

The geometry studied was a $l = 2$, $M = 5$ stellarator with $\varepsilon = 1/7$ and $q = 2$. In the absence of a radial electric field, the trapped particle region of pitch angle space is filled with unbounded collisionless drift orbits. A radial electric field with a potential change across the plasma equal to the particle's energy confines all the drift orbits but excursions remain large ($\Delta \psi / \psi \sim 50\%$ with $\psi$ the magnetic flux). The drift orbits are quite sensitive to pitch angle scattering. Consequently, the radial ion transport, though large, is considerably reduced from naive estimates based on the large radial excursions.

*Work supported by U.S. DoE Contract No. EY-76-C-02-3073.
A SECOND STABILITY REGION FOR A SEQUENCE OF FINITE-$\beta$
FLUX-CONSERVING TOKAMAK EQUILIBRIA

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A sequence of flux-conserving finite-$\beta$ tokamak equilibria is tested for stability against ideal M.H.D. modes (so called ballooning). The existence of a new stability region$^{1-3}$, for $\beta$ beyond a second critical value, is confirmed$^{4}$ for this numerically generated equilibrium sequence.

The equilibria are calculated using a program due to R. Englade$^5$, for boundary conditions consisting of a conducting surface of circular cross-section. The major and minor radii of the torus are 50 and 20 cm; the vacuum toroidal field is 17.3 T; and the toroidal current is 3-6 MA. The rationalized inverse rotational transform $q(\psi)$ varies between 1.01 and 2.82 over the plasma cross-section. The members of the equilibrium sequence are characterized by different values of the ratio of plasma to toroidal pressure, $\beta_T = 8\pi \bar{p}/\bar{B}_T^2$. Each is represented by a curve in the $s,G$ plane, where $s$ is the magnetic shear parameter, $d\ln q/d\ln r$, and $G$ is the pressure gradient parameter, $-8\pi R_0 r^2 dp/d\psi dr/dr$. $r(\psi)$ is a characteristic scale of a flux-surface. For each curve, parameterized by the flux coordinate $\psi$, the range of unstable flux-surfaces is determined from the general eigenvalue equation governing ballooning modes. It is observed that when $\beta_T > 13-15\%$ ($\beta_T > 12-14\%$) the instability effectively disappears for all values of $s$.

In addition, we develop a model equation which permits us to decouple the stability analysis from the equilibrium and to simplify the numerical solution of the eigenvalue problem. The values of the equilibrium parameters are obtained by fitting to the numerically generated equilibria. Agreement was found with the results obtained directly from the equilibria using the general eigenvalue equation.

Normal mode model equations for high toroidal number ballooning modes are studied analytically, in that limit of the relevant parameters corresponding to magnetic surfaces close to the magnetic axis. After taking this limit, the eigenvalue equation becomes much simpler, while still retaining the main features of the full ballooning mode equation, and, in general, shows the two points of marginal stability.

For the model configuration with shifted circular magnetic surfaces, the growth rate is conveniently obtained as a function of the pressure gradient parameter \( G = -8\pi R_o r^2 \frac{d \hat{p}}{d \psi} \frac{d r}{d \psi} \) and the shear parameter \( \hat{s} = \frac{d \ln q}{d \ln r} \). Given a specified equilibrium configuration, as we approach the magnetic axis, \( G \) tends to zero while the ratio \( \hat{s}/G^2 \) remains constant. In this limit, the eigenvalue problem at marginal stability is shown to have an exact analytical solution. For a given unstable configuration, the squared growth rate is proportional to the fourth power of \( G \).
Ballistic Damping - Some Physics Considerations.* R. F. Post, H. L. Berk, Lawrence Livermore Laboratory--A technique called "Ballistic Damping" (BD) has been proposed for the control of ion cyclotron instabilities of the highly coherent type characteristically observed in mirror systems at high plasma density. In BD, ion beams, launched parallel to the field lines, transit the plasma, resonantly gaining perpendicular energy and exiting through the far mirror, thereby extracting energy from the wave. Above a critical current the BD effect can exceed the rate of growth of the unstable wave, leading to suppression of the mode. Critical BD currents may be estimated from simple power balance considerations or by incorporation of BD into the quasi-linear formalism of Berk et al.; values found lie well within existing technology, both for present experiments and for scaled-up systems. Critical physics issues include the growth rates of the modes involved. Estimates will therefore be made of limits on the growth rate for the fundamental mode (stabilizable by BD) and of effects that weaken or stabilize higher order modes (only weakly influenceable by BD).

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.

1 R.F. Post, Lawrence Livermore Laboratory UCID 17876, "Ballistic Damping - A Proposed Method of Stabilizing Resonant Ion Cyclotron Modes" (July 1978).


LINEAR THEORY OF HIGH-M TEARING MODES

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ABSTRACT

A linear kinetic theory of high-m (m = poloidal mode number) tearing modes which is valid for arbitrary \((\omega/\nu_e)\) (\(\nu_e =\) electron collision frequency) is presented. We investigate the "semi-collisional"\(^1\) regime in which only the magnetic perturbation enters. Using a pitch-angle scattering Fokker-Planck electron collision operator, we find that previous results\(^2\) are qualitatively incorrect. Numerical solutions of the eigenmode differential equation for the drift-tearing mode result in stable roots for realistic tokamak parameters.

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Kinetic equations for low frequency, high mode number, electromagnetic perturbations in an axisymmetric magnetically confined plasma are developed. The analysis makes use of the high toroidal mode number expansion to reduce the lowest order system of equations to a set of ordinary (along the field line) intro-differential equations. Included in these equations are the effects of finite Larmor radius, magnetic shear, trapped particles, and nonuniform magnetic curvature drifts. Perturbed fields are represented by a scalar potential and two components of the vector potential. Thus, the effects of the compressional component of the perturbed magnetic field are retained and the equations are valid for arbitrary values of plasma pressure. The formalism used here is the generalization of that used by Rutherford and Frieman\(^1\) for electrostatic modes. In appropriate limiting regimes all known ballooning and drift modes are found as special cases.

\* Work supported by the U.S. Department of Energy
FINITE $\beta$ TRAPPED PARTICLE MODES*

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The kinetic equations for low frequency instabilities in magnetically confined plasmas are examined for modes with $k \lesssim 1$. The familiar electrostatic trapped electron mode is modified by electromagnetic effects when $\beta$ (the ratio of plasma pressure to magnetic field energy density) approaches $\epsilon^2$ where $\epsilon$ is the inverse aspect ratio. Solution of the mode equations in this case requires treating two asymptotic regions: an inner region where the effects of ion inertia are important and electrostatic and electromagnetic field components are strongly coupled, and an outer region where ion inertia can be ignored and the field components are weakly coupled. If $\beta$ is then increased further the drift alfven is excited.

* Work supported of USDOE

1. B. Lane and T.M. Antonsen Jr. this meeting.
CURRENT DRIVE WITH ENERGETIC ELECTRONS

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ABSTRACT

Toroidal plasma current generation by means other than magnetic induction is of great significance because of the possible operation of a tokamak plasma in the steady-state mode. To the extent that one is interested in current drive (rather than heating), it is desirable to minimize the power dissipation necessary to drive the current. A possibly efficient way to drive a current is by deposition of momentum primarily on high velocity electrons, for which the collisional drag is small. In this paper, we consider several alternatives for achieving such current drive by energetic electrons. These schemes are: lower hybrid rf drive on the tail of the electron Maxwellian distribution, current maintenance by "runaway" electrons for which collisional drag is negligible but anomalous effects become important, and current drive by relativistic electron beams (REB).

The basic mechanisms involved are examined, with attention paid to the major limiting factors. For the case of lower hybrid current drive, we consider the quasi-linear and collisional absorption of the mode and obtain self-consistent current drive in the presence of plasma transport, using a one-dimensional numerical code. For the case where the "runaway" tail of the electron distribution is used to generate most of the current, we consider a quasi-linear equilibrium in the presence of anomalous effects due to Doppler-shifted cyclotron resonance. For REB current drive, we consider the possibility that a two-stream instability may be excited which would affect both the resistivity of the plasma and the directional momentum carried by the beam. Finally, we obtain the current to power ratio for each of these different techniques and arrive at numerical estimates using the nominal design parameters of RST, a conceptual steady-state tokamak under study at General Atomic Company.

Supported by the Electric Power Research Institute, EPRI Contract No. RP 323-3.
WKB Theory of the Ballooning Mode Spectrum

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There are in general \( O(n) \) unstable eigenmodes with azimuthal mode number \( n(\gg 1) \) in a ballooning-unstable, axisymmetric torus. The modes with small radial wavenumber \( N \) are the most unstable, but also the most localized. Employing the ordering \( N = O(n) \), expanding in \( n^{-1} \) between turning points, in \( n^{-1/3} \) near turning points and matching the expansions we have derived the quantization conditions

\[
\int k \, dq = \frac{N + (1/2)}{n}
\]

previously postulated. This formula applies for phase space trajectories topologically equivalent to trapped particle orbits. There also sometimes exists a class of modes analogous to passing particles, for which the periodic nature of the ballooning mode dispersion relation makes the WKB analysis highly nonstandard. These modes are quantized according to the formula

\[
\int_{R} k \, dq = \frac{N}{n},
\]

where \( R \) is the region of the \( k - q \) plane bounded by the lines \( q = 0, k = \pm 1/2, \) and \( \omega^{2}(k, q) = \omega^{2} \). The relation to EBK quantization \(^2\) will be discussed, as will the comparison of the above results with PEST.

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Numerical Studies of Resistive Ballooning Modes*

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Numerical solution of the high-n resistive ballooning equations shows the existence of pressure-driven instabilities with complex frequency \( \omega \) scaling as fractional powers of \( \varepsilon \equiv n^2 \tau_A / \tau_R \), with \( n \) the toroidal mode number, \( \tau_A \equiv qR/C_A \) the Alfvén transit time, and \( \tau_R \equiv a^2/\eta \) the resistive skin time. These modes go unstable for \( 0 < \varepsilon < \sim 1 \), with resistive effects negligible for \( \theta < \varepsilon^{-1/3} \) and dominant for \( \theta > \varepsilon^{-1/3} \), with poloidal coordinate \( \theta \) mapped onto an infinite domain by the ballooning representation.

Matched asymptotic expansions for large and small \( \theta \) lead to a dispersion relation similar to that governing low-n resistive interchange and tearing modes. We present here the results of a numerical study comparing the predictions of three different methods. In the first method, we solve the full resistive equations over the whole domain. In the second method, we match numerical solutions for the inner and outer regions. In the third method, the outer region is solved analytically. The numerical studies give confidence in the analytical results, while the analytical results provide greater efficiency and understanding.

* Work supported by the United States Department of Energy Contract No. EY-76-C-02-3073.


3 A. H. Glasser, J. M. Greene, and J. L. Johnson, Phys. Fluids, 18, 875 (1975)
The Role of the Continuous Spectrum in Ideal MHD Ballooning Mode Theory*

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A theory of ideal MHD modes with large toroidal mode number \( n \) in sheared toroidal magnetic fields has recently been developed,\(^1\) motivated by an interest in ballooning modes driven unstable by the interaction of the plasma pressure gradient and magnetic field curvature and stabilized by magnetic tension. By means of a new type of periodic representation, the poloidal coordinate \( \theta \) is mapped onto an infinite domain, with convergence as \( \theta \to +\infty \) replacing toroidal periodicity as a boundary condition. An application of Floquet theory to the behavior of the solution as \( \theta \to +\infty \) shows that, under some conditions, this behavior is exponentially growing or decaying, and the boundary conditions are the vanishing of the coefficients of the growing solutions. Under other conditions the behavior is oscillatory, and convergence cannot be achieved. This behavior is identified with the continuous spectrum of ideal MHD,\(^2\) which is important for initial value problems, dissipation and heat absorption, and resistive effects.

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*This work was supported by the United States Department of Energy Contract No. EY-76-C-02-3073.


\(^2\)M. S. Chance, et al., Nuclear Fusion 17, 1, 65 (1977)
Poloidal Rotation Instability in Tokamaks

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Abstract

For plasmas with significant high-Z impurity ($Z_{\text{eff}} \gtrsim 3$), the dominant force near the center of the plasma tending to cause weak poloidal rotation is the electron viscous force proportional to $m_e v_{Te} q_{||} E_{||} r/eR^2$. The ion viscous force opposing rotation is weakened by electrostatic trapping which, for the ions, is out of phase with the magnetic trapping. The electrostatic potential is caused by the non-uniformity of the $Z$-ions on a magnetic surface, the non-uniformity being particularly enhanced as the poloidal velocity ($v_\theta$) approaches resonance with the slow magnetoacoustic wave velocity

\[ \frac{B_\theta}{B} \left[ \frac{5}{3} + \sum n_z Z^2 \left/ \left( n_i + \frac{T_i}{T_e} n_e \right) \right. \right] \left/ \left( n_i + \frac{T_i}{T_e} n_e \right) \right.]^{1/2} \left( \frac{T_i}{m_z} \right)^{1/2}. \]

Somewhat ahead of resonance the plasma goes unstable to poloidal acceleration, the acceleration being towards a new equilibrium with much higher $v_\theta$. This instability, which is interpreted as the onset of the disruption in the sawtooth oscillations, occurs at a critical $T_e$ given by

\[ \frac{T_e}{T_i} \approx C \left( \frac{m_i}{m_e} \right)^{1/2} n_{io} eV \]

where $C$ is only weakly dependent on the plasma parameters. The critical electron temperatures from this relationship accurately predict the experimental values of $T_{eo}$. For oxygen impurity in hydrogen, $C \approx 0.5$ and the above relationship can be written

\[ J_{||} \approx eE_{||} = 1.6 \times 10^2 \left( \frac{n_e}{10^{13}} \right) \left( 2 \frac{T_i}{T_e} \right)^{1/2} \left( 2 \frac{n_{io}}{n_{e0}} \right) \left( T_i [\text{eV}] / 400 \right)^{1/2} \text{ amperes} \]

and since at high densities the last three factors are approximately unity, this becomes identical with the Murikami, Callen, Berry relationship, $J_{||} = 1.6 \times 10^2 \left( \frac{n_e}{10^{13}} \right)$.
Frozen pellet fuel injection into the ISX-A plasma demonstrated that the pellets have a strong cooling effect on the plasma. This cooling effect is primarily due to dilution of the plasma energy amongst a larger number of particles but also due to energy lost in the ionization process. If the pellets are large enough (local perturbations \( \Delta n/n \gg 1 \)), the cooling can reduce further ablation of the pellet, i.e., the ablation process becomes "self-limiting." Pellets passing through the magnetic axis or crossing a rational flux surface should see a further reduction in the ablation rate due to geometric effects if the ionization process is truly localized in the vicinity of the pellet. Large perturbations in the background plasma may require a kinetic analysis since the electron distribution function can become non-Maxwellian during the ablation process. The relative timescales for the various processes are discussed and compared qualitatively with pellet injection observations in ISX-B.

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ADIABATIC AND STOCHASTIC ION MOTION IN
A CYCLOTRON RESONANT FIELD

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ABSTRACT

Ion motion in a cyclotron resonant field is of crucial importance in understanding mirror containment in such experiments as r.f. containment, r.f. heating, velocity diffusion in loss cone unstable plasmas and stream penetration into mirror traps. To attack these problems we have developed several analytic and numerical methods which in appropriate limits reproduce such effects as the reversible ponderomotive force equations and the criteria for superadiabatic or stochastic motion. We find that there exists a low energy region in phase space where particles can be trapped even in the presence of a repelling ambipolar well. This may have an important effect on establishing saturation amplitude of loss cone mode. Analysis also indicates that a superadiabatic region for particle motion exists above a moderate energy (or order 8keV in a mirror machine with 2XIIB plasma parameters), which appears to be contradicted by experimental data. Numerical studies of the equations in a quadrupole field are being performed to see if the discrepancy can be explained.

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-Eng-48.
It has been previously shown\(^1\) that the motion of an ion in a coherent lower hybrid wave becomes stochastic if the velocity of the ion satisfies \(v_\perp > \omega/k_\perp\) and if the electric field of the wave exceeds a threshold. In order to determine the heating rate of the ions in this case, we compute the velocity-space diffusion coefficient for the ions. We accomplish this by reducing the Lorentz force law for the ions to a set of difference equations\(^2\)

\[
\begin{align*}
\dot{u} &= \theta - \rho, \\
\dot{v} &= \theta + \rho,
\end{align*}
\]

These equations give the Larmor radius (which is related to \(\rho\)) and the phase \((\theta)\) of the ion on the \((j+1)\)th cyclotron orbit in terms of these quantities a cyclotron period earlier (the \(j\)th orbit). The parameters \(A\) and \(\delta\) describe the electric field strength and the proximity of the wave frequency to a cyclotron harmonic. The stochasticity condition for the difference equations is \(A > 1/4\). These equations allow a rapid numerical determination of the correlation function and hence the diffusion coefficient. This is checked against the exact equations of motion by solving the diffusion equation by a Monte Carlo method. Since normally only tail ions can gain energy from the wave in this way, we include collisions as a mechanism for transferring the energy to the bulk ions. The resulting two-dimensional Fokker-Planck equation is reduced to a one-dimensional Fokker-Planck equation in \(v_\perp\) using a method similar to that of Fisch\(^3\). Expressions for the heating rates of the bulk ions and electrons in the steady state are obtained.

A Kinetic Theory of Evolution of Anisotropic Plasmas

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In an anisotropic toroidal plasma, collisions and anisotropy can cause the plasma distribution functions to evolve in time and at the same time induce a motion in the plasma. The present work describes a kinetic theory for evaluating the evolution and motion of the plasma for small collision frequency.

The evolution of the anisotropic plasma can be roughly described as through three successive stages.

(1) First, the anisotropic electron distribution evolves to Maxwellian

\[ \frac{\partial}{\partial t} \sim \frac{u}{\lambda} \sim \frac{1}{\tau_e} \]

where \( u \) is the plasma velocity normal to the flux surface, \( \tau_e (\tau_i) \) is the electron (ion) collision time, and \( \lambda \) is the scale length.

(2) Next, the anisotropic ion distribution evolves to Maxwellian

\[ \frac{\partial}{\partial t} \sim \frac{u}{\lambda} \sim \frac{1}{\tau_i} \]

(3) Finally, neo-classical transport takes over.

In stages (1) and (2), the normal velocity \( u \) gives rise to convective transport of mass and energy. It is this global convective motion rather than the relaxation of the distribution function itself that is of primary interest here.

A procedure is given for determining the plasma velocity \( u \) and the evolution of the distribution functions. It is shown that the plasma velocity is determined by a boundary value problem involving two Fredholm integral equations and a partial differential-integral equation.

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(212) 460-7458
ADIABATIC COMPRESSION OF A ROTATING PLASMA

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Current experiments exhibit plasma mass flow, especially after neutral beam heating. The adiabatic evolution of a rotating ideal MHD plasma through a sequence of steady state equilibria, can be determined from the initial state by the use of constants of the motion. A specification of a rotating axi-symmetric equilibrium requires the supply of 5 arbitrary functions of \( \psi \) - the poloidal flux function. These functions can be obtained from 5 conservation laws for each moving flux tube, namely: conservation of mass, entropy, toroidal flux, fluid circulation along field lines, and the angular momentum of the ignorable direction. A similar problem can be solved for a two-pressure guiding center fluid (double adiabatic model), both with and without flow.

The problem is best described by a formulation using Generalized Differential Equations\(^1\). A plausible numerical algorithm based on the "1-1/2 D" concept\(^1\) of iterating between geometry and plasma profiles can be used in the present problem.

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\( \dagger \) Work supported by U.S. DOE Contract No. EY-76-C-02-3077.

(212) 460-7204
The stability of hydromagnetic equilibria is examined in terms of a variational principle in which the energy is minimized while keeping a number of global integrals of motion, viz., \( K = \int d^3x \; A \cdot B \), \( G = \int d^3x \; \omega \cdot (B + (m/2q)V \times v) \), etc., constant, \( B = V \times A \) and \( \omega = V \times v \) is the vorticity. When only \( K \) is taken into account, the usual force free solutions \( V \times B = \kappa \beta \) are obtained, but if \( G \) is also taken into account then we have in addition \( \gamma = \alpha \beta \) together with \( \rho \sqrt{\gamma^2/2 + (d\rho/d\rho)\rho} = 0 \), where \( \rho \) is the mass density and \( p \) is the pressure.

We have obtained axisymmetric incompressible equilibria which match to vacuum fields by using the method of free boundary. For the special case \( \gamma = B/\sqrt{\beta} \) and \( \rho \) constant, we obtain toroidal, D shaped equilibria where the vacuum field at infinity is uniform. For a magnetic cusp we obtain the interesting case of a plasma with a spindle cusp boundary but the internal fields are equivalent to two vortex rings with oppositely directed toroidal fields.

These equilibria are stable to internal incompressible perturbations and the surface perturbations are examined by the method of Rosenbluth and Bussac.\(^1\) Cusp-shaped equilibria are found to be stable to both surface and internal incompressible perturbations.

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Both (axially symmetric) MHD and GCF \((p^\perp \approx p^\parallel \approx p)\) equilibria with spheroidal plasma-vacuum interfaces have been considered in the past by G.K.M. et al. These equilibria apparently have all the external and internal parameters required for viable plasma confinement configurations; and they cover the full range of physical parameters including singularities which of course must be avoided by a practical device. For example: (1) A small vacuum region surrounding the fat toroidal plasma, where the two (or four) stagnation points (or separatrices) are near the singlet plasma boundary, can lead to a square-root singularity (integrable) of the poloidal current density on the plasma-vacuum interface (related to kinks). This high current behavior can be totally suppressed by a modest amount of external axial current, \(I_o\); in a practical device, access to the plasma is essential so that the spherical small-vacuum-plasma configuration is imbedded in a topologically straight vacuum region which in turn is bounded by carefully shaped confining coils on an appropriate magnetic flux surface. (2) For MHD configurations there is a practical restriction on pressure or \(\beta\) since in the high-pressure diamagnetic regime, the required \(I_o\) increases very rapidly with increasing \(\beta\) and soon becomes excessive; but this behavior is for \(\beta_{\text{poloidal}} > 0.5\) (related to incipient reversed currents in the plasma); and (3) In the GCF plasma equilibria the plasma-vacuum interface can be spheroidal, i.e. prolate, spherical or oblate depending primarily on \(\beta\). For prolate \((p^\perp > p^\parallel)\) or oblate \((p^\perp < p^\parallel)\) configurations the outer confining coils must be modified; otherwise kink behavior is immanent on the plasma-vacuum interface. But the plasma-vacuum interface can be maintained inside of a closed spherical region by appropriate choice of \(\beta\).
Instability Driven by the Electron Return Current in a Field Reversed Ion Ring

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It has been shown that electron return currents can increase the difficulty of field reversal on a collisional time scale.\(^1\) We have been studying field reversal by the pulsed injection of energetic ions. On the relevant time scales we may neglect collisions but must include electron inertia. We consider the return current effects for a large aspect ratio ion ring, accelerated azimuthally through field reversal by a cusp magnetic field. We find that a strong electron return current flows throughout the region of closed field lines. The return current drives an instability having a growth rate

\[
\omega_1 = \frac{\Omega_{eo}}{2} \exp\left(-\frac{\omega_{pe}}{\Omega_{eo}}\right),
\]

where \(\Omega_{eo}\) is the electron cyclotron frequency in the cusp field.

The unstable mode is an electrostatic surface wave, driven unstable by the presence of a resonance,

\[
\omega - kv_e = \pm \omega_{pe},
\]

in the return current region. The subsequent nonlinear evolution is being studied.

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TRANSITION FROM COLLISIONAL TO PASTUKHOV ION CONFINEMENT FOR TMX

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ABSTRACT

The ion confinement time in the central cell of TMX has in the past been calculated assuming that the distribution function is zero on the loss-cone boundary. We term this Pastukhov confinement\(^1\) which is valid for very long mean-free paths. As the collisionality of the plasma increases, the filling time of the loss-cone finally becomes shorter than the axial transit loss time. In this collisional case the distribution function is Maxwellian everywhere except near the end of the confinement region where the loss-cone is depleted.

For TMX the ion mean-free-path is longer than the system, but the volume of the loss-cone is small because of the large mirror ratio. Consequently, ions need to scattering significantly less than 90\(^\circ\) to fill the loss-cone. A simple calculation shows that for initial TMX parameters the loss-cone filling time is somewhat shorter than the axial transit time indicating that the device is in the transition region between collisional and Pastukhov confinement. We shall present the results of detailed calculations of the confinement time for TMX obtained from a Monte Carlo code which is valid for any collisionality. The total confinement time, \(\tau_t\), is found to be approximated by the sum of the collisional and Pastukhov confinement times, \(\tau_c\) and \(\tau_p\); i.e., \(\tau_t \approx \tau_c + \tau_p\). The different scaling of \(\tau_c\) and \(\tau_p\) with density, ion temperature and mirror ratio will be discussed. We find that for initial TMX operation, \(\tau_c/\tau_p \geq 1\). A self-consistent rate equation code is used to calculate the effect this additional confinement has on the steady-state parameters of the system.

\(^{*}\)Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-Eng-48.

Chaotic, Strange Attractor-Type Behavior in Instability Saturation by Mode Coupling

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We describe the results of an accurate and comprehensive numerical study of instability saturation by mode coupling in a three wave system. The (normalized) equations studied are: 

\[ \dot{a}_1 = a_1 + a_2 a_3 e^{-i\delta t}, \]

\[ \dot{a}_{2,3} = -\gamma_{2,3} a_{2,3} - a_1 a_{2,3} e^{-i\delta t}, \]

where \( \gamma_2, \gamma_3, \) and \( \delta \) are real and positive. Particular emphasis is placed upon distinguishing bifurcations leading to motions characteristic of a strange attractor. The tools used in this investigation include power spectral analysis, surface of section plots, and reduction of the latter to one dimensional mappings.

Exploration to date of the two dimensional parameter space (setting \( \gamma_2 = \gamma_3 = \Gamma \)) indicates that a number of bifurcations occur before a strange attractor appears. For fixed \( \delta \), we observe that for \( \Gamma < \Gamma_a \) and \( \Gamma > \Gamma_b > \Gamma_a \), all initial conditions examined lead to unbounded solutions. In the range \( \Gamma_a < \Gamma < \Gamma_0 \), the solutions converge to a simple stable periodic orbit, which manifests itself as a single fixed point in the surface of section. As \( \Gamma \) is increased above \( \Gamma_0 \), bifurcations to more complicated periodic orbits occur. These orbits manifest themselves as periodic points in the surface of section. As \( \Gamma \) is increased past some critical value, \( \Gamma_5 \), the motion becomes chaotic with the characteristics of a strange attractor. (The values \( \Gamma_a, \Gamma_b, \Gamma_0, \) and \( \Gamma_5 \) depend on the value of \( \delta \).) In the chaotic case, the points in the surface of section appear to lie along an arc. However, reduction to a one dimensional mapping shows that this arc must have some thickness (similar considerations have been applied to the Lorenz attractor). Qualitative changes are observed in the structure of the strange attractor as \( \Gamma \) is increased. For \( \Gamma_5 > \Gamma > \Gamma_a \), the power spectra consist of very sharp discrete peaks, while for \( \Gamma > \Gamma_5 \) the power spectrum becomes broad. These results, mapped in the parameter space \((\delta, \Gamma)\), and results for \( \gamma_2 \neq \gamma_3 \) will be presented.

This system of equations has also been examined by Vyshkind and Rabinovich (for \( \gamma_2 = \gamma_3 \)) who, in contrast with the results described above, always obtain stochastic motion if \( \delta \neq 0 \).

*Work supported under U.S. DOE Contract No. EY-76-S-02-3170.
Nonlocal Investigation of the Lower-Hybrid-Drift Instability in Reversed Field Configurations,
J. D. Huba, Science Application, Inc., and J. Drake and N. T. Gladd, Naval Research Laboratory--The lower-hybrid-drift instability is regarded as an important cross-field anomalous transport mechanism in a variety of laboratory confinement systems, including theta pinches, RFP's and mirror machines. This instability may also act as a source of resistivity to drive tearing modes in reversed field systems. We present a fully electromagnetic, nonlocal, kinetic theory of the lower-hybrid-drift instability in a reversed field geometry \( B_2(x) = B_0 \tanh (x/a) \). For moderate sheet pinch widths \((a \sim 2r_{Li})\), a broad spectra of radial wave vectors \((k_x)\) are excited with comparable growth rates. These modes typically occupy a substantial portion of the plasma sheet. Implications of the radial structure in understanding the nonlinear saturation of the instability and influence on tearing modes will be discussed.
Although rapid heating in tokamaks allows attainment of finite $\beta$ while controlling the $q$ profile and with it the stability of the plasma (FCT concept), the long term maintenance of finite $\beta$ equilibria under resistive diffusion has been questioned. The potential problems which could arise on the resistive time scale include $q_{\text{axis}}$ dropping significantly below unity leading to instability, and $\beta$ limiting separatrix forming on the inside edge of the plasma due to the interaction of the vertical field with the plasma's own poloidal field.

We have investigated the resistive diffusion of FCT equilibria using both analytical techniques and a fully toroidal 1-1/2D free boundary transport code. Although the problems mentioned above are real, they can be circumvented by combinations of cross section shaping, profile tailoring, and coil current adjustments. In general broad temperature profiles are required to keep $q_{\text{axis}}$ above unity as $\beta$ approaches 10%. For peaked profiles $q_{\text{axis}}$ can drop to 0.5 or less on the resistive time scale with $q_{\text{edge}}$ of 3-4. Separatrix formation is avoided by using higher order confining fields rather than uniform vertical field; such fields do not appear overly difficult to design. D shaped or other noncircular plasmas are also favorable for avoiding low $q_{\text{axis}}$ and separatrix formation. This is due to toroidal geometric effects which favorably modify the overly simple relations derived for large aspect ratio circular cross section. Significant modifications to the current profile are also observed in the transition from FCT to resistive behavior. Some of these may have implications for stability.

Ion Streaming Instabilities

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In examining carefully the linear electrostatic dispersion relation of a plasma of counterstreaming ions with an electron background in an unbounded uniform magnetic field, we have found a new mode. This mode has a lower threshold and a higher growth rate than other modes of this type previously considered.

Instability for this mode requires that the electron temperature be at least 4 times one of the ion temperatures. For example, if the parallel and perpendicular electron temperatures are equal, $T_e > 4T_i$ is needed, where $T_i$ may be either the parallel or perpendicular ion temperature. Defining a dimensionless parameter $\tilde{V}_i = V_d/\sqrt{2} \tilde{V}_i$ where $V_d$ is the drift velocity of the ion beams and $\tilde{V}_i = \sqrt{T/m}$, their thermal velocity, we find that $\tilde{V}_i > 2.5$ is needed for instability when $T_e > 4T_i$. The growth rates are large, up to $20\omega_i$. This should be compared with the ion-cyclotron mode of Weibel, also purely growing, which has a higher streaming threshold ($\tilde{V}_i > 3$) and a maximum growth rate $\omega_i/\sqrt{3}$. Another mode (with real part $\omega_r = 2\Omega_i/\sqrt{3}$) discussed by Perkins has a threshold about the same as our mode but a growth rate an order of magnitude lower. This new mode has $\omega_r = 0$, $k_i = 6k_i$ and a perpendicular wave length much smaller than the ion larmor radius.

When $T_e > 4T_i$, another mode exists with parallel propagation and the low threshold $\tilde{V}_i > 1.3$. This mode has growth rates $\omega_p_i/100$, or roughly similar to the other mode.

We consider electrostatic ion Bernstein waves driven unstable by a cold ion beam with velocity $u$ directed along the magnetic field. A reactive type instability results from the coupling of the beam mode at $\omega = k_u u$ to the Bernstein mode at $\omega = n\Omega$, with an approximately cubic dispersion relation at the crossover point. The nonlinear analysis assumes a single unstable mode. When the ratio of beam energy density to thermal energy density is small, saturation by beam trapping is expected. However, when this ratio is large (as in beam heating experiments) the nonlinear ion gyrofrequency shift saturates the instability first by detuning the resonance. Saturation can occur at values well below those predicted by beam trapping. To describe the nonlinear behavior in detail coupled equations for the Bernstein wave, $\phi$, and beam wave $n$ are derived:

$$i\phi_t + iv\phi_x + \phi_{xx} + |\phi|^2\phi = n$$

$$n_{tt} + \phi = 0$$

$v$ is related to the group velocity of the waves. The temporal problem is solved numerically and indicates nonlinear oscillations about the expected saturation level. An exact one-soliton solution exists for the problem in one space dimension.

*Work supported by DOE.
STOCHASTIC HEATING IN A LARGE-AMPLITUDE STANDING WAVE

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ABSTRACT

In heating plasmas with high-power radio-frequency waves, a symmetrical launching structure is often used. This leads to large-amplitude standing waves in the plasma. Particles may execute random walks in the two oppositely-directed traveling waves. As a result, stochastic heating of low energy particles can readily occur. The physical origin of the stochasticity may be explained as the scattering off the dense set of unstable fixed points. At the stochasticity boundary, where \( p = \frac{eE_0 k_o}{m \omega_o^2} > 0.456 \), the Fourier spectrum of particle trajectory is characterized by the onset of a "stochastic" mode and broadband noise. The stochastic trajectory is presented by the stroboscopic method.\(^1\) The energy gain for large \( p \) is found from a multiple-time expansion and scales linearly with \( p \). Modification of the plasma dielectric function due to stochastic electron motions is also obtained. The present mechanism should affect the plasma-wave coupling and accessibility conditions. Comparisons with some experiments\(^2,3\) will be discussed. If a standing wave is created in the plasma core by launching two traveling waves from the plasma edge, it may stochastically heat the plasma without any resonance condition.

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\(^1\)G. Smith and N. Pereira, Phys. Fluids 27 (1978) 2253.
At the present time the possibility of generating DC plasma currents by external RF fields is of considerable interest. A potentially useful RF current driver is the travelling-wave variation of the lower-hybrid heating scheme. As part of our extensive simulation of this heating scheme, we have investigated the simultaneous process of heating and DC current generation by a unidirectional lower-hybrid wave of high phase velocity \( \omega/k = 4.1 v_{th} \), which is launched by an external structure that mocks up the role of an "end-fire" waveguide array. Although the present 2-1/2 D electrostatic simulation is rather idealized, a variety of important collisionless effects are isolated; in particular, the role of density profiles and surface heating for the DC current generation process are studied.

*Work supported by USDOE and ONR.
Energetic ion beams appear to provide an ideal driver for pellet fusion. Either light ion beams of $10^7$ amps at 1 MeV or heavy (high z) ions of 1000 amps (equivalent) at 10 GeV supply 10 Tw and have ranges compatible with moderate size $\ll 1$ mm pellets.

We concentrate here on the problem of quasi-ballistic propagation through a 5 m radius target chamber. We consider first what background pressure comprises a vacuum and then discuss the consequences of gradually increasing the background pressure. Self consistent charges and currents will be discussed as well as the effects of a wide group of instabilities.

Our main conclusion is that at pressures $<1$ millitorr, the heavy ion beams can propagate even in a charged mode, as in the accelerating system, and at higher pressures can propagate in a neutralized mode, in spite of a set of possible plasma instabilities.

For the light ion case, however, the beam must be neutralized, and a surface instability usually prevents targetting without the help of a background plasma.

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MAGNETIC FLUCTUATIONS EXCITED BY \( \alpha \)-PARTICLES

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The production of 3.5 MeV \( \alpha \)-particles in a (D-T) fusion reactor can lead to the excitation of various plasma modes that can be compared to those expected from the injection of energetic neutral beams in present day hydrogen or deuterium plasma experiments. The general features of the type of magnetic confinement configurations that are considered, have an important influence on the collective modes that can be excited by a non thermal particle or energy source. For instance, to determine whether modes that would be identified as shear-Alfvén waves in an infinite homogeneous plasma are driven unstable by resonant interaction with the \( \alpha \)-particles, a number of questions must be addressed such as: a) what form do modes of this kind take in a toroidal geometry and in the presence of magnetic shear, as they cannot be found as ordinary waves; b) which is the appropriate form of resonant interaction between these modes and the \( \alpha \)-particles; c) how is the theory of these modes related to that of ballooning modes in toroidal systems.

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In continuation of our work\textsuperscript{1} we have studied the effect of shear on the normal mode structure of a uniform, hot, magnetised plasma. Shear modifies the normal mode structure by introducing an intrinsic damping which is independent of the wavelength for both large and small \( (\rho_1 k) \) (\( \rho_1 \) = ion larmor radius, \( k \) = wave number) and is of the order \( (\rho_1 S) \) (\( S \) = inverse shear length). An electron drift relative to the ions is introduced and the effects of shear on the current driven ion cyclotron instability are studied. Our results do not agree with a previous study by Bhadra\textsuperscript{2} of the same problem. The marginal stability criterion obtained through our treatment differs from that of Bhadra's\textsuperscript{2} quite significantly, and indicates that less shear is required for marginal stability. Treating the problem first at the Weber equation level and for nearly perpendicular propagation, we find that depending on the temperature ratio \( \tau (=T_i/T_e) \), the effect of the shear changes, rather sharply, from a damping of the order \( (\rho_1 S) \) for \( (\rho_1 k) > (\rho_1 k_c) \) to a shift in the real frequency of the same order for \( (\rho_1 k) < (\rho_1 k_c) \). A shooting code is now being employed to study this problem numerically, without making any approximation for the potential. Smoothing of this sharp transition at \( (\rho_1 k_c) \) is expected. Particle orbit modifications\textsuperscript{3} (Shear Kinetic Drift) are also introduced and consequences thereof discussed.

CURRENT PENETRATION STAGE IN A TOKAMAK*

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ABSTRACT

To study the current penetration stage in a tokamak we use G2M with a picture of transport which in addition to neoclassical transport for the ions takes into account the neutral background, wall-plasma effects, and impurities effects. Without the inclusion of resistive tearing modes, runs done in the regime of operation of the Pre-Tex tokamak\(^2\) (\(a = 19\) cm, \(B_T = 10\) kG, \(I \sim 75\) KAmp at 10 \(\mu\)sec) shows skin effect in the current. These profiles are unstable to tearing modes. For \(m > 20\) (early times) we enhance the electron transport in the perpendicular direction \(q_{\perp}^{(e)} = (\delta B/B)^2 q_{\parallel} \exp - a (dq/dr)\). For the remaining modes assuming that the saturated \(\psi\) has the same shape as the linear, we solve \(\nabla^2 \psi = \left(\frac{\partial \psi}{\partial \psi_0}\right) \psi\) determine \(\Delta'(w) = 0\) at saturation and enhance \(q_{\perp}^{(e)}\) in the island width \(w\), \(q_{\perp}^{(e)} = (\delta B/B)^2 q_{\parallel}\), \(\delta B\) being an estimate of the magnetic field in the islands. We have observed the double tearing for which Kadomtsev reconnection should be used.

*Work supported by the U.S. Dept. of Energy.

(1) N. Byrne, private communication.

(2) R. Bengtson, private communication.

(3) B. Carreras, H. Hicks, B. Waddell, ORNL/TM-6570.
Simulation of Axisymmetric Alfvén Resonance Heating of Tokamaks

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It was recently reported\(^1\) that by oscillating the vertical field in a tokamak at a frequency \(\omega\), resonant heating will occur if the resonant condition

\[ \omega = \frac{V_A}{qR} \]

is satisfied at one or more radial points. Here we present the results obtained by simulating this process using a two-dimensional ideal MHD initial value computer code.\(^2\) It is shown that the time evolution of the plasma is markedly different when the resonance condition is satisfied compared to when it is not.

\(^*\)Work supported by U.S. DoE Contract No. EY-76-C-02-3073.


The development of electron beam turbulence heating as a technique for in situ microwave generation and electron heating in EBT has recently been proposed. The theory relevant to the interaction of a moderate energy, parallel electron beam with a mirror plasma is reviewed and compared with experiment. Attention is focused on beam-plasma coupling, rather than electron heating as such. Of particular interest is the case of a mirror ratio of 3-5, with $\omega_{pe}/\Omega_{ce} \approx 4$. 

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+W. B. Kunkel, M. C. Vella and B. Feinberg, LBL.
Global Reversed Field Pinch (RFP) burn simulations\textsuperscript{1,2} have indicated that the RFP is attractive as a reactor concept. The RFP is stable at high $\beta$, ohmically ignited, and appears conducive to quasi-steady operation. In order to further investigate these properties, a one-dimensional model has been devised.\textsuperscript{3}

Results have verified the feasibility of ohmic ignition at parameters near those of the global studies. However, the required magnetic field is higher due to off-axis peaking of the temperature profile. Pressure profiles also peak off-axis which stabilizes Suydam modes in the central plasma region. Inclusion of anomalous transport enhances this effect.


Neoclassical Diffusion in Plasmas of Helical or Toroidal Symmetry

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The neoclassical particle flux across magnetic surfaces has been calculated in such a way that the results apply equally well to helically and toroidally symmetric plasmas. Two cases are considered: First, the collision operator is left completely arbitrary, but the magnitude of the magnetic field is assumed to vary only slightly over a magnetic surface

\[
\frac{B_{\text{max}} - B_{\text{min}}}{B_{\text{min}}} \ll 1.
\]

In this case, the neoclassical transport calculations for the two different symmetries become identical, and the diffusion coefficients calculated previously for the small aspect ratio, axisymmetric tokamak can be carried over to the helically symmetric plasma without change, except for the substitution of appropriate helical parameters for the corresponding toroidal ones. Secondly, a detailed calculation of the particle flux is carried through with the Lorentz collision operator, but with the variation of the magnetic field and the shape of the magnetic surface left completely general, except for the requirements of symmetry, helical or toroidal.

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LOW FREQUENCY WAVE PROPAGATION IN A HOT TOROIDAL PLASMA*

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ABSTRACT

The equations for low frequency wave propagation (ICRH) have been derived in full toroidal geometry with arbitrary cross section. The dielectric tensor used here includes both particle effects (Landau damping) and global properties (inhomogeneities, structure of the magnetic field). To this aim, finite Larmor radius expansion has been performed and homogeneity along the field lines has been supposed. To first order, one gets two second order equations, weakly coupled through the toroidal effect, written in a system of intrinsic coordinates related to the flux surfaces, out of which the leading equation, on $B_\phi$, is an Helmoltz type equation in curved coordinates. Its coefficients are expressed in terms of the components of the dielectric tensor and include, through Z-functions, dissipative effects. Then the singularity in this equation along the ion-ion conversion layer, $\omega = \omega_i$, is smoothed out by these dissipative effects, which compete with higher order dispersive terms in Larmor radius expansion. It is shown, by comparing these terms to the lower order dissipative terms, that along the ion cyclotron resonance layer $\omega = \Omega_i$, these later are always the larger, whereas along $\omega = \omega_s$, they only overtake as long as the ratio of the minority ions density to the main ions density is: $\eta < \eta_{\text{int}} \approx (2k_\perp \sqrt{\nu_i \nu_i} / \nu / Z. B. 10^3) \left[ \frac{3}{4} + (A_{\perp}^2 / 1.7 Z_{\perp}^2 \nu_i) \right] 10^{13} / \text{cm}^3$, $\text{m}^{-1}$, keV, kG which can be fairly large, $\approx 30\%$ for large enough $k_\perp$ are observed. So the picture is that for $\omega_s$ close enough to $\Omega_i$, lower order absorption effects dominate, whereas for $\omega_s$ far enough from $\Omega_i$, they significantly lower the available power at $\omega_s$, where ion-ion conversion effects now compete with stochastic effects coming from the collisions of the small wavelength components of the EM fields with the particles, enhancing the lower order absorption affect. So ion-ion conversion process is masked, if any, and the structure of the field is given by the leading equation for $B_\phi$ to lowest order in aspect ratio.


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†Proposed address.
IN ORDER TO STUDY PLASMA HEATING AND IGNITION BY NEUTRAL INJECTION, A MONTE CARLO NEUTRAL INJECTION COMPUTER CODE\textsuperscript{1} HAS BEEN COUPLED TO A SINGLE FLUID, ONE DIMENSIONAL TRANSPORT CODE AND A TWO DIMENSIONAL FLUX CONSERVING EQUILIBRIUM CODE. WE HAVE SHOWN THAT, BY TAKING ADVANTAGE OF CENTRAL $\alpha$-HEATING, PROFILE EFFECTS, AND FLUX SURFACE SHIFTS IN ELONGATED PLASMAS, IT IS POSSIBLE TO IGNITE A MODELED, PROTOTYPICAL REACTOR PLASMA USING 100-150 keV (D\textsuperscript{+}) NEUTRAL BEAMS. TO DO THIS, THE PLASMA IS STARTED AT FULL BORE BUT LOW DENSITY. THE DENSITY IS THEN INCREASED BY PERIPHERAL FUELING SO THAT THE CENTRAL CORE BEGINS TO IGNITE AT THE TIME WHEN THE NEUTRAL BEAMS NO LONGER PENETRATE TO THIS REGION. THE FUSION $\alpha$-PARTICLES TAKE OVER THE HEATING REQUIREMENTS IN THE CORE REGION. BECAUSE OF THE DECREASING BEAM LINE EFFICIENCY WITH INCREASING ENERGY, IT IS FOUND THAT A NEARLY CONSTANT EXTRACTED POWER IS NEEDED FOR IGNITION IN THE RANGE STUDIED. THERE IS THUS LITTLE ECONOMIC DIFFERENCE IN THIS ENERGY RANGE. HOWEVER, THE HIGHER ENERGIES AROUND 150 keV IMPLY FEWER INJECTORS AND PERHAPS LOWER IMPURITY PRODUCTION RATES DURING HEATING TO IGNITION.

\textsuperscript{*} Research sponsored by the Office of Fusion Energy (ETM), U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

The changes in tokamak plasmas undergoing large adiabatic compression in major radius are examined numerically over the range of aspect ratios: 1.5 ≤ A ≤ 3 in circular, elliptic, and D-shaped cross sections. The numerical approach combines the computation of fixed boundary FCT equilibria and one-fluid, flux surface averaged energy and particle balance equations. The fixed boundary approach allows a precise prescription of the plasma boundary position and shape. During compression the minor radius (a) is adjusted iteratively to ensure the invariance of (\psi_{edge} - \psi_{axis}), q(\psi) and \(F(\psi_{edge})\). It is found that the dependences of I_p (plasma current) and \(\beta_p\) (poloidal beta) on the compression ratio (C) differ significantly from those proposed by Furth and Yoshikawa, while the dependences of a, \(\bar{\beta}_t\) (averaged toroidal beta), and P (pressure) show a milder difference.

The present interpretation is that compression to small A dramatically increases the plasma current which lowers \(\beta_p\) and makes the plasma more paramagnetic. Despite the large \(\bar{\beta}_t\) values (≥ 30% with \(q_{axis} ≈ 1, q_{edge} ≈ 3\)), this tends to concentrate more toroidal flux toward the magnetic axis which requires reduced minor radius to preserve the continuity of F at the plasma edge. For D-shaped plasmas with mild elongation (1.6), the averaged vertical field decay index (-\(\partial z B_z/\partial z nR\)) on the mid-plane changes from -0.7 to +0.2 as A is reduced from 3 to 1.5. This indicates that the vertical stability of the D-shaped plasma column is enhanced with decreasing A.


Interchange Stability of Axisymmetric Field Reversed Equilibria

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Axisymmetric field reversed equilibria are obtained by a new computational method for solving the Grad-Shafranov equation. The method is pertinent to ion rings, field reversed axisymmetric mirrors, and field reversed theta pinches, all in the absence of a toroidal magnetic field. The method allows us to specify the pressure profile with the pressure on the magnetic axis and the pressure on the separatrix as independent parameters. No bifurcations are observed when this method is employed. As part of a stability code under development, a mapping routine has been created which transforms \((r,\theta,z)\) coordinates to \((\psi,\theta,\phi)\) coordinates, where \(\psi\) identifies a flux surface and \(\phi\) is determined by specifying the form of the Jacobian. In particular we have looked at cases where the Jacobian depends only on \(\psi\). The mapping routine is used to compute the interchange criterion assuming unfavorable \(V''\) (i.e., unfavorable curvature). Results of these stability computations are presented for configurations with various pressure profiles and with varying amounts of pressure on the separatrix.

EQUILIBRIUM AND STABILITY OF FINITE-8 MULTipoLEs

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We have developed codes to study the ideal MHD equilibrium and stability of finite-8 plasmas confined by multipole magnetic fields with \( B_\phi = 0 \). The equilibrium code solves the Grad-Shafranov equation for a given \( P(\psi) \) profile and specified values of the coil currents. The stability code solves the second-order differential equation given by Johnson et al.\(^1\) for the eigenvalue \( P'_\text{crit}(\psi) \), the value of \( P'(\psi) \) corresponding to marginal stability of high-\( n \) ballooning modes at the given \( \psi = \text{const} \) surface. We will report our initial results for the UCLA toroidal quadrupole experiment. The numerically-obtained value of \( \theta_{\text{crit}} \) will be compared to that obtained for the linear quadrupole, which can be solved almost completely by analytic methods.


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Turbulent Evolution of the Collisionless Tearing Mode due to Stochastic Magnetic Fields

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Magnetic perturbations due to tearing instabilities can lead to the destruction of flux surfaces and radial diffusion of magnetic field lines. As a result, electrons can diffuse radially due to their rapid transport along the stochastic field lines. Thus, the tearing instabilities modify the electron motion. At the same time, the altered electron motion will self-consistently modify the evolution of the instabilities. In a slab model, it is shown that the magnetic turbulence both broadens the layer of particle acceleration and also causes a ponderomotive renormalization of the background distribution. The influence of these effects on the nonlinear growth and saturation of the modes is estimated. Formally, the $v_{||} \cdot \mathbf{v}$ streaming nonlinearity in the drift kinetic equation is studied, where $\mathbf{v}$ is the direction of the fluctuating magnetic field. A statistical closure approximation, obtained from the Direct Interaction Approximation by neglecting a mode coupling term, is used to derive a nonlinear dispersion relation. The theory depends crucially on two characteristic lengths: $L_o$, an autocorrelation length which is inversely proportional to the spread of the spectrum in parallel wavenumber, and $L_k$, a nonlinear mixing length which describes the rate of exponential divergence of adjacent field lines. We have previously shown that $L_o \approx \xi_s$ and $L_k \approx \xi_s (k^2 D_m \xi_s)^{-1/3}$, where $\xi_s$ is the shear length, $D_m$ is the magnetic diffusion coefficient, and $k$ is a typical wavenumber. For a sufficiently low turbulence level, one has $L_0 < L_k$. In this regime, stochastic diffusion can initially enhance the growth rate. Saturation can occur by quasilinear relaxation of the background current or by sufficient turbulent broadening of the perturbed current layer.


DIFFUSE VLASOV-FLUID SCREW PINCH*

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We present a method for the numerical solution of linearized stability problems for the diffuse Vlasov-fluid screw pinch. The theory behind the technique has been applied with considerable success to the sharp boundary screw pinch\(^1\) and the rotating diffuse theta-pinches.\(^2\) Here we present a version of the method which is much more economical with computer storage, a main difficulty is solving inhomogeneous collisionless plasma stability problems. The reason for this difficulty is due to the manner in which the eigenvalue appears in the equations, requiring that large arrays must be stored for later iteration upon the eigenvalue. We manage to save large amounts of storage by computing the dispersion matrix elements as numerical functions of the eigenvalue. Since the matrix elements are smooth functions of the eigenvalue only a few points in the complex plane are required to give an accurate representation of the dispersion matrix. We form the dispersion matrix by taking the inner product of the force operator with a finite element representation of the electron fluid displacement. A complete discussion of the analytical techniques and the numerical algorithms will be presented.

*Work performed under the auspices of the U.S. Department of Energy.


2. C. E. Seyler, submitted to Physics of Fluids.
In the collisionless regime, particles in EET with magnetic poloidal precession drift velocities that nearly cancel the \( \mathbf{E} \times \mathbf{B} \) drift are limited in their poloidal motion, forming crescent-shaped drift orbits (analogous to the banana orbits in tokamaks). Because of the relatively large radial excursion of these crescent-shaped orbits compared with that of the circulating particle orbits (analogous to the untrapped particle orbits in tokamaks), they make significant contributions to diffusion in the collisionless regime. Previous neoclassical transport calculations have not simultaneously included the effects of these orbits and differential collision operators or are valid in more collisional regimes.

An analytic calculation of the diffusion coefficient due to the crescent-shaped orbits is presented here. We solve the bounce averaged drift kinetic equation in the small collision frequency limit, using techniques similar to those employed in calculations for tokamaks. Two major differences distinguish this from the tokamak banana calculation: (i) the radial width of the crescent-shaped orbit is essential in the lowest order drift kinetic equation; and (ii) the longitudinal adiabatic invariant determines the trapping boundary. The diffusion coefficient in the collisionless regime is found to scale as the square root of the inverse aspect ratio, which is significantly different from the scaling in most collisional regimes. This scaling agrees with the scaling obtained from entropy production arguments. However, the overall scaling depends also on the radial and poloidal ambipolar potential and hence on the transport of the off-resonance species.


\(^1\text{Catto, Rosenbluth, and Tsang, this meeting.}\)
ONE-DIMENSIONAL TRANSPORT SOLUTIONS FOR EBT-II*

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ABSTRACT

Recently [1], one-dimensional radial transport solutions for the
ELMO Bumpy Torus (EBT-I) have been obtained in the collisionless electron
regime as observed in experiments [2]. In these calculations, resonant
diffusion of ions is included in regions where poloidal drift frequencies
are small. For ion temperatures characteristic of EBT-I, this leads to
ion transport coefficients which are approximately independent of colli-
sionality (plateau regime [3]). In this paper, we extend these calcula-
tions to the higher temperature and density regime proposed for the EBT-II
experiment [4]. Results show somewhat hollow density profiles in steady
state due to the presence of off-diagonal neoclassical transport coeffi-
cients. In addition, the sputtered flux of aluminum due to charge
exchange neutrals in EBT-II is increased by a factor of 20-30 over that
found in EBT-I calculations.

*Research sponsored by the Office of Fusion Energy (ETM), U.S. Department
of Energy under contract W-7405-end-26 with the Union Carbide Corporation.
ENHANCED TAIL FOR IONS IN EBT

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ABSTRACT

Experimental observations of EBT suggest that the distribution function for well trapped ions can have an "enhanced tail" [1]. Here we present analytic kinetic calculations based on simple models of the sources and sinks which yields such an "enhanced tail". The two features which are critical to this analysis are that a narrow region of velocity space dominates the diffusive loss processes and that the source of particles (ionization of neutrals) is at lower energies. For well trapped particles \( v^\perp = 0 \) this zone occurs for energies approximately equal to \( e\phi \). The slope of the distribution function is approximately constant (as a function of energy) to either side of this critical energy and is smaller in magnitude for higher energies (i.e. an enhanced tail).

These calculations suggest that an "enhanced tail" on the ion distribution is a natural consequence of neoclassical theory for EBT. An "enhanced tail" has implications for further development of neoclassical theory for EBTS. For example, simple arguments suggest that taking into account this distortion of the lowest order distribution function from a Maxwellian could lead to larger electric fields (factor of 2) than presently obtained from 1-D transport calculations.

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Research sponsored by the Office of Fusion Energy (ETN), U.S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

A ONE-FLUID MODEL OF MAGNETIC FIELD FLUCTUATIONS
IN A MAGNETIZED PLASMA WITH A TEMPERATURE GRADIENT

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The time-dependent thermal fluctuations in a magnetized
($B_0=\vec{B}_0\hat{z}$) plasma with a temperature gradient ($\nabla T_0=\hat{y}$) are considered. A one-fluid MHD model with all damping effects
is utilized. The system under consideration is supposed to
be stationary and spatially homogeneous. The assumptions
of local equilibrium [1] and local charge neutrality are
made. The Fourier-transform $\phi$ of any quantity $\phi$ is intro­
duced by the following equation:

$$\phi(r,t) = \hat{l}^{-1} \int \int \int (2\pi)^{-3} d^3k \omega \phi(k,\omega) \exp(ik\cdot r - i\omega t),$$

where $k = (k_x, k_y, k_z)$, $\omega = (\omega_x, \omega_z)$, $\hat{l}$ is the length of the system in the gradient direction.

The dynamic correlation function of the magnetic field
fluctuations is obtained explicitly. The influence of the
temperature gradient on the propagation and damping of the
modes in the plasma is analyzed. The static correlations
of the magnetic field are also studied [2].

The dynamic and static correlation functions of the
various hydrodynamic quantities may also be investigated
using the same method (for comparison see [3]).

The results obtained will be useful in explaining trans­
port in tokamaks.

The author wishes to acknowledge here the hospitality
of the University of Maryland.

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[1] A. I. Akhiezer, et. al., Plasma Electrodynamics, §11.6,


(1975).
Rotation of a Toroidal Plasma

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The toroidal and poloidal velocity components of an axisymmetric toroidal plasma are calculated by using the classical formulas of stress tensor in the MHD equations.

It is found that the poloidal velocity component is completely determined in terms of the instantaneous plasma variables \((B,P,T)\), independent of the initial conditions or viscosity coefficients, while the toroidal velocity component depends on the initial conditions.

On the other hand, the toroidal velocity component can increase or decay in time. The criterion for determining whether it increases or decays depends only on the instantaneous plasma variables and is independent of the initial conditions of the toroidal velocity and viscosity.

Explicit expressions are obtained for the poloidal and toroidal velocity components as well as the formula for determining the rate of change of the toroidal velocity component. The special case of a large-aspect ratio plasma is also examined.

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THE NONLINEAR EVOLUTION OF RESISTIVE INSTABILITIES
IN FINITE BETA REVERSED FIELD PINCHES*

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ABSTRACT

There is currently renewed interest in the Reversed Field Pinch concept as a means of confining a hot plasma. Such magnetic field configurations are characterized by a reversal of the axial field in the outer regions, and allow for higher values of plasma B than do tokamaks. The use of ohmic heating to achieve thermonuclear temperatures is also a possibility.

The linear stability of these devices is well known. An analytic equilibrium (the Pitch and Pressure model) which is stable to ideal MHD modes has been found with $B \sim 30\%$. This was subsequently found to be unstable to tearing modes$^1$. More recently, equilibria which are stable against both ideal and tearing modes at values of central $B$ up to 18% have been found$^2$. However, these equilibria are unstable to slow resistive interchange modes.

We use a non-linear, two-dimensional, MHD computer code$^4$ to study the non-linear behavior of resistive modes in the equilibria described above. For the Pitch and Pressure model we find that the $m=1$ tearing mode can trigger the slow resistive interchange, which leads to localized interchange vortices near the x-point with subsequent convolution of flux surfaces. For the tearing mode stable equilibria, the $m=1$ interchange remains relatively benign, while the $m=0$ mode can lead to large magnetic islands and highly distorted flux surfaces.


*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48.
MHD Equilibrium and Stability of the Levitated Octupole* M.W. PHILLIPS, University of Wisconsin-Madison—A computer code has been developed for looking at ideal magnetohydrodynamic equilibria in the Levitated Octupole. We use this code to study high beta equilibrium in conjunction with experiments now going on in this device. Results for betas up to 8%, as defined locally on the separatrix in the bridge regions, are considered. We also look at ideal MHD stability with respect to ballooning modes and attempt to set an upper limit for beta for this device.

*Work supported by USDOE.
Interaction between Anomalous Loss and Neoclassical Impurity Transport in Tokamaks.

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Abstract

Neoclassical transport theory for an impure toroidal plasma predicts that high-Z impurities diffuse towards the centre until their density profile becomes sharply peaked. The effect on this transport of anomalous electron loss, such as is observed in all Tokamaks, will be examined. In the standard neoclassical theory the dependence on the radial electric field is eliminated by using the ambipolar condition. Since the total fluxes must be ambipolar, this ambipolar electric field may be changed by the presence of anomalous electron loss. This would strongly influence the diffusion of impurity ions, because of their large ionic charge. The effect may be much larger than a simple addition of the two independent fluxes.

One possible mechanism for anomalous electron loss is the break-up of magnetic surfaces by MHD, resistive, or kinetic instabilities, leading to ergodisation of the field lines. An outward directed ambipolar electric field may be needed to restrain the outward electron flow along field lines, reversing the field found when neoclassical transport is considered alone. The effect of this on impurity transport will be evaluated. This offers a possible explanation of some experimental results.
POWER REQUIREMENTS OF EBT ELECTRON RINGS*

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ABSTRACT

Two related models are developed to describe the rings of energetic electrons necessary for MHD stability of the EBT plasma. Both assume electron cyclotron resonance heating and differ in that: for Model A heating is limited by the rate of pitch angle scattering; for Model B electrons can always reach resonant surfaces even in the absence of pitch angle scattering. The resulting ring particle distributions resemble cosmic ray type spectra, and decrease monotonically from approximately the toroidal electron temperature to some high energy cutoff. The calculated spectra are used to estimate the collisional power loss between the ring and the toroidal plasma. These estimates are insensitive to the Model A/Model B difference, and show loss rates of 10-20 kW in EBT-1 (60 kW microwave power was available), and 1000-1500 MW for the EBTR-48 conceptual reactor (4000 MW thermal output). Most loss occurs at relatively low energies where collisional rates are large. These results suggest that contrary to previous estimates EBT reactor operation may require large recirculating power fractions.

*Work supported by DOE.
QUASILINEAR RADIAL TRANSPORT SIMULATION OF TMX PLUGS

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ABSTRACT

The end plugs of the Tandem Mirror Experiment (TMX) are expected to follow 2XIIB scaling based on a quasilinear theory of DCLC turbulence. It has been realized recently that radial plasma transport due to the DCLC turbulence and charge-exchange on cold background gas near the plasma surface may contribute significantly to particle and energy loss. These effects are important in determining the plug profiles which will affect the DCLC stability as well as confinement. We study this problem using the two-dimensional quasilinear radial transport code. At fixed turbulence level, the code was first verified to produce steady states consistent with 2XIIB experimental observations. We then run the code with the TMX design parameters to obtain steady state density, temperature and confinement times. The results in comparison with the design parameters will be reported. The simulation results with self-consistent calculation of the DCLC turbulence will also be reported.

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-Eng-48.
A systematic (statistical) approach to determining ideal incompressible MHD equilibria subject to given information on the values of a few global constraints is presented. This is not only more realistic experimentally but also in marked contrast to the standard theoretical MHD approach of arbitrarily specifying two profiles and then determining the remaining third profile from the Grad-Shafranov equation. In our approach, utilizing the information theory formulation of statistical mechanics, we obtain (for ID systems) a set of coupled ordinary differential equations whose solution yields the "most probable" MHD equilibrium consistent with the given information on the global constraints: total energy $E$, magnetic helicity $H$, toroidal current $I_{\phi}$, and toroidal flux $\phi_B$ on some subset of these. Typically, if $H$ is prescribed, the most probable states are of the screw pinch or reversed-field-pinch type with finite pressure while if $H$ is unspecified the profiles are those for a low $\beta$ Tokamak. Relaxation of constraints $E$ and $H$ results in uniform $J_{\phi}$ and $B_{\phi}$, consistent with the information theoretic approach.

For fixed $\phi_B$, profiles with varying degree of field reversal are generated by varying the constraints $E$, $H$, and $I_{\phi}$. These are plotted on the usual $F-\phi$ diagram and the stability of these profiles to the $m=1$ kink mode is examined. The relation of most probable ($\beta \neq 0$) states to the force-free $\beta = 0$ Taylor states is also considered.

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Anomalous Current Penetration

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Abstract

Recent experiments on PLT indicate that the current penetration in the start-up phase of a Tokamak is much more rapid than predicted by magnetic field diffusion and neoclassical resistivity. We propose here that this could be due to the turbulent electron viscosity. An expression for the turbulent viscosity is derived, and is used in the electron equation of motion, which is solved along with Maxwell's equations, using a one-dimensional code. Preliminary results show that anomalous viscosity caused by even extremely small fluctuations can provide effective current penetrations. Further application of the newly derived turbulent transport coefficients to Tokamak plasmas is also discussed.


This work is supported by the U.S. Department of Energy Contract DE-AC05-79ET53036.
Theoretical understanding of the nature of ideal MHD instabilities and considerable experience with existing numerical techniques and their limitations has enabled significant improvements in our ability to formulate ideal MHD instability problems computationally. As a first step in extending our computational treatment of ideal linear MHD modes to allow for resistive instabilities in general axisymmetric toroidal configurations, we have constructed what is essentially a second generation PEST code. This uses a scalar version of $\delta W$ similar to that found by Bineau. Although restricted in its ability to find exact normal modes and growth rates, it is directly applicable to the determination of marginal stability and thus addresses many problems of practical importance (including that of $\delta$-limits in tokamaks). Generalizations in the formulation enable it to be applied to configurations like spheromaks and reversed-field pinches for which the earlier version had limitations. These improvements have resulted in considerable reductions in computer requirements and thus, while it is now possible to apply these methods in a more routine fashion with other activities (e.g., plasma modeling codes, comparison with experimental data), it should also be possible to study more complex situations, such as behavior very close to marginal stability and larger toroidal mode number instabilities. Here, we describe the basic features of the formulation, present the results of a comparison study with the previous version of PEST and, if time permits, give some examples of its application.

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Plasma Transport by Stochastic Magnetic Fields
in Axisymmetric Geometries

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A canonical framework developed by Kaufman\textsuperscript{1} for particle diffusion in axisymmetric geometries is adapted to enable a more systematic study of plasma transport due to magnetic perturbations of axially symmetric equilibrium fields. In particular, the particle drifts present in any realistic geometry are built into the formalism. The expression for the diffusion tensor $D$ involves the square of field-particle coupling coefficients $g$. The dominant $g$ is proportional to $J_0(k_p\rho)$ (where $J_0$ is the Bessel function of index 0, $k_p$ is a typical perpendicular wavelength of the turbulent spectrum, and $\rho$ is the gyroradius of any given particle), and thus $D$ is down from the zero gyroradius result of previous theories\textsuperscript{2,3} by a factor $J_0^2(k_p\rho)$. For ions or for runaway electrons in the presence of drift or tearing turbulence, one may have $(k_p\rho) \sim 1$, so $D$ for these particles will be greatly reduced from previous estimates. This may provide an explanation, alluded to in Ref. 2, for the anomalously long confinement times of runaway electrons in tokamaks.

\textsuperscript{*}Work jointly supported by U.S. DoE Contract No. EY-76-C-02-3073 and U.S. AFOSR Contract No. F 44620-75-C-0037.

\textsuperscript{1}A.N. Kaufman, Phys. Fluids 15, 1063 (1972).


NEUTRAL BEAM HEATING CALCULATIONS FOR TORSATRONS*

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It has been demonstrated, both theoretically and experimentally, that tokamak plasmas can be heated efficiently through neutral beam injection. The status of this method is less clear for stellarator plasmas, although recent computational results imply heating efficiencies of more than 97% for tangential injection of the beam in a torsatron of moderate aspect ratio. Significant heating has been observed experimentally for tangential injection in the Cleo stellarator.1

These results have been obtained with a fully three-dimensional code originally developed by Dei-Cas2 for tokamak geometry and modified by S. Rehker3 to take into account stellarator fields and geometries. The method of calculation used in the code is based on the single particle guiding center equations and employs Monte-Carlo techniques to model the collisional transfer of the beam energy to the plasma. For a 16 field period \( \eta = 3 \) torsatron of minor radius 35 cm, major radius 350 cm and toroidal field of 30 kG with plasma density of \( 10^{14} \text{ cm}^{-3} \) and ion temperatures of 1 keV, 97% heating efficiency was obtained with tangential injection. The efficiency decreased to under 20% when the injection angle with respect to the magnetic axis was increased to 65 degrees. The results show that ionized particles born near the outer edge of the plasma tend to escape, but good confinement occurs near the plasma center. As the angle of injection is increased, one observes confinement only for those particles born well inside the plasma.

2J. Dei-Cas, Varenna School of Plasma Physics (1976).

*This work was supported in part by the National Science Foundation under grant ENG 77-14820 and in part by the U.S. Department of Energy under Contract No. ET-73-S-02-5069.
COMPUTER MODEL OF A SLOW RFP*

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ABSTRACT

An old code (G2M) has been adapted to the calculation of the slow diffusion of profiles to be expected of a reversed field pinch of ZT-40 size. The basic equations are those of Ref. 1, though the boundary conditions have been altered to allow for a resistive shell and the transport coefficients are enhanced, following Christiansen and Roberts in Suydam-unstable regions. The roles of impurities and neutrals are examined.


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Effect of Toroidal Curvature on Stability
Windows for MHD Kink Modes *

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As part of a parametric survey to investigate the behavior of ideal MHD instabilities in tokamaks, we are studying the behavior of the stable window for kink modes for different current and pressure distributions and aspect ratios. These should occur near \( nq \sim m - 1 \) with \( q \) the safety factor at the plasma-vacuum interface, \( n \) the toroidal mode number, and \( m \) the dominant poloidal mode number. In the large aspect ratio limit we find agreement with cylindrical calculations. The width of the stable window decreases with decreasing aspect ratio, and also with decreasing shear.

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†On loan from Westinghouse Research and Development Center.

The Goodness of Ergodic Adiabatic Invariants

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For a "slowly" time dependent Hamiltonian system exhibiting ergodic motion, the 2N dimensional phase space volume inside the hypersurface, Hamiltonian equals constant, is an adiabatic invariant. (This invariant has proven to be useful for discussing particle motion in field reversed geometries,\(^1\) and should have application to other plasma fusion problems where ergodic particle motion is prevalent.) It is shown that the error in the constant is diffusive and scales like \((\tau_c/\tau)^{1/2}\), where \(\tau_c\) is a certain correlation time of the ergodic motion, and \(\tau\) is the time scale over which the Hamiltonian changes.

\(^1\)R. V. Lovelace, Phys. Fluids (1979).
Fueling of a long-pulse divertor tokamak is modeled with the 1-D transport code PROCTR. Flow to the divertor plate is at the ion sound speed and cross-field diffusion in the scrapeoff is of the order of Bohm ($\sim 10^4 \text{ cm}^2/\text{sec}$). We consider fueling by neutral beams ($P_{\text{beam}} = 7 \text{ MW}, E_{\text{beam}} = 50 \text{ keV}$), gas and pellets of a machine with major radius $R_o = 150 \text{ cm}$, wall radius $a_w = 55 \text{ cm}$ and separatrix radius $a_s = 45 \text{ cm}$. Neutral beam fueling alone results in a scrapeoff transparent to impurities. Gas puffing thickens the scrapeoff but does not fuel the plasma when the puffing rate is limited by an assumed divertor pumping rate of $10^{22} \text{ sec}^{-1}$. Pellets are shown to fuel the plasma using existing technology ($v_{\text{pell}} = 1 \text{ km/sec}$). We conclude that gas puffing is needed to control the opacity of the scrapeoff to sputtered impurities while pellets are needed to fuel the plasma.

MODULATIONAL THEORY OF THE CUBIC NONLINEAR
SCHRÖDINGER EQUATION

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ABSTRACT

The one dimensional cubic nonlinear Schrödinger equation \((iE_t + pE_{xx} + q|E|^2E = 0)\)
has two well known solutions in the form of uniformly translating cnoidal
waves classified by the sign of the constant \(pq\). These waves are
determined, apart from phase constants, by the values of four parameters.
We use Whitham's averaged variational principle\(^1\) to derive a determined
quasilinear system of evolution equations for the values of these parameters
when they are regarded as slowly varying in space and time. The stability
of the waves is also determined by examining the quasilinear system for
local hyperbolic or elliptic character.

\(^*\)Work performed under the auspices of the U.S. Department of Energy by the
Lawrence Livermore Laboratory under contract number W-7405-Eng-48.

\(^1\)G. B. Whitham, Linear and Nonlinear Waves, (Wiley, New York, 1974)
TOROIDAL PINCH EQUILIBRIA WITH FLOW*

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The Morozov-Soloviev\textsuperscript{1} magnetohydrodynamic equations with ideal compressible
stationary flow are solved for some axisymmetric configurations. The reversed field
theta pinch is examined in particular. Rotation is found to have interesting
consequences on the forms of the equilibrium.

*Work performed under the auspices of the U. S. Department of Energy.

Coupling and Penetration of Whistler Waves in Inhomogeneous Plasma.† K. S. THEILHABER, MIT—Linear Excitation of Whistler waves in the lower-hybrid frequency range is described. These waves are excited at the plasma edge by electric fields perpendicular to those required for the lower-hybrid excitation. We consider coupling from a waveguide array and find power reflection as a function of array design and density gradient at the edge. We then consider propagation into the plasma and calculate field structure as a function of distance of penetration.

† Work supported by U. S. Department of Energy Contract (ET78-S-02-4682).
Orbit-Averaged Particle Codes for Long-Time Simulations
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A new method for efficient computer simulation of long time-scale phenomena has been proposed and has proved successful in one- and two-dimensional magneto-inductive particle codes. The method relies on orbit-averaging charge and current densities in Maxwell's equations before solving for the self-consistent electric and magnetic fields, in order to both filter out high-frequency phenomena and reduce the number of simulation particles necessary to adequately fill phase space. This activity is motivated by the desire to efficiently simulate evolution of plasma over long time intervals compared to particle orbit periods.

We have modified the one- and two-dimensional magneto-inductive particle codes MAGIC\textsuperscript{1} and SUPERLAYER\textsuperscript{2}. These codes use a Darwin model to calculate self-consistent fields from the plasma current provided by finite-orbit ions, assume charge neutrality, and neglect electron dynamics (appropriate assumptions for open magnetic field lines and $T_i \gg T_e$). SUPERLAYER also models neutral beam deposition, r-f, and other mirror physics. In the orbit-averaged codes, the plasma current is averaged over the cyclotron orbits of the ions. To approximately time-center the difference equations, we use a predictor-corrector method. We have achieved numerically stable code operation averaging over several cyclotron periods with one or two corrector iterations. Results of the new codes agree with those of their predecessors which do not orbit-average, but some physics improvements are evident. The new codes very effectively filter high-frequency noise associated with discreteness of the injection models used when studying neutral beam build-up to high beta and field-reversal. This results in cleaner and more realistic ion orbits. A factor of two reduction in the number of particles needed has been achieved, but this is problem dependent. Refinements such as temporal interpolation of the fields on corrector iterations and digital smoothing of the currents will be described, along with details of the equations, algorithms, and tests made so far.

   UCID-17795, Rev. 1 (1978)