ABSTRACTS

OF PAPERS PRESENTED AT THE

SHERWOOD THEORY MEETING

March 23-24, 1972

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

1972 SHERWOOD THEORY MEETING

UNIVERSITY OF CALIFORNIA

LOS ALAMOS SCIENTIFIC LABORATORY

23-24 MARCH 1972

1972 SHERWOOD THEORY MEETING SCHEDULE

WEDNESDAY, 22 MARCH 1972

4:00PM - 9:00PM --- Meeting Registration at Los Alamos Inn

THURSDAY, 23 MARCH 1972

7:45AM ----------	First Bus Departs Los Alamos Inn for Meeting in Physics Building Auditorium
8:00AM ----------	Second Bus Departure from Los Alamos Inn
8:30AM ----------	Meeting Commences
10:15AM ----------	Coffee Break
10:30AM ----------	Meeting Reconvenes
12:15PM ----------	Lunch at South Mesa Cafeteria
1:30PM ----------	Meeting Reconvenes
3:00PM ----------	Coffee Break
3:15PM ----------	Meeting Reconvenes
5:00PM ----------	Meeting Adjourns
5:30PM ----------	Bus Departs Physics Building for Los Alamos Inn
6:30PM ----------	Cocktail Hour at Los Alamos Inn
7:30PM ----------	Dinner at Los Alamos Inn

FRIDAY, 24 MARCH 1972

7:45AM ----------	First Bus Departs Los Alamos Inn for Meeting in Physics Building Auditorium
8:00AM ----------	Second Bus Departure from Los Alamos Inn
8:30AM ----------	Meeting Commences
10:15AM ----------	Coffee Break
10:30AM ----------	Meeting Reconvenes
12:15PM ----------	Lunch at South Mesa Cafeteria
1:30PM ----------	Meeting Reconvenes
3:00PM ----------	Coffee Break
3:15PM ----------	Meeting Reconvenes
5:00PM ----------	Meeting Adjourns
5:30PM ----------	Bus Departs Physics Building for Los Alamos Inn

1972 SHERWOOD THEORY SESSION SCHEDULE

Thursday, 23 March 1972

Session A — Chairman: K. R. Symon

8:30-10:15 AM

H. M. Agnew
"Welcome to Los Alamos"
J. L. Tuck
"Sherwood Welcome"

A1) P.H. Rutherford, H.P. Furth, and D.F. Duchs
"Neoclassical Transport in Tokamaks and the Effect of Neutral Gas"

A2) J. L. Johnson, E.A. Frieman, J.M. Greene, and K.E. Weimer
"Toroidal Effects on Kink Modes in Tokamaks"

A3) F.W. Perkins and J.M. Kindel
"Fast Wave Heating of Tokamak Plasmas"

A4) D. Dobrott and D.K. Bhadra
"Bifurcation and Stability of a Toroidal Equilibrium with Noncircular Cross Section"

A5) M.N. Rosenbluth and C.S. Liu
"Drift Wave in a Sheared Magnetic Field"

A6) A.H. Glasser and W.B. Thompson
"Neoclassical Plasma Transport in Axisymmetric Toroidal Systems"

10:15-10:30 AM ---Coffee

10:30-12:15 PM

A7) B. Coppi, E. Minardi, and D.C. Schram
"Resonant Scattering of Trapped Particles by Toroidal Plasma Modes"

A8) W.E. Drummond
"The Texas Theoretical Program"

A9) F.L. Hinton and M.N. Rosenbluth
"Transport Properties of a Toroidal Plasma at Low-to-Intermediate Collision Frequencies"

A10) J.C. Wiley and F. L. Hinton
"Numerical Solution of the Radial Neoclassical Transport Equations"

A11) T. Gladd, L. Cheung, and W. Horton
"Drift Wave Instabilities in Tokamaks"

A12) A. B. Macmahon
"MHD Stability of Tokamak Plasmas"

A13) D. W. Ross and N. T. Gladd
"Effect of Shear on the Stability of the Dissipative Trapped Ion Mode"

A14) R. D. Hazeltine, F. L. Hinton, A. B. Macmahon, and M. N. Rosenbluth
"Banana Transport Coefficients for a Torus of Arbitrary Aspect Ratio"

A15) H. R. Strauss
"Corrections to Toroidal Diffusion Coefficients"

12:15-1:30 PM--- Lunch

Session B — Chairman: M. N. Rosenbluth

1:30-3:00 PM

B1) D. J. Sigmar
"Neoclassical Resistivity of Toroidal Plasma due to Momentum Transfer from Circulating to Trapped Electrons"

B2) G. Guest
"A Summary of the Theoretical Plasma Physics Program at Oak Ridge National Laboratory"

B3) J. T. Hogan and R. A. Dory
"Energy Flow in Tokamaks"

B4) J. F. Clarke
"Fast Ion Cooling in Tokamaks"

B5) E. C. Crume and H. K. Meier
"Simulation of the Nonlinear Evolution of Loss-Cone Flute Instabilities"

3:00-3:15 PM--- Coffee

3:15-5:00 PM

B6) T. J. Fessenden, R. V. Lovelace, and R. N. Sudan
"Studies of Equilibria and Stability of Strong E-Layers"

B7) J. A. Byers, J. P. Holdren, J. Killeen, A. B. Langdon, A. A. Mirin, and M. E. Rensink
"Results of Computer Simulation of Pulse Trapping and Pulse Stacking in Astron Using the Superlayer (r-z) Code"

B8) J. A. Byers
"Computer Simulation Results from an r-θ Code on the Negative Mass Instability of a Relativistic Electron Beam"

B9) A. B. Langdon, M. E. Rensink, and T. J. Fessenden
"Injection Current Limitation in Astron"

B10) M. E. Rensink
"Thin E-Layer Equilibria"

B11) E. P. Lee
"Kink Mode of a Relativistic Beam"

B12) H. L. Berk and L. D. Pearlstein
"Theoretical Aspects of Astron Build-Up by Multiple Pulses"

B13) H. L. Berk, R. C. Harding, and D. E. Baldwin
"Nonlinear Line Tying of Flute Modes"

B14) D. E. Baldwin
"Enhanced Particle Scattering in 'Loss-Cone' Plasmas"

Friday, 24 March 1972

Session C — Chairman: G. E. Guest

8:30-10:15 AM

C1) H. P. Zehrfeld and B. J. Green
"Stationary Toroidal Equilibria at Finite Beta"

C2) C. L. Hedrick and D. B. Nelson Clarke –
"Finite Beta Equilibria and Stability for the Canted Mirror"

C3) R. L. Morse
"Summary of LASL Theoretical Effort and Topics from the Laser Fusion and Theta Pinch Programs"

C4)-C6) J. P. Freidberg
"Status of Scyllac Theory"

C7) D. W. Forslund, J. M. Kindel, and E. L. Lindman
"Parametric Instabilities of Whistlers"

10:15-10:30 AM---Coffee

10:30-12:15 PM

C8) J. M. Kindel and H. Okuda
"Parametric Instabilities at Low Frequencies Above the Lower Hybrid"

C9) H. Grad
"Spectrum and Low Shear MHD Stability"

C10) G. Bateman
"Long-Wavelength Ion-Acoustic Waves"

C11) H. Weitzner
"Toroidal High Beta Equilibria"

C12) G. O. Spies
"Non-Linear Energy Principle for Closed-Line Configurations"

C13) D. C. Stevens
"Compression of a Tokamak Without Walls"

12:05-1:30 PM---Lunch

Session D **Chairman: L. D. Pearlstein**

1:30-3:00 PM

D1) G. K. Morikawa
"Slightly Anistropic Pressure Toroidal Equilibria with Nearly Spherical Plasma-Vacuum Interfaces"

D2) G.A. Emmert and R. K. Richards
"Role of Dissipation in the Feedback Stabilization of Reactive Instabilities"

D3) C. Mercier
"Localized Criterion of Stability in M.H.D."

D4) R.J. Rahn and J.E. McCune
"Electrostatic Modes of a Non-Uniform Plasma"

D5) G. Laval
Title not availabe

D6) C.K. Chu, H.C. Lui, and R.T. Taussig
"An MHD Code for Plane and Axisymmetric Flow"

3:00-3:15 PM---Coffee

3:15-5:00 PM

D7) O. Buneman
"Subgrid Resolution by Means of Splines"

D8) E. Ott, J.B. McBride, J.P. Boris, and J.H. Orens
"Theory and Simulation of Turbulent Ion Heating"

D9) D. Montgomery
"Two-Dimensional Vortex Motion, Guiding Center Plasmas and "Negative Temperatures" "

D10) G. Knorr
"Poor Man's Plasma Simulation"

D11) J.M. Dawson and W.L. Kruer
"A Possible Plasma Laser"

D12) W.L. Kruer and J.M. Dawson
"Mode Coupling of Electron Plasma Oscillations by Finite Amplitude Ion Fluctuations"

D13) N.K. Winsor
"Fokker-Planck Investigations of the Wet Wood Burner Concept"

D14) H. Okuda
"Plasma Diffusion Across a Magnetic Field"

Supplemental Abstracts

S1) R.L. Dewar
"An Analytic Investigation of the Saturation of Kinetic Plasma Instabilities by Particle Trapping"

S2) B. Newberger and P.H. Rutherford
"Finite Beta Effects on Low-Frequency Instability in Toroidal Plasma"

S3) R. Dagazian, B. Coppi, and R. Gajewski
"High Current Density Plasma Confinement Configuration"

ABSTRACTS

Neoclassical Transport in Tokamaks and the Effect of Neutral Gas*

P. H. Rutherford, H. P. Furth, and D. F. Düchs†
Plasma Physics Laboratory, Princeton University
Princeton, N. J. 08540

ABSTRACT

A complete neoclassical code, including mass transport and both banana and plateau regimes is now in operation. The numerical results agree fairly well with a simplified 3-regime code,[1] which neglects mass transport, (The main differences result from the rounding of the transition from banana to plateau regime in the present code which reduces ion thermal transport.) As reported for the simple code, the degree of peaking of the temperature and current profiles is controlled by the thermal instability: depending on initial conditions, one can achieve either a peaked, flat, or dished T_e-distributions. The "trapped-particle pinching" does not emerge as a discernible additional mechanism for the contraction of the current channel. Results of the present code differ substantially from those reported in Ref. 1 for a 1-regime (banana) code with mass transport, particularly near the axis, where the pure banana transport equations become singular.

The addition of a "cold" neutral gas influx of the experimentally observed magnitude causes substantial peaking of the temperature and current profiles. The present code allows for four generations of "hot" charge-exchange neutrals; however a single generation gives the correct plasma profiles to a good approximation. The effect of various anomalous enhancements of the transport coefficients has been studied with a view to fitting the detailed experimental profiles.

Thermal instability dominant

* Work supported by U. S. Atomic Energy Agency Contract AT(11-1)-3073. Use was made of computer facilities supported in part by National Science Foundation Grant NSF-GP 579.

† Present address: Institut für Plasmaphysik, Garching, Germany.

1 D. F. Düchs, H. P. Furth, and P. H. Rutherford, Fourth Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, June 1971, paper CN-28/C-4.

Toroidal Effects on Kink Modes in Tokamaks*

J. L. Johnson†, E. A. Frieman, J. M. Greene, and K. E. Weimer
Plasma Physics Laboratory, Princeton University
Princeton, N.J. 08540

ABSTRACT

We previously reported[1] that finite-pressure effects can affect the value of the safety factor at which a toroidal column becomes unstable. To further investigate this effect we have carried through a calculation where a/b, the ratio of the radii of the current-carrying column and the container, is of order $\varepsilon^{-2(m+1)}$ with ε the inverse aspect ratio. With this ordering curvature effects couple modes in which the zeroth-order ξ_r's vary as $\sin(m\theta-n\phi)$ and $\sin[(m+1)\theta-n\phi]$ if q, the safety factor, is equal to $m/n+O(\varepsilon)$. This mode coupling enhances the instability. Since curvature effects are significant in third order in this calculation, it provides a better model for studying the physical mechanisms occurring in kink modes than the usual ordering where one must go to fourth order or higher.

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*Supported by the U.S.A.E.C. Contract AT(11-1)-3073.

†On loan from Westinghouse Research Laboratories.

[1] E. A. Frieman, K. E. Weimer, J. M. Greene, and J. L. Johnson, Bull. Am. Phys. Soc. II, 16, 1223 (1971).

Fast Wave Heating of Tokamak Plasmas*

F. W. Perkins and J. M. Kindel†
Plasma Physics Laboratory, Princeton University
Princeton, N.J. 08540

ABSTRACT

The ST Tokamak represents plasma that is large enough and has sufficiently high density so that it can be a resonant cavity for the magnetoacoustic wave at the ion cyclotron frequency. The large Tokamaks (such as ST) are the first low-β toroidal confinement devices to achieve this condition. We demonstrate that the magnetoacoustic wave suffers ion cyclotron damping when $\omega=\omega_{ci}$ as a result of ion thermal effects. The damping decrement is $\gamma=(3\sqrt{\pi})^{-1}\ (k_\perp c/\omega_{pi})^4\ k_\parallel (2T_i/M)^{1/2}$. The magnetoacoustic cavity-resonance thus becomes an efficient way to heat ions in the center of Tokamak devices. We will discuss coil structures to launch the magnetoacoustic wave and compare the relative heating rates of electrons and ions.

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*Work supported by U.S. A.E.C. Contract AT(11-1)-3073.

†Now at Los Alamos Scientific Laboratory.

Bifurcation and Stability of a Toroidal Equilibrium with Noncircular Cross Section

D. Dobrott and D. K. Bhadra

Gulf General Atomic Company
San Diego, California 92112

An explicit MHD toroidal equilibrium with noncircular cross section is examined for the case where the current density is a nonlinear function of the poloidal magnetic flux. The model is appropriate for a sharply peaked pressure profile. The resulting nonlinear eigen-value problem is solved by perturbation methods which demonstrate that a multiplicity of neighboring equilibria can co-exist for the same poloidal beta, current gradients, and boundary conditions. The clustering of these solutions increases with the "eccentricity" of the plasma cross section. Stability analysis of these equilibria have been made. The equilibria are unstable to arbitrary displacements in the cross-sectional plane, when the current density gradient is a highly concave function of the poloidal flux.

Work supported by the U.S. Atomic Energy Commission, Contract No. AT(04-3)-167, Project Agreement No. 38.

Drift Wave in a Sheared Magnetic Field

M. N. Rosenbluth and C. S. Liu

ABSTRACT

The stability conditions for the drift wave in a sheared magnetic field are derived for the modes driven unstable by the finite ion gyroradius effect, the electron temperature gradient and the parallel current, respectively. The nonlinear effect of the temperature gradient mode is discussed as a possible explanation for the anomalous skin effect.

NEOCLASSICAL PLASMA TRANSPORT IN AXISYMMETRIC TOROIDAL SYSTEMS*

Alan H. Glasser and William B. Thompson

Department of Physics
University of California, San Diego
La Jolla, California

The variational method for calculating transport processes in the neoclassical regime, developed for Tokamak by Rosenbluth, Hazeltine and Hinton, is generalized to arbitrary axisymmetric toroidal systems. The use of the scalar magnetic potential, as in Rutherford's treatment of the Lorentz model, is eliminated, allowing treatment of non-vacuum systems. As an example, results are given for Doublet II.

*Supported by the United States Atomic Energy Commission under Contract No. AT(04-3)-34 Project Agreement 85-15.

Abstract Submitted

for the Sherwood Theoretical Meeting

Los Alamos Scientific Laboratory

23-24 March 1972

Resonant Scattering of Trapped Particles by Toroidal Plasma Modes

B. Coppi, E. Minardi and D. C. Schram

MIT-FOM* Collaboration

Ion temperature gradient driven modes can be excited in two-dimensional plasma configurations under less restrictive conditions than in one dimensional plasmas.

The resonant interaction of modes of this kind with magnetically trapped ions is analyzed and shown to significantly affect their orbits. It is suggested that process of this kind may provide an effective particle scattering mechanism in regimes where the single-particle collision mean free paths are considerably longer than the periodic particle excursions.

*Euratom-FOM Association, FOM Instituut voor Plasma-Fysica, Jutphaas, Nederland

THE TEXAS THEORETICAL PROGRAM

W. E. Drummond

Center for Plasma Physics

University of Texas at Austin

Transport Properties of a Toroidal Plasma at Low-to-Intermediate Collision Frequencies

F. L. Hinton
Center for Plasma Physics
University of Texas at Austin

and

M. N. Rosenbluth
Institute for Advanced Study
Princeton University

The neoclassical plasma transport coefficients, for large aspect ratio toroidal magnetic confinement systems have been calculated in the regime of low-to-intermediate collision frequency. The problem of solving the linearized drift kinetic equation, and calculating the transport coefficients, has been formulated as a variational principle. A maximal form of the variational principle has been used to solve the kinetic equation by a finite difference technique. A correction to the asymptotic banana regime solution has been obtained with the Wiener-Hopf technique, to solve the kinetic equation in the boundary layer between the trapped and untrapped regions of phase space. The transport coefficients are monotonic functions of collision frequency, with a gradual transition from the banana regime to the plateau regime. Analytical formulas have been fitted to the numerically computed results.

Numerical Solution of the

Radial Neoclassical Transport Equations

J. C. Wiley and F. L. Hinton
Center for Plasma Physics
University of Texas at Austin

A computer code has been developed to study the decay of a quiescent plasma in an axisymmetric toroidal confinement system. The radial transport equations of neoclassical theory are solved numerically to give the time evolution of the density, electron and ion temperatures, and the poloidal magnetic field. A predictor-corrector implicit finite difference technique was used, which affords considerable flexibility in the choice of transport coefficients. The consequences of the trapped particle pinch effect and the bootstrap current have been examined. The use of the full banana-to-plateau coefficients is shown to give somewhat different results than with banana coefficients only.

Drift Wave Instabilities in Tokamaks

Tom Gladd, Lung Cheung, W. Horton
Center for Plasma Physics
University of Texas at Austin

The critical density and temperature gradients for drift wave instability are found for a variety of plasma conditions. The scaling laws for the threshold conditions are obtained, and some consequences for stable plasma confinement are discussed. We show that the critical gradient for instability is decreased in the second order by a small tokamak current, but for currents exceeding the condition for ion-sound instability there is a marked first order decrease in the critical density gradient. The effect of finite beta is to couple the drift and Alfven wave which acts to stabilize the drift wave at sufficiently large beta. The relationship of beta and the shear for stability is given and contrasted with a recent publication by Mishin.[1]

The effects of trapped particles are investigated by returning to the Vlasov equation and developing a systematic stability theory based on the appropriate expansion of the Vlasov equation. The resonant interaction of the low bound frequency electrons with the drift wave is calculated to obtain growth rates and instability conditions.

1. E. V. Mishin, JETP 32, 1218 (1971).

Effect of Shear on the Stability of the

Dissipative Trapped Ion Mode

David W. Ross and Nevel T. Gladd,
Center for Plasma Physics
University of Texas at Austin

Numerical solutions of the radial equation for the collisional trapped ion mode are presented. Previous work[1,2] is extended to include a detailed treatment of the untrapped ion resonances near rational flux surfaces. Model profiles of the rotational transform ($2\pi/q$), the density, and the temperature are chosen to simulate those of a large Tokamak. It is found that, owing to the untrapped particles, the modes tend to be localized between the rational surfaces, and their eigenvalues doe not depend strongly on the boundary condition at the edge of the plasma. For large azimuthal wave numbers (lq/r) the rational surfaces become closely spaced. The combined effect of the resulting enhanced Landau damping and short radial wavelength tends to reduce the growth rates. Effects of the finite banana widths of untrapped ion orbits and the structure of the azimuthal integral equation are briefly discussed.

1. D. W. Ross and N. T. Gladd, Bull. Am. Phys. Soc. 16, 1240 (1971).
2. D. W. Ross and C. W. Horton, Jr., Phys. Rev. Lett. 28, 484 (1972).

Banana Transport Coefficients for a

Torus of Arbitrary Aspect Ratio

R. D. Hazeltine, F. L. Hinton, A. B. Macmahon
Center for Plasma Physics
University of Texas at Austin

and

M. N. Rosenbluth
Institute for Advanced Study
Princeton University

Previous calculations of neoclassical transport coefficients have been accurate only to lowest order in the square root of the inverse aspect ratio, $(r/R)^{\frac{1}{2}}$, although this parameter is as large as 1/3 on present tokamaks and may be larger on future devices. We remove the restriction to large aspect ratio by calculating, from neoclassical theory, the transport coefficients for a torus of arbitrary aspect ratio, in the banana regime. Our procedure is to consider the limit $r/R \rightarrow 1$, in which virtually all particles on the outer surface of the plasma are trapped, and in which the transport calculation is particularly simple. From the limit it is possible to infer reliable values at intermediate aspect ratio for all the transport coefficients except the electrical conductivity; the exceptional case is treated by a more detailed analysis of both $\left(\frac{r}{R} \rightarrow 0 \text{ and } \frac{r}{R} \rightarrow 1\right)$ aspect ratio limits. Although the transport coefficients were calculated using magnetic surfaces obtained from particular solutions of the MHD equilibrium equations, we believe that the results are not sensitive to these particular choices.

Corrections to Toroidal Diffusion Coefficients

H. R. Strauss
Center for Plasma Physics
University of Texas at Austin

The toroidal diffusion coefficients are corrected to take into account the effects of magnetic surfaces that are not circular and concentric, but displaced and elliptical.

In the plateau regime, for example, the diffusion coefficient is corrected by a factor

$$\left(1 + \frac{\rho}{2} \frac{d\varkappa}{d\rho} - 2K\right)$$

where $\varkappa$ is the ellipticity and ρ is the average radius of the flux surface. At finite β, this reduces the diffusion.

$$r = \rho(1 - \Delta\cos\theta - K\cos 2\theta)$$

Abstract Submitted
for the Sherwood Theoretical Meeting
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Neoclassical Resistivity of Toroidal Plasma due to Momentum Transfer from Circulating to Trapped Electrons

D. J. Sigmar*

The d-c- resistivity of a near collisionless Tokamak plasma has recently been the subject of several papers[1),2),3)] with as many different results. In Ref. 3, the conductivity derived from an entropy principle seems not to coincide with that derived from the quoted electron distribution. Based on physical ideas proposed by Galeev[1)] and by Coppi[2)], the guiding center kinetic equation for the electrons in the banana regime is solved analytically including the effect of collisions between circulating and trapped electrons. This effect distorts the electron distribution and the ensuing conductivity is

$$\sigma_{\parallel} = \sigma_{\text{Spitzer}} \left[1 - 1.90\sqrt{\frac{r}{R}}\right].$$

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*Supported by USAEC Contract AT(11-1)-3070 at the Research Laboratory of Electronics at M.I.T.

1) Hinton and Oberman, Galeev, Salat, Daybelge in 1969-71.

2) Callen, Coppi et al, IAEA Conference in Madison 1971.

3) Rosenbluth, Hazeltine and Hinton, Phys. Fluids 15, 116 (1972).

$\sigma_{\parallel} = \sigma_{sp}(1 - 1.95\sqrt{\frac{a}{R}})$ where $n_T/n_0 = \sqrt{2r/R}$

Coppi - $\eta_{\parallel} = \eta_{sp}\left(1 + \alpha \frac{n_T}{n_0}\right) / \left(1 - \frac{n_T}{n_0}\right)$ $\alpha > 0$

A SUMMARY OF THE THEORETICAL PLASMA PHYSICS PROGRAM AT OAK RIDGE NATIONAL LABORATORY

Gareth Guest

Three Programs

1. ORMAK - 3½ aspect ratio - allows access to collisionless regime - 240 kilowatts of power
John Clarke auxiliary heating - injection of fast neutral atoms

2. Magnetic Mirror Confinement

3. Elmo Bumpy Torus - single particle confinement
guiding center equilibrium
Stability a) of the (mirror - confined) annulus - relativistic, high-β, anisotropic, line-tied
b) of the circulating plasma

ENERGY FLOW IN TOKAMAKS*

J. T. Hogan and R. A. Dory
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

We will discuss the influence of several factors on non-turbulent energy and particle flow in the Tokamaks of today's experiments and tomorrow's hypothetical reactors. The factors are: 1) the most recent transport coefficients calculated by Hinton and Rosenbluth, 2) radiative cooling and nuclear heating, 3) heating by energetic neutral injection, and 4) effects of neutrals and impurities.

Calculations have been done for both the initial turn-on of the discharge (post-ionization) and the long term steady-state conditions.

As a side study we have run comparisons with results reported by Rutherford at the IAEA 1972 Meeting.

Prominent features in 5 – 10 KEV region
1. Large heat capacity of big machine
2.

*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

FAST ION COOLING IN TOKAMAKS*

J. F. Clarke
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

Injection of energetic neutrals[1] into tokamak plasmas creates a large energy reservoir for plasma heating. The transfer of this energy to the plasma particles (i,e) can be shown to depend on the effective electric field seen by the fast ions (α). This effective electric field E^* depends on the externally applied heating field E, the trapped particle resistivity, η_{TP} and the bootstrap current J_B. To order $\sqrt{r/R}$

$$E^* = E \left\{1 - \frac{Z_\alpha^2}{Z_i} \frac{\eta_S}{\eta_{TP}}\right\} + \eta_S J_B$$

where η_S is the Spitzer resistivity. The first term represents the competition between the applied field and the friction due to the electron ohmic current.[2] The second term represents the friction produced by the gradient driven currents in the plasma.

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*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1] G. G. Kelley, O. B. Morgan, L. D. Stewart, W. L. Stirling, and H. K. Forsen, ORNL-TM-3594 (October 1971).

[2] H. P. Furth and P. H. Rutherford, to be published.

SIMULATION OF THE NONLINEAR EVOLUTION
OF LOSS-CONE FLUTE INSTABILITIES*

E. C. Crume and H. K. Meier
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

The evolution of linearly unstable loss-cone flute modes is studied by computer simulation in a model with one spatial coordinate and two velocity coordinates.[1] These modes depend on the coupling of Bernstein waves, supported by a hot, mirror-confined plasma, and the lower hybrid wave of a warm Maxwellian plasma component.[2] Results are given for cases where the relative density and temperature of the warm plasma component are systematically varied. The modes are stabilized by a rapid heating of the warm component giving saturation amplitudes substantially lower than those predicted by older analytic theories[3] but in reasonable agreement with a recent analytic theory developed at ORNL.[1,4] This result is encouraging since the containment of multicomponent plasmas in mirror machines is strongly affected by the maximum amplitude of these and closely related instabilities.

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*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1]E. C. Crume, H. K. Meier, and O. Eldridge, to be published. (Presently available as ORNL-TM-3675.)

[2]G. E. Guest, W. M. Farr, and R. A. Dory, Phys. Fluids 13, 3038 (1970).

[3]While no older theories are strictly applicable in the ranges of plasma parameters studied, see R. E. Aamodt and S. E. Bodner, Phys. Fluids 12, 1471 (1969) and D. J. Sigmar and J. D. Callen, Phys. Fluids 14, 1423 (1971). Also see the simulation studies in J. A. Byers and M. Grewal, Phys. Fluids 13, 1819 (1970).

[4]O. Eldridge, E. C. Crume, and H. K. Meier, to be published. (Presently available as ORNL-TM-3642.)

Studies of Equilibria and Stability of Strong E-Layers*,

T. J. Fessenden**, R. V. Lovelace, and R. N. Sudan, Laboratory of Plasma Studies, Cornell University, Ithaca, New York

The dependence of the gross dimensions of long E-layers on the layer strength ζ, external magnetic fields, conducting tank walls, and the process of formation is discussed in terms of overall conservation of energy, radial and axial pressure balance, and flux conservation. The role of the single particle canonical angular momentum is discussed.

We examine the stability of strong E-layers ($\zeta \simeq 1$), including the effect of the self-fields, to modes of the form $\exp(im\theta - i\omega t)$. A model for the E-layer previously found applicable for the precessional m=1 mode[1] is now extended to the kink mode, m=2. Because of the higher frequency of this mode it is necessary to include the induced electric field previously neglected. The effect of (a) currents induced in the background plasma in screening the perturbed fields and (b) stabilizing effects of an external toroidal magnetic field are also taken into account.

* Work supported in part by U. S. Atomic Energy Commission Contract AT(30-1)-4077.

** On leave from Lawrence Livermore Laboratory, Livermore, California.

1 R. V. Lovelace and R. N. Sudan, "Precession of a Strong E-Layer", Cornell University, Laboratory of Plasma Studies Report LPS No. 82, January, 1972 (to be published).

RESULTS OF COMPUTER SIMULATION OF PULSE TRAPPING AND PULSE STACKING IN ASTRON USING THE SUPERLAYER (r-z) CODE*

J.A. Byers, J.P. Holdren, J. Killeen, A.B. Langdon, A.A. Mirin and M.E. Rensink

Lawrence Livermore Laboratory, University of California
Livermore, California

March 3, 1972

ABSTRACT

Recent work has extended and confirmed the results on pulse stacking reported at Madison. The results in detail are dependent on the resistor configuration, degree of charge neutralization and the external well shape, but for all of the runs completed to date, the ζ increase runs into a definite saturation due to a lengthening or axial expansion of the layer. Examination of v_z-z phase space shows that there is a phase space exclusion principle operating: the incoming pulse can enter an already occupied region only by pushing out all of the trapped particles. Whatever radial motion is occuring is apparently averaged out. For neutralized cases this general behavior is insensitive to optimization attempts. The axial expansion appears to be a fundamental process, usually more important in the trap region than the interaction of the pulse with the resistors. Furthermore, there is little dependence of $\zeta_{final}/\zeta_{1st}$ on the amplitude of the injection current; the maximum ζ achieved for 600A, 800A, 1800A was 0.45, 0.60, 1.2, each approximately 2—3 times ζ_{1st}. Also, the presence of a large external B_θ can only modify the detail and does not prevent saturation. Christofilos has suggested that for an unneutralized layer the axial expansion may become less important than the resistor interaction. Following pulse stacking in vacuum the layer would be neutralized adiabatically, hopefully leading to a higher final ζ than in a neutralized case due to the reduction in axial expansion. This is under investigation with the code. Injection of an unneutralized pulse with a z velocity of 0.1c results in a very long layer, several meters in extent. In contrast, with an injection velocity of 0.01c-0.02c an unneutralized pulse is observed to pinch very tightly with a length (FWHM) of less than 50 cm. This behavior is very sensitive to the precise resistor configuration. Initial results on stacking with these tightly pinched unneutralized layers indicate that stacking is limited by radial expansion and consequent particle loss to the resistors.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

COMPUTER SIMULATION RESULTS FROM AN r-θ CODE ON THE NEGATIVE MASS INSTABILITY OF A RELATIVISTIC ELECTRON BEAM

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March 4, 1972

ABSTRACT

The model is a 2-D(r-θ) superparticle model, fully electromagnetic and relativistic. The electron beam is centered at a radius r_o, but energy spread is allowed, giving a radial extent to the beam. The system is bounded at both inner and outer radius by perfectly conducting walls. The resistors at intermediate radii, as in Astron, are not included, but theoretically they cannot help to stabilize the mode. The code reproduces, in the appropriate limits, all features of the linear theory. For large $\zeta(\gtrsim 0.1)$ and for zero frequency spread the linear growth rate α, in the 2-D model approaches order of $\omega_o=eB/(m\gamma)$. The amount of frequency spread required to linearly stabilize the modes goes as $\delta\omega/\omega_o \approx 2\alpha/(n\omega_o)$ where n is the azimuthal mode number . Thus, in the Astron geometry, where the maximum $\delta\omega/\omega_o = (r_2 - r_1)r_o < 1$, frequency spread alone cannot stabilize the high ζ cases. The code results confirm this. Observation of nonlinear development in the code shows wall loss for $\zeta \gtrsim 0.10$. External B_θ will not stabilize the mode but can prevent the wall loss. Cold plasma, $n_p > 10^{10}$ can stabilize the instability. The predictions of the code then are that only a sufficiently dense cold plasma or a sufficiently strong external B_θ will prevent wall loss from high ζ negative mass modes. Wall loss should be an expected result from this model since it has high linear growth rates for the large wavelength modes and little room is allowed for radial expansion. The question remains as to the deficiencies of the r-θ model as it is applied to Astron which is z-dependent to an extreme degree in some phases— at injection in particular. At the present we have only conjectured that the r-θ model probably overestimates the wall loss, but quantitative information is lacking. In this regard, even a linear theory of a z-dependent model, including the response of plasma along B lines, is still lacking. If a z-dependent theory were to predict considerably lower linear growth rates then the wall loss could also be expected to be much less or even completely absent. Also, in this case lower values of frequency spread would stabilize the mode. For example, if we allow a maximum frequency spread $\delta\omega/\omega_o \approx 0.5$ for Astron geometry, then for stability to be obtained z-dependent effects would be required to reduce α by a factor of 1.2 for $\zeta = 0.1$ or by a factor of 4 for $\zeta \gtrsim 1.0$. It should be pointed out that in Astron a settled unneutralized layer with a $\zeta \approx 0.07$ has been achieved, with a $\delta\omega/\omega_o = 0.25$, at most. This data indicates that z-dependent effects are reducing α by at least a factor of 2. A linearized z-dependent computer model appears quite feasible and will be constructed shortly.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

INJECTION CURRENT LIMITATION IN ASTRON*

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Livermore, California

March 3, 1972

ABSTRACT

In a vacuum there appears to be a limit on the current which can be injected into the Astron tank. To see this we consider a model which treats only the axial motion of injected rings of current and charge. As a ring enters at one end of the cylindrical tank, two opposing forces act on it: (1) a magnetic attraction by other current rings already in the tank, and (2) an electrostatic repulsion due to the uncompensated space charge of the other rings in the tank. For some resistor and wall configurations there is a net repulsive force which tends to slow down or reverse the directed axial motion of the newly injected electrons. The theory gives a maximum injection current comparable to the accelerator currents being used in the experiment. The limiting mechanism can be destroyed by focussing due to resistors, which partially cancel the space-charge repulsion. The results of simulation experiments on these phenomena agree with the theory.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

THIN E-LAYER EQUILIBRIA*

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Livermore, California

March 1, 1972

ABSTRACT

We study the equilibrium of an infinitely long, thin E-layer under the influence of external and self-fields. Three constraints are imposed: (1) the canonical angular momentum P_θ is constant, (2) the total energy per particle is constant, and (3) there are perfectly conducting cylindrical walls both inside and outside the E-layer. The results can be summarized as follows: (1) with complete charge neutralization, as the number of particles N increases, the E-layer radius R_e tends toward the value $R_e = \sqrt{R_W R_C}$ where R_W and R_C are the radii of the outer and inner conducting walls, respectively; (2) the total kinetic energy per particle decreases as N increases; (3) the axial temperature increases with N so as to maintain stability with respect to the tearing mode; and (4) the current per unit length saturates at a value well beyond that required for field reversal.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

KINK MODE OF A RELATIVISTIC BEAM*

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March 6, 1972

ABSTRACT

In order to evaluate the possibilities of confining and heating a plasma with a relativistic electron beam, an analysis of the stability of its kink (m = 1) mode is carried out. The essential feature is the treatment of the beam as a distinct component with finite inertia rather than as a plasma current. To simplify the analysis the plasma background is assumed to be a uniform and pressureless fluid obeying the laws of perfect magnetohydrodynamics, while the beam is treated as a rigid body. To simulate a toroidal geometry periodic boundary conditions are imposed in the axial (z) direction of a straight cylindrical volume. The walls of the cylinder have perfect conductivity. A strong field B_z is imposed, but the beam supplies the only poloidal field B_θ. It is found that the stability condition of Kruskal and Shafranov ($\iota < 2\pi$) obtains, so the beam intertial effects have not improved stability in this case.

*Work performed jointly under the auspices of the U. S. Atomic Energy Commission and the Advanced Research Projects Agency, Order No. 1377.

grad – Los Alamos –

– Low shear MHD instabilities – go between Suydam and finite-beta

linear in parameter instead of exponentially small if you pick
– the wrong normalization to (e.g. $\int \xi_1^2$

–

local

small growth rate

$$\Gamma^2_{max} = \frac{-p'}{\left(\frac{p'}{\gamma p} + \frac{q'}{q}\right)} \frac{B_z^2}{B^2 + \gamma p} (k + m\mu)^2$$

→ shear

→ Suydam

$\Gamma^2 = -\omega^2$
look at small ω rather than magnitude –

$p' < 0$ unstable = p', Suydam, Interchange unstable

$p(r)$, $q(r) = \int \frac{dz}{B_z}$, $\mu = \mu_0 + \epsilon \mu_1(r)$

increased shear for these localized (?) makes things worse –

some nonexponential instabilities – occur in all toroidal except pressureless plasma $\nabla p = 0$
Growth ~ $A + Bt$ ξ_{tt}

Diffusion
1.) curl $E = 0$ – even at low β is not justified
Nurnberg – recent paper – says Furth & Rosenbluth model can't be justified

THEORETICAL ASPECTS OF ASTRON BUILD-UP BY MULTIPLE PULSES

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March 6, 1972

ABSTRACT

The time asymptotic response to an incoming current source in the presence of a partially built up E-layer in the Astron geometry is examined. A one-dimensional Vlasov model which considers only motion in the axial direction (perpendicular to the current direction and tank radius) is studied . The approximation is valid provided $\omega_{\beta z} \ll \omega_{\beta r}$ (the respective betatron frequencies) a condition which generally is expected to prevail during the early stages of pulse stacking. It is shown that: (1) A backcurrent is induced in the E-layer for incoming velocities (V) in excess of the maximum E-layer (V_c). This backcurrent which peaks at $V = V_c$ attenuates the interaction with resistors. (2) For $V < V_c$ a new dissipation mechanism (Landau damping) occurs so that the incoming pulse transfers energy to the E-layer as well as to the resistors. (3) The transfer of energy to the layer leads to axial expansion which competes with contraction due to the addition of current. The net effect for a neutralized layer can lead to a saturation in E-layer strength, a behavior encountered in the numerical experiment. For the unneutralized layer, however, this situation is not nearly as severe and there appears to be a parameter range where stacking should occur.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

NONLINEAR LINE TYING OF FLUTE MODES

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Livermore, California

March 6, 1972

ABSTRACT

It has been suggested that the nonlinear field saturation levels of flute modes in mirror machines is determined by the electron temperature. This limitation arises because there are induced strong nonlinear end currents to a grounded wall when the perturbed field potential becomes comparable to the electron temperature. To investigate this effect, we have built a one-dimensional plasma simulation model, wherein electron dynamics along a field line is treated nonlinearly, while the ions, which are loss cone unstable, are mocked up by a linear partial differential equation. Our system is linearly unstable, and saturation can only occur through the action of the electron end currents. To date, we have found that the end currents temporarily prevent growth when the perturbed potential becomes comparable to the electron temperature. However, after some time instability reappears. Currently, we are searching for parameter ranges that can totally quench the instability.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

ENHANCED PARTICLE SCATTERING IN "LOSS-CONE" PLASMAS*

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March 6, 1972

ABSTRACT

A kinetic theory of a finite plasma subject to the Rosenbluth and Post[1] convective loss cone instability is developed. As a calculation of the fluctuation level in an unstable plasma, it has the advantage over such a calculation for absolute instabilities in that fluctuation amplitudes are limited without involving a change in the particle distribution function. Thus there exists a steady state response to a source of particles such as a mirror machine filled by neutral injection. We find that when $\omega_{pe}^2 << \omega_{ce}^2$ the ion-ion scattering time scales like $\tau_{ii} \sim \tau_{sp}(1 + (T_i^3 m_e/T_e^3 m_i)^{1/2} A)^{-1}$, where $A = \exp(2|\mathrm{Im}\int k_{||} dz|)$ is the amplification of the convecting wave crossing the system, and τ_{sp} is the Spitzer value. Thus for $T_e/T_i \lesssim (m_e/m_i)^{1/3}$, particle loss rates will be enhanced over the classical value even if less than a single e-folding of the instability exists in the finite system. These results are discussed in terms of the 2X results which exhibited a scaling similar to the above.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. N. Rosenbluth and R. F. Post, Phys. Fluids 8, 547 (1965).

Stationary Toroidal Equilibria at Finite Beta

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We investigate the effects of plasma flow on axisymmetric, self-consistent equilibria in toroidal geometry. This problem is of considerable interest in relation to hot plasmas confined in toroidal systems with longitudinal current. On the basis of the one-fluid MHD plasma model, we use a concise formulation to elucidate important features of the equilibrium. In contrast to previous flow calculations which were treated almost exclusively in the low beta approximation, we retain, together with flow, all beta effects.

As in the treatment of the full flow problem at low beta, we find conditions for equilibrium. The form of our description allows a quite general discussion of their nature and existence for finite beta. This shows that there is a close relationship between the solvability conditions for equations arising from integrals of the system, and the nature of the characteristics of the partial differential equation describing the radial force balance. In the case of large aspect-ratio these considerations lead to a generalized Bennett relation and to an expression for plasma displacement exhibiting beta and flow effects.

FINITE BETA EQUILIBRIA AND STABILITY FOR THE CANTED MIRROR*

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The purpose of this investigation is to calculate self-consistent, high-beta, equilibria which agree with the experimental parameters observed in the canted mirror device at ORNL. Here we report mainly results from the axisymmetric case of zero cant angle. In addition to the usual equilibrium quantities of interest (magnetic field, currents, and pressures) we have numerically evaluated various interchange stability criteria for these equilibria.

We have assumed a separable pressure model of the form

$$\underset{\approx}{P} = g(\psi)\ \underset{\approx}{\hat{p}}(B)$$

Hence, a sufficient criterion for interchange stability is

$$g'(\psi) \int \frac{d\ell}{B^2} \left[\hat{p}_\perp \frac{\partial B}{\partial \psi} + \hat{p}_\parallel \frac{\kappa}{r}\right] < 0$$

A somewhat more complicated expression is used to provide a necessary condition for interchange stability. We find that the bulk of the plasma is stable against simple interchanges and only the very outer edge need be line-tied to provide stability.

*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

Summary of LASL Theoretical Effort and
Topics from the Laser Fusion and Theta Pinch Programs

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250,000 gauss mirror field

MHD Stability of Diffuse Two-Dimensional Equilibria*

by

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Los Alamos, New Mexico

ABSTRACT

A method for determining the stability properties and growth rates of a general diffuse two-dimensional MHD equilibrium has been developed and tested. For a given equilibrium either analytic or numerical, we expand the perturbations in a double Fourier series of admissible functions. The problem reduces to minimizing the quadratic form δW in the unknown Fourier coefficients subject to the real normalization condition. The matrix coefficients are obtained numerically by integrating products of the equilibrium quantities with the Fourier functions over the two-dimensional domain. The eigenvalues and eigenfunctions of this matrix, which typically runs about 50 × 50, are computed numerically to give the growth rates and eigenmodes. A simple test of accuracy is performed by increasing the number of Fourier functions. The accuracy of the method has been verified for the specific case of the $\ell = 0$ bumpy pinch, which is compared with the diffuse calculations of Weitzner[1] in the appropriate limits.

no wall stabilization for diffuse profiles

ka

$\delta \sim 1$

$\beta \sim 1$

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[1] H. Weitzner, "Growth Rates and Spectra for a Particular Axially Symmetric Equilibrium," to be published.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

$$\xi(r, z) = \sum_{n,\ell} C_{n\ell} J_0(\lambda_n r) e^{i\ell z}$$

$$\omega^2 \sim \delta(1/\delta L)$$

Stability of a Finite β, $\ell = 2$ Stellarator*

by

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ABSTRACT

It has been pointed out by Grad and Weitzner[1] that low β stellarator theory would predict critical β's ~ .5 for $\ell = 2$ for reasonable values of the helical field strength. Admittedly these high critical β's are somewhat artificial, since the analytic results used lie outside the range where the expansions are valid; that is the stellarator expansion assumes both β and the helical field strength to be small. We have computed the stability of a sharp boundary, $\ell = 2$ system for arbitrary β and arbitrary helical field strength to determine the scaling of the critical β. The results of the compution are not as optimistic as the simple extrapolation of low β stellarator theory given above. In fact the highest critical β for any helical field strength is $\beta \approx .14$ (for the $m = 1$ mode). Thus an $\ell = 2$ system is not particularly attractive (over an $\ell = 1$ system) for pulsed θ pinches where $\beta \sim .8$ because of shock heating.

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[1]H. Grad and H. Weitzner, Phys. Fluids 12, 1725 (1969).

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Application of the Vlasov-Fluid Model to the Screw Pinch*

by

J. P. Freidberg and H. R. Lewis

ABSTRACT

The Vlasov fluid model,[1] in which ions are treated collisionlessly and electrons are treated as a massless fluid, is being solved numerically for screw pinch equilibria. The equilibria and marginal stability criteria for this model are the same as for ideal MHD. Growth rates for $m = 1$ perturbation are basically the same as for ideal MHD, but growth rates for $m > 1$ perturbation can be much smaller with the Vlasov fluid model. The equation to be solved numerically can be derived from a variational principle[2] in a form that does not require explicit knowledge of unperturbed particle orbits.

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[1] J. P. Freidberg, to be published in Phys. Fluids.

[2] H. Ralph Lewis, Phys. Fluids 15, 103 (1972).

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Parametric Instabilities of Whistlers*

by

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ABSTRACT

It is shown by theory and numerical simulation that a finite amplitude whistler propagating along a DC magnetic field is unstable to decay into a second whistler at nearly the same frequency and at a wave vector at nearly the same magnitude but oppositely directed from the pump, and a low frequency ion wave at twice the pump wave number. The driving mechanism of the instability is the following: the electron $\underline{V}_w \times \underline{B}_1 + \underline{V}_1 \times \underline{B}_w$ motion, where w refers to the pump and 1 refers to the linear whistler, can resonantly drive up an ion wave propagating along the DC magnetic field and in turn, an electron density fluctuation of the ion wave can couple with the electron's zero order motion in the pump to produce a current which resonantly drives up the linear whistler. The threshold for this process is quite low i.e., $B_w/B_o > 2\, k_o R_{Le}(\gamma_e \gamma_s/\omega_o \omega_s)^{\frac{1}{2}}$ where B_w and ω_o are the pump amplitude and frequency, γ_e and γ_s the damping of the excited whistler and acoustic wave. For amplitudes much above threshold, the maximum growth rate is given by $\gamma_{max}/\omega_o \approx \sqrt{3}(\Omega_i/\omega_o)^{\frac{1}{3}}(B_w/2\, B_o)^{\frac{2}{3}}$; this agrees quite well with the simulation results. In fact, the theory is valid up to very large amplitude, $B_w/B_o > 1$.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Parametric Instabilities at Low Frequencies Above the Lower Hybrid*

by

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ABSTRACT

Large amplitude electrostatic waves above the lower hybrid frequency are of interest in shock heated plasmas and in many LH heating experiments of Tokamak application. We discuss theory and numerical simulation which show that a lower hybrid wave at an angle θ to a DC magnetic field B_o, is subject to a decay instability into a second lower hybrid wave also at θ but at much shorter wavelength and into a low frequency ion wave. The threshold electric field can be such that the electron's $\underline{E}_o \times \underline{B}_o$ velocity, which drives the instability, is only a fraction of the ion thermal velocity. We calculate growth rates both near and much above threshold; the latter ones scale as $E_o^{\frac{1}{2}}$. There is also a purely growing instability whose threshold is the corresponding linear one lowered by the resonance and whose growth rate can be comparable or even exceed that of the decay instability.

Numerical simulations with a $1\frac{1}{2}$ dimensional finite size particle model verify the occurrence of the above parametric instabilities and show that the instabilities can result in substantial electron and ion heating. For angles deviating from 90° to $\underline{B}_o$ by more than the square root of the mass ratio, electron heating exceeds ion heating.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

†Princeton University Plasma Physics Laboratory.

Spectrum and Low Shear MHD Stability

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New York University

ABSTRACT

Many qualitative and quantitative equilibrium and stability properties are elucidated by a study of the spectrum, normal modes plus continua. For example there are unstable non-δW modes, changes of topology when perturbing a marginal equilibrium, and [Grossman and Tateronis-Garching] resistive marginal stability criteria contained in the perfectly conducting theory.

Four distinct continua have been identified in the cylindrical screw pinch. In two distinct cases there is an infinite point spectrum accumulating at the end of a continuum (Suydam instability and closed line interchange instability). More generally, local instability (Mercier etc.) can exist only when the unstable eigenvalues have an accumulation point. Growth rates for local modes (and in Stellerator and Scyllac expansions) can be smaller than expected.

A new class of low shear modes in the cylindrical screw pinch gives an explicit smooth transition between Suydam and interchange instability. Growth rates increase with increased shear and with increased deviation from resonance ($k \cdot B=0$). The limit of zero shear is smooth in some respects and exceptional in others.

ALSO BRIEF SHORT SUBJECTS FROM THE NYU PROGRAM.

Long-Wavelength Ion-Acoustic Waves

Glenn Bateman*

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New York University

ABSTRACT

The stability of ion-acoustic waves with wavelengths much longer than an electron mean-free-path (more precisely $1 \gg k\lambda_e \gg \sqrt{m_e/m_i}$) is determined for a uniform plasma under a variety of conditions. The moment equations, summarized by Braginskii, are used exclusively. The time dependence of the background (temperature and drift) and the form of the heat sources and sinks included in the calculation have a crucial effect on stability. If there is only a temperature difference ($T_i \neq T_e$ and no drift) evolving without sources or sinks, these waves are found to be stable for all temperature ratios. The instability found by Rognlien and Self (Phys. Rev. Letters, 27 792-795 (1971)) appears when there is a steady state x temperature difference (with source and sink). Their stability criterion can be reproduced by assuming an electron heat source and ion heat sink independent of the background state. Other models for the sources and sinks are considered here. For a plasma in a small uniform, constant electric field, with no sources or sinks, an Ohmic heating term prevents instability. In order to find a stationary state which is stable to runaway temperature and drift, it is necessary to consider heat sinks with a strong temperature dependence or systems in which the applied current is held fixed. The long-wavelength ion-acoustic wave stability of a number of models is tabulated.

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Toroidal High Beta Equilibria

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ABSTRACT

Previous studies of toroidal high beta equilibria based on piecewise constant pressure profile ideal magnetohydrodynamics are being extended to the distributed pressure case. The system is approximately $\ell = 1$ helically symmetric fields. The basic expansion parameter is the ratio of plasma radius to helical wave length. The equilibrium depends on the specific assumptions made on the helically symmetric currents. For an easily applied assumption on the nature of this current, although not the best, equilibria are computed and exhibited.

$\epsilon = k r_0 = h a$. Expand in ϵ $B_r \sim \epsilon$, $B_\theta \sim \epsilon$, $B_z \sim 1$
$K \sim \epsilon^2$, Perturb by adding $l = 0 \sim O(\frac{K}{\epsilon^2})$ ordering of fields
$\Rightarrow$ unique set of equilibria. Constant - no current being driven in or out of flux surfaces ($J_{\parallel} = 0$) - then a 1 parameter family of eq. result -

Non-Linear Energy Principle for Closed-Line Configurations

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New York University

ABSTRACT

Previously formulated variational principles for equilibrium and non-linear stability of toroidal configurations remain valid for equilibrium, but not for stability, if all magnetic field lines are closed. In this degenerate special case non-linear stability is, on the other hand, more vital than it is in configurations with ergodic field lines because conservation of topology ceases to prevent plasma from flowing across magnetic surfaces. In the present paper the appropriate variational principle is formulated for closed-line configurations. The potential energy, when varied subject to specified constraints, is stationary for equilibria, has a non-negative second variation for exponentially stable equilibria, assumes a relative minimum for nonlinearly stable equilibria, and assumes its absolute minimum for absolutely stable equilibria. The first two of these statements are proved, while the last two have to be considered as definitions. As an application the full minimization is done with fixed field lines. This yields a concise derivation and a complete interpretation of previously known conditions (viz. those for stability to interchanges and to flutes).

* on leave from Max-Planck-Institut für Plasmaphysik, Garching, Germany

Compression of a Tokamak Without Walls

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ABSTRACT

The equilibrium parameters of a tokamak without walls are found by a computer. It is assumed that there is circular symmetry about the main axis, the boundary is sharp, pressure is constant inside the plasma, and that both magnetic fluxes enclosed by the boundary are conserved. Graphs of various parameters versus vertical magnetic field will be presented.

Slightly Anisotropic Pressure Toroidal Equilibria with Nearly Spherical Plasma - Vacuum Interfaces

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ABSTRACT

Axially symmetric toroidal equilibria with nearly spherical plasma-vacuum boundaries are obtained with a guiding-center plasma having slightly anisotropic pressures. The lowest-order scalar-pressure case has spherical plasma-vacuum boundaries. In the anisotropic case the free plasma boundary becomes either slightly prolate or oblate depending on whether $p_\perp > p_\parallel$ or $p_\perp < p_\parallel$.

Role of Dissipation in the Feedback Stabilization of Reactive Instabilities*

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A general theory of linear feedback stabilization of electrostatic instabilities has been given by J. B. Taylor and C. N. Lashmore-Davies.[1] They found that, to stabilize reactive instabilities in a dissipationless plasma, the phase shift of the feedback circuit must be unique, and at best, the system can only be made neutrally stable; positive damping is not possible. We have extended the theory to include dissipation in reactive instabilities; the dissipation is chosen such that the waves are damped when the plasma is reactively stable. The reactive instability can still be stabilized by linear feedback. The minimum gain needed for stabilization is essentially unchanged, but now there is a small but nonzero range for the phase shift and the system exhibits positive damping. Furthermore, too much gain causes the system to go unstable. These general features have been observed in a number of experiments. The maximum allowable range for the phase shift is $\Delta\phi_{max} = 2\gamma_f/\gamma_o$ where γ_o is the original growth rate and γ_f is the maximum damping rate with feedback. This is a general result independent of the plasma and the particular instability.

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[1] J. B. Taylor and C. N. Lashmore-Davies, Phys. Rev. Lett. 24, 1340 (1970).

*Research supported by U. S. Atomic Energy Commission

LOCALIZED CRITERION OF STABILITY IN M.H.D.

C. MERCIER

Département de Physique du Plasma et de la Fusion Contrôlée

Centre d'Etudes Nucléaires

B.P. N° 6 - 92. FONTENAY - aux - ROSES

ABSTRACT

The localized perturbations of MHD equilibrium states are considered. In the general case of a toric configuration, two kinds of localized perturbations are to be distinguished : one kind is localized principally in the direction perpendicular to magnetic surfaces, while the second kind is localized along the magnetic surfaces.

Perturbations of the first kind lead to the well known localized criterion. The stability against perturbations of the second kind is studied in this paper and gives two different results according to the region of plasma where it is considered. In the neighbourhood of the magnetic axis, the two criteria are rigorously the same in the most general toric configuration - But, inside the plasma away from the magnetic axis, it is shown by considering a simple case (geometry of revolution symetry, magnetic surface with circular cross section and without shear) that the criterion of the second kind is quite different and may be more unstable than the usual localized criterion.

Electrostatic Modes of a Non-Uniform Plasma

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Abstract

The electrostatic modes of a plasma with a spatially non-uniform equilibrium state have usually been discussed in the framework of WKB approximations to the governing integral equations, or multiple-slab models, or by computer studies. Using approximate integral equations derived from the fully non-uniform orbit-integral equation for self-consistent fields, we derive, by perturbation-interaction techniques, closed-form expressions for the plasma response to an arbitrary source. This approach obviates the necessity for the localization assumptions of the WKB theory and for the uniformity assumptions of multiple slab models. These solutions give the response as a sum of modes, related to infinite medium absolute modes, with wave numbers (and/or frequencies) depending on the strength of the nonuniformity. Each term in the sum has, in turn, a relative amplitude depending on the strength of the non-uniformity. Interpretation of the solution is aided by comparison of the (electron) Landau damping, at a given $\underline{k}$, with infinite medium results. The overall solution is consistent with the "outgoing-wave" boundary conditions of Pearlstein, and Berk and Book.

Work supported by U.S. Air Force under Grant No. AFOSR 69-1697.

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Los Alamos Scientific Laboratory

<u>An MHD Code for Plane And Axisymmetric Flow</u>

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Abstract -

The Fluid In Cell method (FLIC) [(1)] is modified for compressible, time-dependent magnetohydrodynamic flow to include a magnetic field perpendicular to the flow directions. The basic equations used for finite difference method are two-dimensional, ideal, single fluid MHD equations coupled with Maxwell's equations. The finite difference equations with variable cell volumes and widths, and the boundary conditions on a curved rigid wall and on the moving vacuum-plasma interface are described. Cell-mixing on the moving contour is used for stability reason. The computations for a plane shock wave reflecting from a circular wall are made by using slip boundary conditions at the curved wall. The calculations for shock formation by a constant driving current in an axisymmetric annular shock tube are obtained by means of a Lagrangian contour on the moving vacuum-plasma boundary. Then the generated shock wave is utilized to study its reflection from a flat end wall in the shock tube. The results of the computations are checked with those obtained from other method and with experimental data.

(1) Journal of Computational Physics 1, 87-118 (1966) An Eulerian Differencing Method for Unsteady Compressible Flow Problems. Richard A. Gentry, Robert E. Martin, and Bart J. Daly

Subgrid Resolution by Means of Splines

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Charge "sharing" or "weighting", combined with field interpolation from a table of potentials, will yield high resolution and low noise if done by means of quadratic splines. Two-dimensional experiments will be shown that indicate resolution down to one-eighth grid mesh size. One need not make the grid smaller than a Debye length if one uses quadratic splines. Efficient spline algorithms have been written. The Poisson operators used in these tests were the 5-point, the 9-point and Lewis operators.

With a view to simulating a CTR device, these studies have been extended to 3D, and to electromagnetics. A propagation experiment with a 7-point 3D wave operator (numerically equivalent with the space-time leap-frog scheme for Maxwell's equation) showed that non-physical imperfections died down due to radiation damping.

Since quadratic splines are too expensive for a full 3D e-m code, a search was made for the linear splines which, in the sense of Lewis' variational principle, are compatible with the 7-point wave operator or Maxwell leap-frog scheme. An extremely simple weighting and field evaluation scheme was obtained, based on a tetrahedral mesh.

Theory and Simulation of Turbulent Ion Heating

by the Modified Two-Stream Instability

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We present results of an analytical and numerical study of the fluid-like modified two-stream instability, which is driven by relative drifting between electrons and ions across a magnetic field and has characteristic frequency and growth rate comparable to the lower hybrid frequency, $\omega_{pi} (1 + \omega_{pe}^2/\Omega_e^2)^{-\frac{1}{2}}$. This instability is important from a fusion standpoint because, as our computer simulation results show, it leads to strong ion heating. The linear theory of the instability is discussed both in the electrostatic and fully electromagnetic cases, and we present a detailed numerical study of the dependence of the unstable roots of the dispersion relation for a wide range of plasma parameters. Our nonlinear theory includes discussions of (1) quasilinear theory, (2) trapping, (3) a derivation of a fully nonlinear scaling law which shows how results scale with electron-ion mass ratio, (4) the effect of cross-field vortex-like motion caused by turbulence induced $\underline{E} \times \underline{B}$ drifts. One- and two-dimensional computer simulations with dense $\underline{k}$-space spectra are presented and related to theory. Since strong ion heating occurs in this instability, we discuss methods by which the modified two-stream instability can be used advantageously in fusion devices, e.g., tokamaks and mirrors.

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TWO-DIMENSIONAL VORTEX MOTION, GUIDING CENTER PLASMAS, AND "NEGATIVE TEMPERATURES"

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The mathematical description of N two-dimensional guiding-center plasmas of charged rods aligned parallel to a dc magnetic field is identical to that for point vortices in two-dimensional, incompressible inviscid hydrodynamics. Both are Hamiltonian systems with the property that $\Phi(E)$, the total phase space volume with energies less than E, goes from zero at $E = -\infty$ to a finite constant as $E \to +\infty$. Thus the structure function, $\Omega(E) \equiv d\Phi(E)/dE$, has a maximum at a finite value of the energy, $E = E_m$. The formulas $S = K \ln \Omega(E)$ and $(KT)^{-1} = \Omega'(E)/\Omega(E)$ for the macroscopic entropy and temperature can be derived quite generally from the classical ensembles. The second formula predicts that for $E > E_m$, the temperature is negative. For a system whose component parts have a finite interaction, $T < 0$ implies the absence of a stable thermodynamic equilibrium state. For sufficiently high energies, the spatially uniform state, in particular, will be unstable. The volume in phase space with very large energies corresponds mostly to configurations in which charges of like sign are crowded preferentially together at one side of the confining volume and those with opposite sign are crowded toward the opposite side. All such states correspond to a pair of large counter-rotating vortices, as far as the fluid velocity is concerned. A phenomenon similar to this prediction (Onsager, 1949) has recently been observed by Deem and Zabusky (1971). However, the Deem-Zabusky computation uses smooth and continuous fluid variables instead of point vortices, so the correspondence is not perfect. Additional numerical simulations are indicated. An interesting feature of this instability is that it is apparently not obtainable from a linearized treatment of the dynamical equations.

Poor Man's Plasma Simulation

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In the field of plasma simulation particle codes usually work with particle numbers which are much smaller than in a real plasma. As a consequence noise rapidly builds up. However, a simulation particle can also be interpreted as representing a finite volume in phase space. We try to simulate the spreading of the phase volume which occurs according to the Vlasov equation. The resulting code is similar to Denavit's and the noise associated with it is grately diminished. We can therefore simulate Landau damping with as few as 400 particles. The resulting savings in computer time are not only helpful for research groups with a low budget, but may be instrumental in doing plasma simulation in more dimensions.

A Possible Plasma Laser*

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ABSTRACT

Ion density fluctuations can provide a negative resistance for electromagnetic waves in a plasma, leading to their amplification. This occurs when the ion fluctuations couple the high frequency field to a plasma oscillation which has a phase velocity in a region where the slope of the distribution function is positive. The amplification can be achieved by drifting plasma electrons past prepared ion density fluctuations. A simple calculation describes the principal features of this finite amplitude instability, and computer simulations confirm its existence.

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Mode Coupling of Electron Plasma Oscillations by Finite Amplitude Ion Fluctuations*

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ABSTRACT

Finite amplitude ion fluctuations efficiently and rapidly couple energy from high-phase velocity electron plasma oscillations into much slower ones and then into the particles, even when this mode coupling is off-resonant. The magnitude of the ion density fluctuation ($\delta n/n$) which gives efficient off-resonant coupling is typically the frequency mismatch between the plasma oscillations normalized to the electron-plasma frequency. This off-resonant coupling allows energetic beams to rapidly transfer their energy to a dense background plasma. Computer simulations confirm the simple predictions.

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Fokker-Planck Investigations of the Wet Wood Burner Concept*

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ABSTRACT

Neutral beam injection promises to be a useful heating method for the next generation of plasma confinement devices. For a sufficiently energetic beam, fusion reactions may occur between the beam ions and confined plasma ions. In a "wet wood burner,"[1] a target plasma with density and temperature below the Lawson criterion interacts with an energetic neutral beam. Under suitable conditions, the expected energy production by nuclear reactions exceeds the beam energy. The results of numerical studies of this process, using the Fokker-Planck equation, will be presented.

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* Work supported by U. S. Atomic Energy Agency Contract AT(11-1)-3073. Use was made of computer facilities supported in part by National Science Foundation Grant NSF-GP 579.

[1] J. M. Dawson, H. P. Furth and F. H. Tenney, Phys. Rev. Lett. 26, 1156 (1971).

Plasma Diffusion Across a Magnetic Field*

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ABSTRACT

Numerical Simulation of plasma diffusion across a magnetic field in three dimensions has been performed for cases with closed and nonclosed field lines of force. For the case of closed field lines, the diffusion essentially takes the form for two dimensions showing three different regions of diffusion; classical, intermediate, and Bohm diffusion studied previously.[1,2] The physical origin of the enhanced diffusion is due to plasma convection which is thermally excited. For the case of nonclosed field lines which corresponds to a system with a rotational transform, the diffusion follows the classical theory to stronger fields. However, there is always a critical strength of magnetic field beyond which the diffusion is enhanced above the classical level. As an application to a real plasma, a simulation is performed showing good agreement with the enhanced diffusion observed in a semiconductor plasma.[3]

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1 J. M. Dawson, H. Okuda, and R. L. Carlile, Phys. Rev. Letters 27, 491 (1971).

2 H. Okuda and J. M. Dawson, Princeton Plasma Physics Laboratory Report PPL-AP 35.

3 M. G. Gurne, W. M. Hooke, G. J. Smith, and M. H. Grenhan, Phys. Rev. A5, 158 (1972).

An Analytic Investigation of the Saturation of Kinetic Plasma Instabilities by Particle Trapping*

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ABSTRACT

The amplitudes at which unstable Langmuir and ion acoustic waves saturate due to electron trapping are estimated using the adiabatic orbit approximation and momentum conservation, and are shown to satisfy the relation $\omega_{Be} \simeq 2.9\ \gamma_{lin}$. The effect of nonadiabaticity of the electron orbits is estimated by using also the sudden approximation distribution function, giving $\omega_{Be} \simeq 3.9 \gamma_{lin}$. The result of Fried et al.[1] lies between these limits. The saturation amplitude of an ion acoustic wave is found to be very low, and it is also found that it is usually possible for resonant ions to reabsorb all the wave momentum by trapping, thus damping out the wave.

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* Work supported by U.S. AFOSR Contract F44620-70-C-0033.

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Finite Beta Effects on Low-Frequency Instability in Toroidal Plasma*

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ABSTRACT

The kinetic study of linear instabilities in toroidal systems is generalized to include the effects of perturbations in the magnetic field. The perturbed vector potential is taken to possess a component solely along the equilibrium magnetic field. The model, in general, retains the full "radial" dependence of the perturbed quantities.

Trapped particle modes have been studied using an eikonal approximation for the perturbed potentials but allowing a more general behavior of the perturbed distribution function in order to preserve the effects of the finite bananas. A small perturbed current parallel to the equilibrium magnetic field is found. This effect is second order in the banana width and arises from the lack of complete cancellation of the Lorentz interaction $\vec{v}\cdot\vec{A}$ over the finite banana orbit, and the fact that the current at a point is made up of contributions from particles whose banana guiding centers are on different magnetic surfaces. These effects are distinct from the stabilizing effect due to the "self-dug" well.

We also examine the question of the persistence of the tearing mode in tokamaks in the limit of long mean-free-path. Finite parallel electron enertia is responsible for the perturbed parallel electric field. The tearing layer can be broadened by the finite banana effects.

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High Current Density Plasma Confinement Configuration†

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ABSTRACT

A magnetic plasma confinement configuration aimed at sustaining high current densities is proposed. It involves small triangular and elliptic deformations imposed over a circular cross section of the plasma column by a surrounding conducting shell at close distance.

The equilibrium of such a configuration is analyzed paying particular attention to the vacuum magnetic surfaces around the plasma. This analysis is combined with an examination of the localized flute mode stability criterion to find numerically the maximum value of the current density that is consistent with the minimum variation of the distance of the conducting shell to the plasma column.

The stability against kink modes is analyzed and shown to be the same as that of an equivalent plasma column with equal area, but circular cross section.

An assessment of the overall stability in this configuration points to a significant improvement, in terms of the ohmic heating resulting from the induced current, over the known configurations with circular cross section.

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