# SHERWOOD THEORETICAL MEETING

## APRIL 23-25, 1970
## PRINCETON, NEW JERSEY

DEI SUB NUMINE VIGET

VET. TESTAMENTUM NOV. TESTAMENTUM

# PLASMA PHYSICS LABORATORY

Contract AT(30-1)—1238 with the
US Atomic Energy Commission

## PRINCETON UNIVERSITY
## PRINCETON, NEW JERSEY

# SHERWOOD THEORETICAL MEETING

# APRIL 23-25, 1970
# PRINCETON, NEW JERSEY

hosted by

PRINCETON UNIVERSITY
PLASMA PHYSICS LABORATORY

# SCHEDULE

Wednesday Evening April 22, 1970

| | |
|---|---|
| 8:30 - 10:00 | Registration at the Holiday Inn |

Thursday April 23, 1970

| | |
|---|---|
| 8:40 a.m. | Bus leaves Holiday Inn for Woodrow Wilson |
| 9:00 | Registration |
| 9:20 | Welcome - M. B. Gottlieb |
| | Session A - Mostly Tokamak Equilibrium and Stability |
| 2:00 p.m. | Session B - Mostly Anomalous Transport and Heating |
| 6:00 | Cocktail Hour at Prospect |
| 7:00 | Banquet at Prospect |
| 9:15 | Bus leaves for Holiday Inn |

Friday April 24, 1970

| | |
|---|---|
| 8:40 a.m. | Bus leaves Holiday Inn for Woodrow Wilson |
| 9:20 | Session C - Stellarators and Mirrors |
| 2:00 p.m. | Session D - Miscellaneous |
| 5:25 | Bus leaves for Holiday Inn |

Saturday April 25, 1970

| | |
|---|---|
| 8:50 a.m. | Cars leave Holiday Inn for Plasma Physics Laboratory. Informal discussions where interest develops. |

Session A. Mostly Tokamak Equilibrium and Stability
(R. J. Hastie, Chairman)

A 1. "MHD Equilibria in Tokamaks and Doublets," R. A. Dory and R. H. Fowler

A 2. "Tokamak Equilibrium," J. M. Greene, J. L. Johnson, and K. E. Weimer

A 3. "Toroidal Hydromagnetic Equilibrium Solutions with Spherical Vacuum Boundaries -- II," G. K. Morikawa and T. Yeh

A 4. "Time Constants for Resistive Diffusion in a Tokamak" and "Time-dependent Resistive Diffusion," J. Hogan and D. Stevens

A 5. "Thermal Equilibrium and Stability of Tokamak Discharges," H. P. Furth, M. N. Rosenbluth, and P. H. Rutherford

A 6. "Conditions for Drift Wave Instabilities in Tokamak Systems," W. Horton and R. K. Varma

A 7. "The Importance of Toroidal Contributions to Shear and the $J_{\parallel}$ - Kink Instability in Tokamak Type Plasmas," A. A. Ware

A 8. "High-$\beta$ Stability in a Three-Dimensional Diffuse Pinch," J. P. Boris

# MHD Equilibria in Tokamaks and Doublets*

R. A. Dory and R. H. Fowler

Oak Ridge National Laboratory, Oak Ridge, Tennessee

## ABSTRACT

We present results for the heuristic stability-determining parameters $q = 2\pi/\iota$ , $V''$ , and $V^{**}$ for toroidal current carrying configurations whose cross sections range from circles (tokamak) to ovals of Cassini (doublet).

The calculations are for ideal MHD equilibria. There is no approximate expansion in toroidal curvature or plasma pressure. The equations solved are $\nabla P = \vec{J} \times \vec{B}$ and curl $\vec{B} = 4\pi \vec{J}$ with boundary condition $\vec{B} \cdot \hat{n} = 0$ at the conducting wall. Azimuthal symmetry is assumed. The equations are well-posed when two auxiliary functions are specified; viz: the pressure variation from each isobar (magnetic surface) to the next, and the toroidal plasma current within each isobar. For this study, these functions are chosen to make 1) the plasma current consistent with $\eta \vec{J} = \vec{E} + \vec{v} \times \vec{B}$ where $E_{toroidal}$ is the Ohmic heating field from a transformer core threading the torus, and 2) zero net plasma flux through each isobar.

Because these requirements give a plasma current density that is linear in the poloidal flux function, analytic solutions may be found in the form of Coulomb Wave Functions, as tabulated in Abramowitz and Stegun. The next step, detailed stability analysis, might then be susceptible to analytic treatment.

---

*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

Tokamak Equilibrium.* John M. Greene, John L. Johnson†, and Katherine E. Weimer, Princeton University.-- To determine the fields necessary for confinement in a proposed Tokamak Compression Experiment, we calculated tokamak equilibrium on a static ideal fluid model, going to third order in an inverse aspect ratio expansion. We formally separated the problem into an inner region and an outer region and asymptotically matched the solutions. In the inner region we generalized Shafranov's model to one in which the magnetic surfaces can be elliptically distorted as well as nonconcentric. In the outer region we used toroidal coordinates to obtain the fields due to currents in the plasma and spherical coordinates for the externally applied fields. Scaling relations for the compression ensure that the magnetic fluxes and number of particles inside each surface are conserved. Then the externally imposed field necessary to contain a parabolic pressure distribution with a uniform rotational transform inside a circular plasma boundary, which is compressed by a factor $\alpha^{-1}$ of its initial radius $a_o$, is

$$\frac{\underset{\sim}{B}(x,\phi,z)}{B_o(x=R_o)} = \frac{R_o\alpha^2 \underset{\sim}{e}_\phi}{(R_o - \alpha r_o\cos\theta + \dots)} - \frac{\alpha^4 a_o^2 \underset{\sim}{e}_z}{2R_o^2 q}\left(\ln\frac{8R_o}{\alpha a_o} - \frac{5}{4} + \frac{\alpha^{2/3}\beta_o q^2 R_o^2}{2a_o^2}\right)$$

$$+ \frac{3\alpha^5 a_o^2 r_o(\underset{\sim}{e}_x \sin\theta - \underset{\sim}{e}_z\cos\theta)}{8R_o^3 q}\left(\ln\frac{8R_o}{\alpha a_o} - \frac{17}{12}\right) + \dots \quad .$$

* Work performed under the auspices of the U.S. Atomic Energy Commission, Contract no. AT(30-1)-1238.

† On loan from Westinghouse Research Laboratories.

# Toroidal Hydromagnetic Equilibrium Solutions with Spherical Vacuum Boundaries - II

G.K. Morikawa and T. Yeh

Courant Institute of Mathematical Sciences

New York University

Numerical calculations are made to find the explicit relationship among the physical parameters for a family of toroidal hydromagnetic equilibrium solutions with spherical vacuum boundaries.* A combination of an inner vacuum hole (produced by a dipole current loop) and an axially symmetric outer vacuum region confined by a shaped current 'sheet' distribution contains the plasma. The geometrical parameter range goes between the limits of no vacuum hole up to a spherical plasma sheath; the range of the plasma pressure parameter is between zero (force-free) and one. Also, a complimentary class of solutions, which requires the presence of an axial current is described. All solutions are exact.

---

*G.K. Morikawa and E. Rebhan, Phys. Fluids 13, (1970)

## Time Constants for Resistive Diffusion in a Tokamak

J. Hogan

Courant Institute of Mathematical Sciences

New York University

We present calculations of time constants for resistive diffusion in Tokamak. Classical, time-dependent, resistive MHD equations are used. A general theory for these processes has been constructed by H. Grad and certain of its predictions are discussed here. The theory distinguishes two time scales: the faster is the classical time of the skin effect $\tau \sim L^2/\eta$ (L: length, $\eta$: plasma resistivity) and the slower is $\tau \sim L^2/\beta\eta$ . Unique values for physical quantities are given by the theory, in particular, a unique resistive equilibrium. We discuss estimates of time constants characterizing the approach to this equilibrium on the slow time-scale.

## Time-dependent Resistive Diffusion

Donald Stevens

Courant Institute of Mathematical Sciences

New York University

The MHD equations with resistivity are solved for a cylindrical model with Tokamak scaling. The time scale is the faster of the two distinct diffusion times and the discharge current is assumed to vary on this scale. Computed results are given for behavior of the magnetic fields and for the resistive diffusion velocity on the fast scale.

Thermal Equilibrium and Stability of Tokamak Discharges.* H. P. Furth, M. N. Rosenbluth,† and P. H. Rutherford, Princeton University--Steady-state temperature and magnetic field profiles are derived in cylindrical geometry, including classical electron-ion equilibration, neoclassical ion thermal transport, and anomalous ohmic heating. Agreement with the Thomson scattering profiles is obtained. Using the measured resistivity anomaly factor, the observed $T_i$ and $T_e$ magnitudes can be fitted only by introduction of a comparable electron heat-loss factor.

The equilibrium without direct electron heat loss can be thermally stable only if $T_i/T_e > 2/3$ for classical resistivity, or $> 1/2$ for extreme anomalous resistivity. (Instability consists of radial contraction, or formation of local current shells.) Radiation cooling is further destabilizing; but for T-3 parameters stability can be achieved by a dominant anomalous electron thermal transport, preferably one that does not diminish with rising $T_e$.

The growth time of the modes is generally limited by the resistive skin time; however, the presence of a run-away component of the current allows shorter growth-times. This type of explanation for the high-density limiting phenomena in T-3 was first proposed by Stodiek. The aggravation of the ordinary skin effect by thermal instability has been pointed out by Artsimovich.

---

* Work performed under the auspices of the U.S. Atomic Energy Commission Contract no. AT(30-1)-1238.

† Also, Institute for Advanced Study.

# CONDITIONS FOR DRIFT WAVE INSTABILITIES IN TOKAMAK SYSTEMS

W. Horton and Ram K. Varma
University of Texas, Austin, Texas

Conditions for plasma instability and the effects of the resulting turbulence are investigated for axisymmetric toroidal systems with the rotational transform provided by a toroidal plasma current, $j_{\phi}(r)$, such as in Tokamaks. Using the inverse aspect ratio $\delta = r/R$ as an expansion parameter, the drift-acoustic waves are derived including the toroidal current in the collisionally decaying equilibrium. Two limiting regimes are distinguished: (1) a collisional regime where $\lambda_{mfp} << R$ and the resistive/viscous effects determine stability and (2) the collisionless regime ($\lambda_{mfp} >> R$) where particle trapping is included.

In the resistive/viscous regime the mode equations are solved for the $\theta$-dependence of the fluctuating potential, $\tilde{\varphi}_{\nu}(r,\theta)\, e^{i\nu\phi - i\omega t}$ where $\phi$ is the angle around the major axis and $\theta$ is the angle around the minor axis, and for the eigen frequencies by a perturbation expansion in $\delta$. The modes are unstable for election drift velocities $u^e_{\parallel} > u^e_{\parallel crit}$ where the critical velocity is determined by the stabilizing effects of ion sound and parallel viscosity. The conditions for marginal stability are given; and, for unstable conditions, the anomalous resistivity and the rate of E x B convection are calculated. Eliminating the unknown level of density fluctuations between the anomalous transport coefficients, we express the enhanced resistivity, $\eta_a / \eta_{cl}$, in terms of $D_a/D_{cl}$.

The considerations in the collisionless regime are similar to those of Rutherford et al.[1] In both regimes the minimum $k_{\parallel}$ is determined by the shear which is sufficient to stabilize the modes for current profiles which are rapidly decreasing functions of radius.

---

[1] P. Rutherford, M. Rosenbluth, W. Horton, E. Frieman, B. Coppi, Plasma Physics and Controlled Nuclear Fusion Research, IAEA, Vienna, Vol. I, p. 367-387 (1969).

# THE IMPORTANCE OF TOROIDAL CONTRIBUTIONS TO SHEAR AND THE $j_{\parallel}$-KINK INSTABILITY IN TOKAMAK TYPE PLASMAS

A. A. Ware
University of Texas, Austin, Texas

An expression has been obtained for the shear ($d\iota/dr$) to second order in the aspect ratio r/R. The higher order toroidal contributions to the shear will be important when $\beta$ is large or under conditions where the zero order shear $(2\pi R_o/B_{\phi o})\,(d\,(B_\theta/r)\,/\,dr)$ is small. For example, the outer region of the screw pinch, which previously has been thought to have zero shear, is shown to have a large shear resulting from the high $\beta$ toroidal contribution. The toroidal corrections to the shear are important in the $j_{\parallel}$-kink instability which depends strongly on the magnitude of the shear and its character. (A zero in the shear at a non-zero radius can be worse than no shear.) Assuming $\beta$ is small, the MHD energy integral has been calculated to order $(r/R)^6$, which is the order required to detect the $j_{\parallel}$-kink instability terms. Destabilizing terms arising from toroidal effects have been found which are $q^2$ greater than the linear pinch destabilizing terms. ($q = rB_{\phi o}/R_o B_{\theta o}$).

High-$\beta$ Stability in a Three-Dimensional Diffuse Pinch,* by

J.P. Boris, Princeton University.

Three-dimensional MHD simulations of the diffuse pinch have been performed using a single-fluid model containing terms representing viscosity, resistivity, and thermal conductivity. The plasma-current contributions to the total magnetic field are treated in a fully self-consistant manner so that high-$\beta$ configurations can be studied. Thus meaningful equilibrium and stability results for linear pinch and toroidal tokamak-like plasmas can be obtained. The sausage instability, for example, will limit the permissible plasma current in pinches and tokamak plasmas. Calculations of the sausage instability in the nonlinear regime and the stabilization effect of a strong axial magnetic field have shown that modes of a given wavelength are unstable only when the magnetic field lines on the surface of the plasma column circle the axis at least once in traversing a wavelength. Effects of plasma- curvature and feedback-stabilization calculations will also be reported.

---

* Work performed under the auspices of the Naval Research Laboratory, Contract no. N00014-67-A-0151-0021; and U.S. Atomic Energy Commission, Contract no. AT(30-1)-1238).

Session B. Mostly Anomalous Transport and Heating
(D. R. Dobrott, Chairman)

B1. "Plasma Diffusion in Two Dimensions -- I," J. B. Taylor

B2. "Strong Turbulent Heating," W. E. Drummond, J. R. Thompson, L. Sloan

B3. "Anomalous High Frequency Resistivity and Heating of a Plasma," P. K. Kaw, W. L. Kruer, J. M Dawson, C. R. Oberman, and E. J. Valeo

B4. "Anomalous Resistance Due to Low Frequency Fluctuations in a Plasma in a Uniform Magnetic Field," O. K. Miwardi and M. N. Rosenbluth

B5. "Turbulent Resistance in the Stellarator," J. P. Boris, J. H. Orens, and J. M. Dawson

B6. "Heating and Counterstreaming Ion Beams in an External Magnetic Field," K. Papadopoulos, R. Davidson, J. M. Dawson, I. Haber, D. Hammer, R. Shanny

B7. "Numerical Simulation of CTR Related Plasma Phenomena," R. L. Morse, C. W. Nielson, and T. A. Oliphant

B8. "Nonlinear Theory of the Drift Dissipative Instability," A. M. Sleeper and A. Simon

# PLASMA DIFFUSION IN TWO DIMENSIONS - I

J.B. Taylor
U.K.A.E.A., Culham Laboratory, Abingdon, Berkshire, U.K.

## ABSTRACT

Diffusion of plasma in two dimensions is studied in the guiding centre model. It is shown that, in this model, diffusion must always exhibit the anomalous 1/B variation with magnetic field. The correlation function and diffusion coefficient are calculated in detail using functional probabilities. In addition to the 1/B field dependence the coefficient is unusual in that it depends on the size of the system. The significance of the results for real plasma, for Bohm diffusion, and particularly for computer experiments, is discussed.

# STRONG TURBULENT HEATING

William E. Drummond*, J. Robert Thompson,† Lee Sloan†

The experimental results of Hamburger will be reviewed in order to provide guidelines for a theoretical understanding of strong turbulent heating. The resulting constraints require a model for strong turbulence in which most of the electrons are fully trapped and with potentials of many kilovolts per Debye length. BGK modes which have these properties will be displayed along with computer simulation results.

* University of Texas, Austin, Texas

† Austin Research Associates

Anomalous High Frequency Resistivity and Heating of a Plasma,*

by P. K. Kaw, W. L. Kruer, J. M. Dawson, C. Oberman and E. J. Valeo,

Princeton University.

When a sufficiently large amplitude oscillating electric field with a frequency close to the electron plasma frequency is applied to a plasma, it excites instabilities which drive up the low-frequency ion density fluctuations and high-frequency electron plasma oscillations. The presence of the large amplitude ion density fluctuations leads to an enhancement in the high-frequency resistivity of the plasma, around the plasma frequency. The result is an efficient heating of the plasma. We have carried out computer experiments on a one-dimensional plasma, which illustrate this effect. A nonlinear treatment of the anomalous resistivity, which is in reasonable agreement with the numerical experiment, is presented.

---

* Work performed under the auspices of the U.S. Atomic Energy Commission, Contract no. AT(30-1)-1238.

# Anomalous Resistance Due to Low Frequency Fluctuations in a Plasma in a Uniform Field*

Osman K. Mawardi and Marshall N. Rosenbluth
Institute for Advanced Study, Princeton, N. J.

Calculations are presented for the parallel resistivity of a magnetized plasma perturbed by low frequency fluctuations. The fluctuations considered are caused by ion-acoustic waves destabilized by the thermal conductivity of the electrons and are of the type recently discussed by Coppi. The resistivity is obtained by means of the non-linear theory developed by Simon† for the estimation of transport coefficients in a plasma. In the two fluid model used here the viscosity and thermal conductivity of the ions have been taken into account. The local value of the resistivity which has been obtained is expressed as a function of the drift velocity of the electrons, their thermal velocity and the ratio of the mean free path for electron-ion collisions to the parallel wave length of the fluctuations. This latter parameter is assumed to be small since a fluid model is used.

---

*Research sponsored by the U. S. Atomic Energy Commission.
†A. Simon, Phys. Fluids 11, 1181 (1968).

Turbulent Resistance in the Stellarator,* by J. P. Boris, J. H. Orens, and J. M. Dawson, Naval Research Laboratory, Washington, D. C., and Princeton University.

A turbulent (anomalous) resistivity has been invoked as a phenomenological description of the rapid plasma heating in shock experiments, stellarators, and tokamaks. The ohmic heating due to large induced toroidal electric field is orders of magnitude larger than can be accounted for by a classical collisional resistivity so other mechanisms must be used.[1, 2] An interpretation of the anomalous resistivity and related phenomena is given in which the nonlinear saturation of electron-ion streaming instabilities provides an exceedingly simple law relating the average heating rate to the applied electric field. Recent one-dimensional and quasi-one dimensional calculations support these hypotheses[2-4] in simulation plasmas and display large, low-frequency ion fluctuations apparently arising from nonlinear mixing of the electron-ion modes. The presence of strong fluctuations as an intrinsic part of this picture may lead to rapid cross-field diffusion in real plasmas and thus anomalous heating could bring on large scale plasma loss. Results of current two-dimensional calculations with the electrons tied to magnetic field lines are presented and agree well with measurements of the plasma current on the model C-stellarator, and with experimental values of $V_{de}/V_{the}$ when the diffusive loss of particles is included.

References


[1] B. Coppi and E. Mazzucato, Princeton University Plasma Physics Laboratory, report no. Matt-720 (1969).

[2] J. M. Dawson, J. Orens, K. V. Roberts (to be published).

[3] J. Boris and K. V. Roberts, 3rd European Conf. on Controlled Fusion and Plasma Physics (Walters-Noordhogg Publ. Co., Utrecht, Neederlands, 1969).

[4] C. Nielson and R. Morse, Conf. on Numerical Simulation of Plasma, Stanford (1969).


* Work performed under the auspices of the U. S. Atomic Energy Commission, Contract no. AT(30-1)-1238.

# HEATING OF COUNTERSTREAMING ION BEAMS IN AN EXTERNAL MAGNETIC FIELD

K. Papadopoulos
R. Davidson
J. M. Dawson
I. Haber
D. Hammer
R. Shanny
Naval Research Laboratory

We consider ion heating by a strong ion-ion two stream instability perpendicular to a magnetic field in the presence of a relatively cold electron background ($T_e \ll M_i V_d^2$). The magnetic field strength is such that the ion trajectories are straight, whereas the electrons are bound to the field lines ($kR_{Le} \ll 1 \ll kR_{Li}$). Theory is presented for both quasi-linear and non-linear stages of the evolution of the system and is compared with a series of computer simulation experiments. It is found that the quasi-linear theory gives a fairly accurate description of spatially averaged plasma properties until the ion beams have been sufficiently modulated for ions to be trapped by the waves. In the subsequent non-linear stage, stabilization occurs when the ion trapping period is equal to the reciprocal growth rate associated with the instability[1]. The directed ion beam energy was converted mainly to random ion energy. This instability might be of extreme interest to turbulent ion heating.

1. W. M. Manheimer, Bull. Am. Phys. Soc. 14, 1041, (1969)

# Numerical Simulation of CTR Related Plasma Phenomena*

by

R. L. Morse, C. W. Nielson, and T. A. Oliphant
Los Alamos Scientific Laboratory, University of California
Los Alamos, New Mexico

ABSTRACT

Several plasma simulation efforts now in progress are intended to give further understanding of current CTR problems. These include studies of turbulent heating with realistic mass ratios, investigations of the effect of binary collisions on the development of microinstabilities, and further work on the turbulence caused by velocity space anisotropy in high β plasmas. The present state of these investigations will be presented.

* This work performed under the auspices of the U. S. Atomic Energy Commission

Nonlinear Theory of the Drift Dissipative Instability. ARTHUR M. SLEEPER and ALBERT SIMON, Univ. of Rochester.--The nonlinear behavior of the drift dissipative instability in a weakly ionized plasma is studied in the threshold regime. The linear analysis of Self[1] is developed and extended to include finite boundaries. Analytic expressions are obtained for the unstable eigenmodes, the instability onset criterion, and all critical linear parameters. A region of parameter space (background pressure and magnetic field) is chosen so that only one (degenerate) mode goes unstable as the pressure is decreased below the critical value. The time-asymptotic behavior of the system is then analyzed using the general method developed by one of us.[2] We calculate the saturated mode amplitudes, frequency shifts, and anomalous flux as functions of the fractional decrease of the pressure below the critical value. The frequency shift is found to be very small for this instability. Mode coupling is shown to be negligible. The calculation is first performed in idealized slab geometry in which the plasma is created continuously at one plate and absorbed at the other. The calculation is then extended to coaxial geometry in which the plasma is created at the inner cylinder and absorbed at the outer, and to true cylindrical geometry in which the plasma is either created at a fixed rate at end plates (Q-machine) or by an RF field. The qualitative features of the instability (onset criterion, small frequency shift and flattening of the initial density gradient) are similar for all of the geometries and plasma production processes studied.

[1] S. A. Self, "Ion-Waves, Drift Waves and Instability in a Weakly Ionized Magnetoplasma", SUIPR Report #265, November 1968, (unpublished).

[2] A. Simon, Phys. Fluids 11:1181(1968).

## Session C. Stellarators and Mirrors
(K. E. Weimer, Chairman)

C 1. "High-Beta Equilibrium with Large Helical Wavelengths," H. Weitzner

C 2. "Stability of a High-$\beta$, Helically Symmetric Pinch," J. P. Friedberg and B. M. Marder

C 3. "The Resistive Ballooning Mode in Stellarator-like Geometries," R. M. Kulsrud

C 4. "Loss-Cone in a Nonaxially Symmetric Toroidal Device," D. Dobrott, R. J. Hastie, and H. Fishman

C 5. "Double-Hump Instabilities at the Upper Hybrid Frequency in Hot-Electron Plasmas," D. J. Sigmar and G. E. Guest

C 6. "Finite Beta Resonant Negative Energy Instability," H. L. Berk and L. D. Pearlstein

C 7. "Inverted Population Instabilities," L. S. Hall

C 8. "An Integral Equation Model for Microinstabilities in Inhomogeneous Plasmas," C. O. Beasley, H. L. Berk, W. M. Farr, and L. D. Pearlstein

# High-Beta Equilibrium with Large Helical Wavelength

H. Weitzner

Courant Institute of Mathematical Sciences

New York University

The free boundary magnetohydrodynamic model with piecewise constant pressure is employed to explore equilibrium and stability properties of simple plasmas with helical symmetry.

In contrast with the earlier Stellarator[(1)] and Scyllac[(2)] scalings, the fundamental small parameter is the helical wavenumber times plasma radius, while the distortion of the plasma column from a true cylinder is much larger. Previous unpublished work[(3)] indicated the existence of useful equilibria stabilized against M=1 perturbations by wall effects for a system with predominantly N=1 helical magnetic fields. An admixture of a small amount of N=2 helical magnetic fields improves the stability characteristics for certain ranges of $\beta$ and in certain cases permits the system to be stabilized against M=1 modes without an outer conductor. The distortion of the equilibrium into a toroidal system is also discussed.

---

(1) J. Greene, J. Johnson, K. Weimer, Plasma Physics 8, 145 (1966)

(2) A.A. Blank, H. Grad, H. Weitzner, Plasma Physics and Contr. Nucl. Fusion Research, IAEA, Vol. 2, p. 607 (1969).

(3) H. Weitzner, Bull. Am. Phys. Soc. Series II, 14, 1049 (1969)

# Stability of a High $\beta$, Helically Symmetric Pinch*

by

J. P. Freidberg and B. M. Marder

Los Alamos Scientific Laboratory, University of California
Los Alamos, New Mexico

## ABSTRACT

The equilibrium and stability of a high $\beta$ theta pinch with a helical $\ell = 1$ field has been studied in connection with the proposed Scyllac experiment. The stability against the m = 1 mode was investigated by expanding $\delta W$ in two small parameters: the ratio of the plasma radius to the wave length of the helix, and the relative helical displacement of the plasma produced by the $\ell = 1$ winding. It was shown that the configuration is stable to this mode for all values of $\beta$ and all values of wall radius. Furthermore, the stability is sufficiently strong that the system can be bent into a torus without destroying the equilibrium.

*This work performed under the auspices of the U. S. Atomic Energy Commission.

The Resistive Balloning Mode in Stellarator-like Geometries, *

by Russell M. Kulsrud, Princeton University.

An attempt is made to numerically evaluate the growth rate of the resistive balloning mode in the Model C geometry for temperatures of T = 10 - 100 eV. The growth rate is exhibited as a function of mode number m. This diagram is essentially characterized by these numbers; $\gamma$ represents the average curvature, $m_s$ a characteristic mode number related to shear and connection length, and $s_o$ a typical growth time. These can be evaluated for any toroidal device and the results apply. Comparing the tokamak and the stellarators in this regime one finds the most important difference is probably in the value of $\gamma$ which determines the growth rate at small m. If the tokamak $\gamma$ is as large as one might expect, the lower m are stabilized in the tokamak at 10 eV while they are still unstable in the stellarator at this temperature.

---

* Work performed under the auspices of the U.S. Atomic Energy Commission Contract no. AT(30-1)-1238.

Loss-Cone in a Nonaxially Symmetric Toroidal Device,* by

D. Dobrott, R. J. Hastie, and H. Fishman, Princeton University.

The loss-cone for particles in a nonaxially symmetric toroidal device is calculated by means of the stellarator expansion.[1] We focus on an optimally ordered[2] $\ell = 3$ stellarator, wherein the depth of the helical and toroidal field modulations are comparable. The effects of multiple trapping states and of the transition between states are included. In the limit of small gyro-radius, the loss-cone is found to depend upon the ratio of the helical to toroidal field modulations and the pitch angle of the individual particles as in the simple mirror.

References

[1] J. L. Johnson, C. Oberman, R. M. Kulsrud, and E. A. Frieman, Phys. Fluids 1, 281 (1958).

[2] D. Dobrott and E. A. Frieman, Magnetic and Drift Surfaces Using a New Stellarator Expansion, Matt-767 (Submitted for publication to Phys. Fluids).

---

* Work performed under the auspices of the U. S. Atomic Energy Commission, contract no. AT(30-1)1238.

# Double-Hump Instabilities at the Upper Hybrid Frequency in Hot-Electron Plasmas*

D. J. Sigmar and G. E. Guest

Oak Ridge National Laboratory, Oak Ridge, Tennessee

ABSTRACT

Electrostatic instabilities in non-Maxwellian mixtures of hot- and cold electrons[1] can lead to enhanced loss of electrons from MHD-stable mirror traps.[2] Predictions of low threshold density, based on marginal stability analysis and roughly corroborated under some experimental conditions, cast doubt on the feasibility of achieving highly ionized target plasmas by resonant electron cyclotron heating. We have made a parametric study of this instability to determine the marginal stability boundaries and conditions under which the mode is absolutely unstable for a large class of distribution functions and plasma parameters. The results suggest that if the two groups are not too different in temperature and density, $T_C/T_H \geqslant .01$ , $N_C \sim N_H$ , the transition to absolute growth can occur only if $\omega_{pe}^2(\text{total}) \geqslant \Omega_{ce}^2$ , although marginal stability thresholds remain low. It is thus important to minimize reflection of convectively unstable waves at the plasma boundary.

---

*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1] L. S. Hall, W. Heckrotte, and T. Kammash, Phys. Rev. **139**, A1117 (1965).

[2] W. A. Perkins and W. L. Barr, Phys. Fluids **11**, 388 (1968); R. A. Blanken, N. H. Lazar, and W. J. Herrmann, Bull. Am. Phys. Soc. **14**, 1062 (1969).

# FINITE BETA RESONANT NEGATIVE ENERGY INSTABILITY*

H. L. Berk and L. D. Pearlstein

Lawrence Radiation Laboratory, University of California

Livermore, California

The unstable negative energy waves of a mirror machine[1] can be described by a set of fluid equations, and stability boundaries exist for all but a highly resonant mode oscillating at a harmonic of the central cyclotron frequency.[2] At high density, when $\beta > (\frac{m}{M})^{1/2}$, this resonant mode can be described analytically. As a consequence of outgoing wave boundary conditions, the mode is unstable at any scale length in the fluid model. By including thermal effects, critical length boundaries are determined, and the critical scale length, $L_{cr}$, is roughly given by,

$$L_{cr} < 5/\beta^{8/3} .$$

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] H. L. Berk, L. D. Pearlstein, J. D. Callen, C. W. Horton and M. N. Rosenbluth, Phys. Rev. Letters, 22, 876 (1969).

[2] H. L. Berk, L. D. Pearlstein and J. G. Cordey, Bull. Am. Phys. Soc. 14, 1018 (1969).

Submitted to CTR Annual Theory Meeting, April 23-24, 1970, Princeton, N. J.

# INVERTED POPULATION INSTABILITIES*

Laurence S. Hall

Lawrence Radiation Laboratory, University of California, Livermore, California

Velocity space inhomogeneities of mirror confined plasma or "loss-cone" distributions imply the presence of two important free energy reservoirs: the inverted population of perpendicular velocities and the angular anisotropy.[1,2] These reservoirs provide the drive for a whole host of microinstabilities and, for electrostatic modes at least, a quantitative examination of the driver terms can be condensed so that instability criteria are given in terms of simple phenomenological coefficients[3] describing the coupling via Poisson's equation. Moreover, when the angular anisotropy is small or when the electric field of the instability lies very nearly perpendicular to $\underset{\sim}{B}_o$, only the inverted population term is important. The coupling associated with the inverted population has been computed for the interpolating distributions $f \sim v_\perp^{2j} \exp\{-(j+1)v_\perp^2/\langle v_\perp^2\rangle\}$.[4]

The utility of this analysis lies in the physical description it provides for a given instability, and in the immediate quantitative evaluation of stability boundaries. Examples will be given as time permits, but of particular interest are new results describing previously predicted[1] but only recently observed[5] one-species modes whose frequencies are shifted appreciably above the cyclotron harmonics.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] L. S. Hall, et al., Phys. Rev. 139, A1117 (1965).

[2] R. F. Post and M. N. Rosenbluth, Phys. Fluids 9, 730 (1966).

[3] L. S. Hall and W. Heckrotte, Phys. Fluids 9, 1496 (1966).

[4] R. A. Dory, et al., Phys. Rev. Letters 5, 131 (1965).

[5] C. F. Kennel, et al., TRW Systems Report 05402-6017-RO-00 (1970).

Submitted to CTR Annual Theory Meeting, April 23-24, 1970, Princeton, N. J.

# AN INTEGRAL EQUATION MODEL FOR MICROINSTABILITIES IN INHOMOGENEOUS PLASMAS*

C. O. Beasley,Jr., H. L. Berk†, W. M. Farr†† and L. D. Pearlstein†

Oak Ridge National Laboratory, Oak Ridge, Tenn.

In the inhomogeneous plasma theory of Beasley, Farr and Grawe particles are contained electrostatically. This leads to artificial bounce-resonance effects when the electron bounce frequency $\omega_{be} > \gamma$, $\gamma$ the growth rate. This theory has been modified to allow for damping of electron bounce orbits, thereby permitting one to examine instabilities for non-zero frequencies $\omega$ for essentially all $\gamma$ and $\omega_{be}$. Results will be given for unstable eigenmodes corresponding in the infinite plasma to Dory-Guest-Harris modes.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Permanent address: Lawrence Radiation Laboratory, Livermore, California.

††Permanent address: Dept. of Nuclear Engr., The University of Arizona, Tucson, Arizona.

Submitted to CTR Annual Theory Meeting, April 23, 24, 1970, Princeton, N. J.

Session D. Miscellaneous
(P. K. Kaw, Chairman)

D 1. "Low-frequency Electron Oscillations in Bounded Plasmas," D. E. Baldwin

D 2. "Drift Loss-Cone Simulation Experiments and Theory for Large Orbits, $a_i \sim R_p$," C. K. Birdsall, D. Fuss, and A. B. Langdon

D 3. "Numerical and Analytical Study of Collisionless Drift-Universal Modes," J. McCune and K. von Hagenow

D 4. "The Correspondence of Two Approaches to Drift Waves in Sheared Fields," J. N. Davidson and T. Kammish

D 5. "Nonlinear Interaction of a Weak Cold Beam and a Plasma," T. M. O'Neil, J. F. Winfrey, and J. H. Malmberg

D 6. "Transition from Adiabatic to Stochastic Behavior," M. A. Lieberman and A. J. Lichtenberg

D 7. "Astron E-Layer," M. E. Rensink and T. K. Fowler

D 8. "Particle Dynamics in a Direct Converter," R. F. Post

Abstract of paper to be presented at Sherwood Theory Meeting, Princeton, April 23-24, 1970.

Low-frequency Electron Oscillations in Bounded Plasmas,* D. E. Baldwin, Yale University and LRL, Livermore - The effects of finite plasma length, finite electron temperature, and the existence of low density external plasma upon low-frequency electron oscillations in a strongly magnetized plasma are considered. The purpose is to consider damping mechanisms available to these modes in mirror machines when they may be unstable due to coupling to ions with a loss-cone distribution. It is found in zero-temperature that the rate of decay of the mode due to leakage from the high- to low-density regions is little altered by changing a discontinuous density transition to a narrow but distributed one. Finite but small electron temperatures introduce traditional Landau damping and an altered version of electron-wave interaction which occurs in the presence of the static potentials of the density transition region. Finally, when the temperature is high enough that the electrons spend a small part of a period in the transition region, but low enough that the Debye length is small compared to the thickness of this region, it is found that a wave is almost perfectly reflected by the transition region, resulting in only weak damping of the electron oscillations.

* This work was supported in part by the Atomic Energy Commission.

# DRIFT-LOSS-CONE SIMULATION EXPERIMENTS AND THEORY FOR LARGE ORBITS, $a_i \sim R_p$ *

C. K. Birdsall†, D. Fuss and A. B. Langdon†

Lawrence Radiation Laboratory, University of California

Livermore, California

Simulation experiments have been done in two dimensions with single speed ions, cold electrons, $k_\perp$ only, uniform $B_o$, with guiding center density initially given by $n(x) = n_o(1 + \varepsilon\cos k_o x_{gc}), 0 \leq \varepsilon \leq 1$. For sufficiently large $\varepsilon$ and $(\omega_{pi}/\omega_{ci})^2$ and for small orbits $(a_i = R_p/2 \approx L/8 = (2\pi/k_o)/8)$, the expected growth from small amplitudes was seen $(0 < k_y a_i < 4,\ \omega_{imag} \approx \omega_{real} \approx \omega_{ci}/2)$. Both $T_i$ and $T_e$ increase, up to $q\phi \approx kT_i$, accompanied by rapid flattening of the density. For larger orbits $(a_i = R_p)$, at the same $(\varepsilon\ a_i)$ product, the plasma was found to have negligible growth in the range $k_y a_i < 4$; doubling either $n_o$ or $\varepsilon$ brought back the instability. Thus large orbits, encircling the plasma, appear to shift the stability boundary in $n_o$, $\varepsilon$ and,or $k_y a_i$.

A linear theory has been set up for this model, allowing arbitrarily large orbits, including those encircling the plasma. The general formulation uses the zero order distribution $f_o = n_o(1 + \varepsilon \cos k_o x_{gc})F(\frac{1}{2}v_\perp^2, v_z)$ with first order potential given by $\phi(\underline{x},t) = \exp i\ (k_y y + k_z z - \omega t)\sum_p \phi_p \exp i(pk_o x)$. These assumptions lead to a tridiagonal equation for $\{\phi_p\}$ which can be solved by truncating the series at $p_{max}$ terms, producing a cluster of $p_{max}$ roots near $n\omega_{ci}$. Solutions which have been obtained for the simulation $f_o \sim \delta(v_\perp - v_o)$, with $k_z = 0$, for large and small orbits, will be discussed.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission

†Permanent address: Electrical Engineering and Computer Sciences Department, University of California, Berkeley, California

To be submitted to the CTR Annual Theory Meeting, April 23, 24, 1970, Princeton.

# Numerical and Analytical Study of Collisionless Drift-Universal Modes*

J. McCune, M.I.T., and K. von Hagenow, IPP, Garching, Germany

Drift universal modes which do not meet the Kadomtsev-Krall-Rosenbluth sufficient condition for stability are studied using the "slab" model of an inhomogeneous plasma. Marginal stability information is provided over a wide range of $k_y a_i$ , $k_z/\varepsilon'$ and $k^2\lambda_D^2$ , both for proton and heavy-ion plasmas. Growth rates and real frequencies are obtained numerically in the unstable regime and compared with various approximate analytical formulas. The modes frequently violate the "classical" condition $v_e > \omega/k_z > v_i$ , particularly for proton plasmas. Maps of growth rates and frequencies in the $k_y - k_z$ plane are constructed for typical experimental conditions and comparison with selected experiments discussed. The non-resonant drift instability of Kadomtsev and Timofeev is studied for possible application to toroidal devices with long effective connection lengths along $\underset{\sim}{B}$ .

*Work partially supported by AFOSR (Grant No. 69-1697)

"The Correspondence of Two Approaches to Drift Waves in Sheared Fields"*

J. N. Davidson and T. Kammash
University of Michigan, Ann Arbor, Michigan

Two methods have been used to study the effect of sheared magnetic fields on drift waves. One follows a highly localized wave packet which moves across the plasma(1) and the other solves for normal modes of oscillation with the boundary condition of outward energy flow.(2) However, the two methods lead to stability conditions which are virtually identical. In this paper, an explanation for this correspondence is provided.

It is shown that in the wave packet approach: 1) the "wave packet" generally considered is in fact a small portion, confined between characteristics of the eikonal equation, of a perturbation of undetermined extent; 2) the normal mode is included in the eikonal solutions; and 3) the most unstable case occurs precisely when the initial perturbation has the same characteristics as the normal mode over some portion of the plasma.

By identifying intermediate equations in both methods with the small amplitude power theorem, it is shown that although the physics of the two methods is the same the slight difference in the form of the stability conditions is due to considering two different quantities. In the normal mode approach, only the size of the perturbation potential is considered in determining stability, while in the "wave packet" approach the small amplitude energy density, which contains both wave and particle energies, is considered.

*Work supported by the U.S. Atomic Energy Commission.
1.) P. Rutherford & E. A. Frieman, Phys. Fluids, 10, 1007 (1967).
2.) L. D. Pearlstein & H. L. Berk, Phys. Rev. Letters, 23, 220 (1969).

# Nonlinear Interaction of a Weak Cold Beam and a Plasma

T. M. O'Neil, J. H. Winfrey, and J. H. Malmberg

University of California at San Diego

La Jolla, California

ABSTRACT

Recently a simple model was proposed for the nonlinear interaction of a weak monoenergetic beam and an infinite homogeneous one dimensional plasma[1]. The essential feature of this model is the observation that after several e-foldings the bandwidth of the growing waves is so narrow that the electrons interact with a very nearly pure sinusoidal field.

In terms of this single wave model, we present a properly scaled solution which depends analytically on all the basic parameters of the problem (i.e. plasma density, beam density, and beam drift velocity). Our solution shows that the single wave grows exponentially at the linear growth rate until the beam electrons are trapped. At that time the wave amplitude stops growing and begins to oscillate about a mean value. During the trapping process the beam electrons are bunched in space and a power spectrum of the higher harmonics of the electric field is produced. Both the oscillation in wave amplitude and the power spectrum are given a simple physical interpretation and compared to prior experimental results[2].

[1]Nonlinear Development of the Beam-Plasma Instability, W. E. Drummond, J. H. Malmberg, T. M. O'Neil, submitted to Phys. Fluids.

[2]J. H. Malmberg and C. B. Wharton, Phys. Fluids 12, 2600 (1969); and J. R. Apel, Phys. Fluids 12, 640 (1969)

Supported in part by AEC Contract AT(11-1)-34 and in part by NSF grant GP-9461.

# TRANSITION FROM ADIABATIC TO STOCHASTIC BEHAVIOR

by
M.A. Lieberman and A.J. Lichtenberg
University of California, Berkeley, Calif.

The Fermi Problem of a ball bouncing between a fixed and an oscillating wall is reexamined. Numerical studies show that the phase plane consists of a complicated but regular structure of adiabatic islands embedded in a non-adiabatic sea. The adiabatic regions can be described analytically by expansions about elliptic singular points. A velocity below which no adiabatic regions exist is predicted from Floquet theory. It is shown that below this transition velocity the random phase assumption holds and the particle motion can be described by a Fokker-Planck equation. Above the transition velocity higher order correlations exist in the non-adiabatic portion of the phase plane. Computations demonstrate that, in some cases, an adiabatic wall forms an upper limit to particle diffusion in velocity space. Introduction of an external random component modifies, but does not destroy, the basic results. The application of the results to a wide class of problems, including cyclotron resonance heating, is discussed.

# ASTRON E-LAYER EQUILIBRIA*

M. E. Rensink and T. K. Fowler

Lawrence Radiation Laboratory, University of California

Livermore, California

The Astron E-layer is characterized by a set of parameters which describe the macroscopic current distribution and the microscopic phase space distribution of the particles. Approximate equations relating these parameters have been derived. The conditions necessary for obtaining equilibria with certain desirable properties will be discussed.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

Prepared for presentation at the Annual CTR Theory Meeting, Princeton, N. J. April 23, 24, 1970.

Abstract- 72384

# PARTICLE DYNAMICS IN A DIRECT CONVERTER*

R. F. POST

Lawrence Radiation Laboratory, University of California
Livermore, California

Studies have been made of some aspects of the flow of ions and electrons through the structure of a direct converter that uses adiabatic magnetic expansion followed by charge separation and electrostatic deceleration of the separated streams in its operation. Computer codes have been used to obtain quantitative data on the various processes that occur in the converter. There seem to be no significant limits on achieving as high an efficiency as desired, although space charge effects ultimately introduce limits on the energy fluxes that can be handled.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Submitted to CTR Annual Theory Meeting, April 23-24, 1970, Princeton, N. J.

Session E. No Oral Presentation

E 1. "Runaway Electrons and the Intensity of Plasma Oscillations," G. Bateman

E 2. "Finite Larmor Radius Effects on Toroidal Low-$\beta$ Confinement," E. Bowers and N. K. Winsor

E 3. "Quasi-Three Dimensional Partical Code for Simulating an Anistropic Plasma," R. N. Carlile and J. M. Dawson

E 4. "Numerical Solution of the Vlasov's Equation," C. K. Chu, P. Sakanaka, R. T. Taussig, and J. C. Whitney

E 5. "Computation of the Equilibrium of a Plasma with Helical Symmetry," N. Friedman

E 6. "Temperature Relaxations in Plasmas with Magnetic Field," S. Ichimaru and M. N. Rosenbluth

E 7. "Stability of a Mirror Machine on the Drift Time Scale," A. Kadish

E 8. "Propagation of Relativistic Beams in Magnetised Plasma," R. Lee and R. N. Sudan

E 9. "Linearized Variational Analysis of One-Dimensional Vlasov Plasma," H. R. Lewis

E 10. "Anomalous Transmission and Reflection of Cyclotron Waves," J. Marsh

E 11. "Singular Perturbation Analysis of Theoretical Models for Warm Inhomogeneous Plasmas," R. M. Miura and E. M. Barston

E 12. "Near-Steady Oblique Shock Waves in a Collisionless Plasma," L. M. Olson

E 13. "Vlasov Equilibria of Finite Beta Axisymmetric Toroidal Configurations," E. Ott and R. N. Sudan

E 14. "Energy Transfer from a Whistler Wave into Electrostatic Modes via Trapped Particle Instability," G. Schmidt

E 15. "Numerical Investigation of Electrostatic Plasma Waves and Their Role in 'Anomalous Resistivity'," C. Spight, F. W. Perkins, and C. R. Oberman

E 16. "On the Hamiltonian Character of the Leapfrog Algorism," K. R. Symon

E 17. "A Theory for the Non-adiabatic Effects in Magnetic Traps," R. K. Varma

E 18. "Ignition of Thermonuclear Reactions by an Intense Spark Discharge," F. Winterberg

E 19. "Rotation of Tokamak Equilibria," R. D. Hazeltine, Edward P. Lee and M. N. Rosenbluth

E 20. "Quasilinear Analysis of Interaction between Normal Modes and Particles in a Plasma Cylinder," Allan N. Kaufman

# Runaway Electrons and the Intensity of Plasma Oscillations

Glenn Bateman
Courant Institute of Mathematical Sciences
New York University

A quasi-steady state distribution is found for the runaway electrons of a uniform plasma in a small externally imposed electric field (but no magnetic field). The intensity of electron plasma oscillations is greatly enhanced above the thermal intensity. Hence, the resonant interactions of runaways with the waves are more important than scattering off thermal particles. An asymptotic solution of the Balescu-Lenard equation is found, for times short on the time-scale for the depletion of thermal electrons, and for velocities nearly parallel to the field. Consistent with this distribution, the spectral intensity (square of the fluctuating potential) near the direction of the field is

$$I_{\hat{k}}(v_p) \sim E^{-2} v_p^5 \sqrt{\pi}\, m_e v_{The}^2 / \hat{k}\cdot\hat{E} \left\{ 1 + \frac{1}{\ln v_p} + \mathcal{O}\left(\frac{1}{\ln v_p}\right)^2 \right\} .$$

Here, $E << 1$ is the electric field measured in units of the Dreicer runaway field ( $\frac{m}{e} \omega_{pe} v_{The} \varepsilon$ , $\varepsilon \equiv 1/n_e \lambda_{De}^3 << 1$ ) and $v_p >> 1$ is the phase velocity of the electron plasma oscillations in units of the runaway velocity ( $E^{-1/2} v_{The}$). This spectral intensity is much larger than that generated by a Maxwellian distribution ( $\frac{1}{2} E^{-1} v_p^2$ ) and may serve as a source of heating energy. These results must be altered when higher-order corrections (in $\varepsilon$ ) dominate in the kinetic equations -- e.g. when collisional damping dominates over Landau damping.

Finite Larmor Radius Effects on Toroidal Low-$\beta$ Confinements,*

by E. Bowers and N. K. Winsor, Princeton University.

Assuming the inverse of the aspect ratio to be small Larmor radius corrections[1] are included in a low-$\beta$ fluid model.[2] In inhomogeneous plasma the characteristic frequencies of parallel acoustic and geodesic acoustic[3] modes are significantly altered. The behavior of Stringer rotation[4] is modified by inclusion of finite Larmor radius terms. These terms have been included in a numerical simulation model. The behavior of this model is very sensitive to boundary conditions. The pump-out time with quasi-static conditions and no guiding center velocity present is not significantly different from results obtained in the absence of drift terms. However, as expected from theory the buildup of potential in the presence of gyroviscous terms is significantly altered.

References

[1] A Simon and W. B. Thompson, J. Nucl. Energy, Pt. C 8, 373 (1966).

[2] N. K. Winsor, Thesis (Princeton University, 1969).

[3] N. K. Winsor, J. L. Johnson, and J. M. Dawson, Phys. Fluids 11, 2448 (1968).

[4] T. E. Stringer, Phys. Rev. Letters 22, 770 (1969).

* Work performed under the auspices of the U. S. Atomic Energy Commission Contract no. AT(30-1)1238; use was made of the computer facilities supported in part by National Science Foundation Grant NSF-GP 579.

Quasi-Three Dimensional Particle Code for Simulating an Anisotropic Plasma.*

R. N. Carlile, University of Arizona, and J. M. Dawson, Princeton University.

This paper describes a particle code, which is an extension of that described by Kruer and Dawson,[1] in which particles are simulated by charged rods constrained to be parallel to the z axis, but otherwise free to move, so that each rod is completely described by coordinates $(x,y,v_x,v_y,v_z)$. The forces which a rod may experience are the self-consistent coulomb force, and a lorentz force due to an externally imposed, homogeneous magnetic field of any desired three dimensional orientation. The chief limitation of this code is that the coulomb force is two dimensional; however, it is ideally suited to simulate plasmas whose behavior can be described by electrostatic waves with coplanar wave numbers. We have investigated the linear behavior of this code, and have found that it will support Bernstein modes in at least the three lowest bands. The usual theory of Bernstein modes predicts the modes found in our code within experimental error. We have started to investigate the non-linear behavior by driving two Bernstein modes at $(\omega_1, \underset{\sim}{k}_1)$ and $(\omega_2, \underset{\sim}{k}_2)$ to large amplitudes. We have found resonant wave-wave scattering in which a third Bernstein mode appears at $(\omega_3, \underset{\sim}{k}_3)$, and for which the relations $\omega_3 = \omega_1 + \omega_2$ and $\underset{\sim}{k}_3 = \underset{\sim}{k}_1 + \underset{\sim}{k}_2$ are satisfied.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] W. L. Kruer and J. M. Dawson, Bull. Am. Phys. Soc. 14, 1025 (1969).

ABSTRACT

Numerical Solution of the Vlasov' Equation

by

C. K. Chu, P. Sakanaka, R. T. Taussig and J. C. Whitney
Columbia University

Two direct finite difference methods for solving the one-dimensional Vlasov equation have been proposed and tested. The first treats the $E\frac{\partial f}{\partial v}$ term in the Vlasov equation as if it were a collision term, and integration proceeds along a constant characteristic direction in $x - t$ space for each $v$. The other method is essentially a "charge sharing" in phase space. Values of the distribution function $f$ are assigned to grid points in $x - v$ space, the grid points are moved according to $dx/dt = v$, $dv/dt = E$, the $f$ values are convected, and they are then proportionately reassigned to the four corners of the new cell. These direct numerical solutions should be applied to problems which have moderately long problem time, smooth distribution in $v$, and not too simple spatial dependence.

Two problems have been solved by these methods. The first is the classical two stream instability problem which because of the discontinuous nature of the distribution function, represents a severe test for the methods. However, results compare rather well with that from the water bag model, in both phase plane contours and field energy variations with time. The other problem studied is the generation of ion acoustic waves from an initial density discontinuity. Results are in good agreement with the particle in cell calculations of Mason.

*This work was supported by AEC Contract AT(30-1)-3954 and partly by NSF Grant GK-1391-x-2 .

# Computation of the Equilibrium of a Plasma with Helical Symmetry

Neal Friedman
Courant Institute of Mathematical Sciences
New York University

This paper is concerned with calculating the equilibrium configuration of a plasma under magnetohydrodynamic assumptions. We assume that the magnetic field which confines the plasma is helically symmetric and that it is zero inside the plasma ($\beta = 1$). This entails solving an elliptic partial differential equation on a domain with a free boundary. A rectangle with fixed boundary is introduced as an auxiliary domain and the differential equation is solved there. Two variational problems are considered. One is the minimization of the Dirichlet-Douglas integral for conformal mapping; the other is the minimization of an energy integral connected with the free boundary. The method of steepest descent is used to develop an iterative procedure suggested by the problem of minimizing these two functionals. Convergence of this algorithm yields a conformal mapping of the rectangle onto the original domain. The equations in this method are approximated by a finite difference scheme which is accurate to second order. A program to implement it has been written for the CDC 6600 computer and has been run for a variety of magnetic fields suggested by Scyllac parameters.

# TEMPERATURE RELAXATIONS IN PLASMAS WITH MAGNETIC FIELD†

Setsuo Ichimaru and Marshall N. Rosenbluth
Institute for Advanced Study, Princeton, New Jersey 08540

We calculate the Fokker-Planck coefficients of a plasma in the magnetic field in terms of the spectral function of the electric-field fluctuations and the dielectric response function; the results are used for the investigation of temperature relaxation processes. We find that the relaxation rate $1/\tau_{ei}$ between the electron and ion temperatures contains an anomalous term $1/\tau^{*}_{ei}$ in addition to the usual Spitzer rate $1/\tau^{(0)}_{ei}$; when $\Omega_e^2 >> \omega_e^2$ ($\Omega_e$: electron cyclotron frequency; $\omega_e$: electron plasma frequency),

$$(1/\tau^{*}_{ei})/(1/\tau^{(0)}_{ei}) = \ln (\Omega_e^2/\omega_e^2) \ln (m_i/m_e)/4 \ln \Lambda \quad ,$$

where $m_i/m_e$ is the mass ratio between the ion and the electron, and $\ln \Lambda$ is the Coulomb logarithm. The physical origin of the anomalous term is the strong coupling between the spiral motion of the electrons and the long-wavelength, low-frequency fluctuations produced by the ions. Assuming $\ln \Lambda \simeq 15$ and $\Omega_e^2/\omega_e^2 \simeq m_i/m_e$, we find the above ratio to be approximately unity for a deuterium plasma so that the actual relaxation rate may become twice as fast as the Spitzer rate. In the cases of the temperature relaxation between the parallel and perpendicular directions to the magnetic field, the existence of the fluctuations due to the well-defined Bernstein modes is found to provide additional relaxation processes just as effective as the usual mechanism due to the screened Coulomb interaction.

†Work supported in part by the AEC under Contract AT(30-1)-3927.

# Stability of a Mirror Machine on the Drift Time Scale

Abraham Kadish
Courant Institute of Mathematical Sciences
New York University

Using a model for slowly varying drift equilibria, the stability of a symmetric finite $\beta$ mirror machine has been studied. Magnetic field curvature has been included without simulation. Sufficient conditions for stability have been derived. These suggest that finite $\beta$ mirror machines may be easier to stabilize than low $\beta$ devices. The possibility of such a stable configuration is contrasted with drift instabilities of other theories.

# PROPAGATION OF RELATIVISTIC BEAMS IN MAGNETISED PLASMA*

Roswell Lee and R. N. Sudan
Cornell University

We have extended the model developed by Hammer and Rostoker[1] to the injection of high current relativistic beams into a cold magnetized plasma. In this model, the beam electrons are assumed to be undeflected from their zero order orbits and the fields associated with the beam are switched on at time $t = 0$. The return current flowing in the plasma, and the associated electromagnetic fields are obtained. We show that the return current does not extend to infinity but dies away inversely as the distance from the head of the beam with a characteristic distance $L=v_o a^2/\nu\lambda_E^2$, where $v_o$ and $a$ are the beam velocity and radius, $\nu$ is the collision frequency of the plasma electrons, $\lambda_E=c/\omega_p$, and $\omega_p$ is the plasma frequency of the plasma electrons. When the beam is injected parallel to a static magnetic field B, it will be magnetically neutralized by a return current over a length of order L if $a \gg \lambda\Omega/\omega_p$ where $\Omega=eB/mc$. When the beam is injected perpendicular to a static magnetic field the beam is found to be magnetically neutralized for a length of order L if $a \gg \lambda_E\omega_h/\omega_p$ where $\omega_h=(\omega^2_p+\Omega^2)^{1/2}$.

*Work supported by AEC contract AT(30-1)-4077.

[1]D. Hammer and N. Rostoker, Cornell University, Laboratory of Plasma Studies, Report No. LPS 16, June 1969.

# Linearized Variational Analysis of One-Dimensional Vlasov Plasmas*

by

H. Ralph Lewis

Los Alamos Scientific Laboratory, University of California
Los Alamos, New Mexico

## ABSTRACT

The Lagrangian formulation for Vlasov plasmas has been applied to the linearized analysis of neutralized, one-dimensional, single-species plasmas with periodic boundary conditions. This formulation is very suitable for investigating the eigenfrequencies and the solution of the initial-value problem with arbitrary equilibrium velocity distributions. For any degree of approximation, the initial-value problem is reduced to the solution of a system of ordinary differential equations in time with constant coefficients and time-dependent driving terms, and an exact particular solution has been found. Numerical examples are presented for a Maxwellian equilibrium velocity distribution and for several nonanalytic equilibrium velocity distributions for which the velocities are bounded.

The method has been generalized to the general case of Vlasov plasmas.

*This work performed under the auspices of the U. S. Atomic Energy Commission.

# Anomalous Transmission and Reflection of Cyclotron Waves*

J. Marsh†

Plasma Physics Laboratory, Princeton University

A formalism is developed to describe the propagation along a slowly varying field of purely transverse cyclotron waves. Starting from the coupled set of linearized Vlasov-Maxwell equations, expansions are made in terms of the two small parameters $\frac{\Omega_{max}-\omega}{\omega}$ and $(k_\infty L)^{-1}$, where $\Omega_{max}$ is the maximum value of the gyro frequency, $\omega$ is the wave frequency, $k_\infty$ is the wave number at infinity and L is the scale length of variation of the magnetic field. This is applied to the case where the field configuration is a magnetic mirror with small field variation which includes points of cyclotron resonance, and with a low-$\beta$ plasma which includes both trapped and untrapped particles. The trapped particles are found to cause anomalous transmission and reflection of the wave by a nonlocal mechanism. The trapped particles participate in the damping of the incident wave in the propagation region before cyclotron resonance and, after free-streaming across the central region of the mirror where the wave is evanescent, excite transmitted and reflected waves by coherently releasing the electromagnetic energy they have previously acquired from the incident wave in the damping process.

* Work performed under the auspices of the U.S. Atomic Energy Commission, Contract no. AT(30-1)-1238.

† Now at Courant Institute of Mathematical Sciences, New York University.

# Singular Perturbation Analysis of Theoretical Models for Warm Inhomogeneous Plasmas

R.M. Miura and E.M. Barston

Courant Institute of Mathematical Sciences

New York University

Due to the complexity of the Vlasov-Poisson equations for inhomogeneous plasmas, it is of general interest to investigate simpler approximate models. We compare three specific models proposed in the literature(1-4) by means of a singular perturbation expansion in powers of $\epsilon = (\lambda_D/L)^{2/3}$ (L = characteristic plasma length). We find that, whereas the electric-field eigen-functions differ in zero order for the macroscopic and microscopic models, the eigen-frequencies are identical to order $\epsilon$.

---

(1) G. Dorman, J. Plasma Phys. 3, 387 (1969)

(2) J. V. Parker, J. C. Nickel, R. W. Gould, Phys. Fluids 7, 1489 (1964)

(3) B. B. O'Brien, Jr., Plasma Phys. 9, 282 (1967)

(4) D.E. Baldwin, J. Plasma Phys. 1, 289 (1967)

# Near-Steady Oblique Shock Waves in a Collisionless Plasma

Lynn M. Olson

The University of Wisconsin, Madison

Waves of small amplitude $\varepsilon$ which travel at an oblique angle $\theta$ to a magnetic field in a cold collisionless plasma and whose head approaches steadiness are studied. The relevant double limit $\lim_{\varepsilon \to 0} (\lim_{t \to \infty})$ is treated in terms of equivalent sets of single limits. Near-steady waves which can lead from one equilibrium to another are shown to be possible only at propagation speeds corresponding to Alfvén Mach number 1 or $\cos \theta$. In the first case, either the magnetic pressure at the head of the wave decreases monotonously from its initial value and the wave spreads linearly with time, or else the head of the wave consists of a near-periodic train of conoidal waves, the first crest of which approaches a solitary wave solution. For $M \cot \theta > 1$, where $M = (m_+/m_-)^{\frac{1}{2}} - (m_-/m_+)^{\frac{1}{2}}$, the magnetic pressure in the near-periodic wave train always exceeds its initial equilibrium value. But for $M \cot \theta < 1$, it always falls short of that equilibrium value. A transition with precursor wave is shown to be impossible for a cold collisionless plasma. At an Alfvén Mach number $\cos \theta$, the only near-steady solution consists of a main wave-front followed by an oscillatory tail; on the length scale considered, the electric field transverse to the propagation direction oscillates without bound at the tail.

VLASOV EQUILIBRIA OF FINITE BETA AXISYMMETRIC TOROIDAL CONFIGURATIONS*

E. Ott and R. N. Sudan

Cornell University

The equilibrium of a finite beta Tokamak has been obtained from the Vlasov equation by a perturbation technique with the aspect-ratio r/R as the small parameter, where r and R are the minor and major radii respectively. We have considered two cases where in zero-order the solutions are: a) the linear Bennett pinch, b) the case of a hollow current linear pinch which results from a distribution function that is a delta function in both energy and momentum[1].

*Work supported by AEC contract AT(30-1)-4077.

[1]D. Hammer and N. Rostoker, Cornell University, Laboratory of Plasma Studies, Report No. LPS 16, June 1969.

# Energy Transfer From a Whistler Wave into Electrostatic Modes via Trapped Particle Instability.

George Schmidt

Stevens Institute of Technology

It has been shown[1] that cyclotron damping of a large amplitude electromagnetic wave propagating along a magnetic field leads to amplitude oscillations and finally to an undamped finite amplitude wave. In analogy to the electrostatic case[2], this behaviour is largely due to trapped particles, trapped in this case in "magnetic wells", executing oscillations parellel to the main field, in a frame traveling with the velocity $u = \frac{\omega - \Omega c}{k}$ The corresponding bounce frequency is $\omega_\beta = \sqrt{k\Omega_1 V_\perp}$, where $\Omega_1 = eB_1/m$, $B_1$ is the wave magnetic field.

It is shown that the resulting "magnetic BGK mode" is subject to an electrostatic trapped particle instability, that couples energy from the transverse electromagnetic wave into growing electrostatic oscillations with the maximum growth rate $\gamma_{max} = 1,35\ \omega_p (\mathcal{T}\omega_T)^2$. Here $\omega_T$ is the plasma frequency of trapped particles, $\mathcal{T} = (k\Omega_1 V_T)^{-1/2}$, $V_T$ is the thermal velocity in a Maxwellian plasma. The dispersion relation for this instability differs in character from the one obtained for electrostatic BGK modes[3], and can be evaluated analytically by conventional methods.

1. P. Palmadesso, G. Schmidt, Bull. Am. Phys. Soc. 14, 1035, (1969).6E8.

2. T. O'Neil, Phys. Fluids, 8, 2255 (1965).

3. W. L. Kruer, M. Dawson, R. N. Sudan, Phys. Rev. Letters, 23, 838 (1969).

<u>Numerical Investigation of Electrostatic Plasma Waves and Their Role in "Anomalous Resistivity"</u>.* C. Spight, F. W. Perkins, and C. Oberman, <u>Princeton University</u>. -- A computer code has been written which calculates the wave frequency and growth rates for linearized electrostatic modes of an electron-ion plasma in a uniform magnetic field. The electron and ion distribution function are essentially arbitrary in the code. Cases have been investigated in which the ions are assumed to have a Maxwellian distribution, and the electrons are assumed to have a Maxwellian distribution drifting relative to the ions and a "Spitzer-Harm" distribution. The low frequency ion waves and the high frequency electron-plasma wave have been investigated over wide variations in magnetic fields, wave propagation direction, temperature ratios, and relative drift velocities. These calculations are being used to support an analytic investigation of "anomalous resistivity."

*Work supported by the U. S. Atomic Energy Commission and by the U. S. Air Force Grant AF49-(638)-1555.

On the Hamiltonian Character of the Leapfrog Algorism

K.R. Symon, University of Wisconsin

It is possible to write a Hamiltonian, periodic in the time, which yields equations of motion whose solutions pass through the points calculated by using the leapfrog algorism used in numerical computations of particle orbits. A variety of analytical methods are then available for studying the effects of the numerical method on the computed orbits. In particular, the smooth approximation provides an effective potential, independent of time, which yields equations whose solutions approximate the results of the leapfrog algorism. By comparing the smoothed potential with the potential in the real problem one can study the errors introduced by the numerical algorism. It is even possible to modify the potential used in the computations so that the effective smoothed potential more closely approximates the potential in the real problem. The analytical methods developed for studying non-linear equations with periodic coefficients can be used to estimate the effects of linear or non-linear resonances between the periods of the orbital motion and the time step used in the computations.

# A THEORY FOR THE NON-ADIABATIC EFFECTS IN MAGNETIC TRAPS

Ram K. Varma
University of Texas, Austin, Texas

A theory is obtained which describes the average non-adiabatic behaviour of a charged particle in a magnetic trap. The average is taken over an ensemble generated as a result of the non-adiabatic fluctuations of the first action invariant $\mu = \frac{1}{2} mC_{\perp}^{2} / \Omega$, $\Omega = eB/mc$, as the particle moves in the inhomogeneous magnetic field. The theory is capable of giving the mean life times of charged particles in magnetic traps which can be compared with the experimental life times.

It is found that the average behaviour is described by a Schrödinger-like equation

$$- i \bar{\mu} \frac{\partial \Psi}{\partial t} = - \frac{\bar{\mu}^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi$$

where the role of $\hbar$ is played by $\bar{\mu}$, the value of the action invariant at injection, and the potential V entering the equation is precisely the potential which describes the guiding center motion in the zero Larmor radius limit, $V = \bar{\mu}\Omega$. It is shown that $\Psi^* \Psi$ indeed represents the density, so that $\Psi$ has the meaning of a probability amplitude. The non-adiabatic escape of particles from mirror traps thus appears to be in the nature of "tunnelling effect" in quantum mechanics.

The life time is calculated, using the standard techniques of quantum mechanics, for the parameters of two experiments,[1,2] and the agreement with the experimental results is found to be surprisingly good.

$T_{th}$ (sec) $\simeq 10^{-7} \exp(0.29\, B_o)$ against $T_{exp} = 3 \times 10^{-7} \exp(0.145 B_o)$ (Balebanov and Semashko[1])

$T_{th}$(sec) $\sim \exp(0.09 B_o)$ against $T_{exp} \sim \exp(0.10 B_o)$ (Dubinina et al[2])

where $B_o$ is the magnetic field at the point of injection.

---

[1] V.M. Balebanov and N. N. Semashko, Nucl. Fusion 7, 207, (1967)

[2] A. N. Dubinina et al. Plasma Phys. 11, 551, (1969)

# IGNITION OF THERMONUCLEAR REACTIONS BY AN INTENSE SPARK DISCHARGE

F. Winterberg, University of Nevada, Las Vegas

It is shown that a high voltage, high current spark discharge in a dense thermonuclear material may create conditions suitable for a small thermonuclear detonation. The spark discharge draws its energy from a Marx surge generator at the end of a low inductance transmission line. The spark discharge develops as a streamer and is associated with a strong magnetic field. The material within the spark channel will attain thermonuclear temperatures by turbulent heating resulting from unstable electron ion plasma oscillations. Because of the high current and the small size of the spark channel, a very high magnetic field of many megagauss can be expected to occur. This high magnetic field will retard the radial expansion of the spark channel and will greatly reduce the electronic heat conduction losses. It will also quench the range of the charged fusion products to the vicinity of the spark channel to such an extent that a small thermonuclear detonation is ignited. The problem of beam focússing occurring in a similar proposed system in which an intense relativistic electron beam is used to ignite a thermonuclear micro-explosion by bombardment does not occur here. In the spark discharge envisioned here, strong focussing can be achieved and, as a result of this in conjunction with the strong magnetic field, a much smaller energy input for ignition seems possible. A lower limit for the energy input to trigger a small fusion explosion is computed to be $10^5$ joules. Because the electrons will assume relativistic energies within the spark channel, the channel should remain stable with regard to kinks. A controlled sequence of such thermonuclear micro-explosions might be used for useful energy conversion.

# ROTATION OF TOKAMAK EQUILIBRIA

R. D. Hazeltine, Edward P. Lee, and
M. N. Rosenbluth

Institute for Advanced Study

## ABSTRACT

It is found that finite resistivity causes the self-consistent tokamak equilibria to be unstable with respect to the appearance of rotation about the magnetic axis. The growth rate is $\tau^{-1} \approx \frac{8\pi^2}{\iota^2} \frac{\eta}{a^2} \frac{R^2\beta}{a^2}$ which is comparable with the skin penetration rate $\eta / a^2$. We employ simple M. H. D. theory with resistivity--possible stabilization by other dissipative effects is not considered. The quantities $\eta$, $\beta$ and $a/R$ = (aspect ratio)$^{-1}$ are regarded as small, but the flux surfaces depart from circular cross section by an amount of order $a/R$. This result is analogous to previous low $\beta$ results obtained for an externally fixed field.

# QUASILINEAR ANALYSIS OF INTERACTION BETWEEN NORMAL MODES AND PARTICLES IN A PLASMA CYLINDER

Allan N. Kaufman

Department of Physics and Lawrence Radiation Laboratory
University of California, Berkeley, California

April 3, 1970

ABSTRACT

A "plasma cylinder" is an equilibrium configuration with cylindrical symmetry: the scalar and vector potentials are functions of radius only. No restrictions are placed on magnetic shear, radial electric field, or gyroradius. The equilibrium particle distributions are functions of the three invariants $J_r$, $P_\theta$, $P_z$ (radial action, canonical angular momentum, canonical z-momentum). The normal modes are solutions of the linearized Maxwell equations, and have the form

$$\underline{E}(r) \exp(im\theta + ikz - i\omega t).$$

A normal mode has energy, angular momentum, z-momentum in the ratio $\omega$:m:k.

With the three conservation laws for particle-mode interaction, the quantum rate equations lead (as $\hbar \to 0$) to coupled kinetic equations for the particle distributions and the mode energies. These equations satisfy an H-theorem, indicating monotonic evolution to thermal equilibrium. The mode-particle coupling coefficient is derived from a classical study of the emission process, and satisfies the resonance condition

$$\omega - k\langle\dot{z}\rangle - m\langle\dot{\theta}\rangle = \ell\,\omega_r ,$$

where $\langle\ \rangle$ indicates average over one radial bounce, $\omega_r$ is the bounce frequency, and $\ell$ is an integer. From this coefficient are found the linear growth rate of a normal mode, the conductivity kernel $\underset{\sim}{g}(\underline{r},\underline{r}';\omega)$, and the diffusion tensor $\underset{\sim}{D}(J_r, P_\theta, P_z)$.