Highlights from Sherwood 2016

International Sherwood Fusion Theory Conference

April 4th-6th
Hosted by the University of Wisconsin-Madison
The Madison Concourse Hotel
Madison, WI
David Anderson (U. of Wisconsin-Madison) kicked off the Sherwood meeting with his review talk, “The Role of Theory and Computation in Advancement of the Stellarator Concept”. The other review talk was given by Eddy Carmack (Institute of Ocean Sciences, Canada) on “The Big New Arctic: The Non-Linear Future Has Arrived”.

Altogether, there were 117 presentations including 14 invited talks that span the field of fusion theory on topics such as plasmoid-mediated reconnection, error field penetration, broadband-MHD and gyrokinetic turbulence, field-reversed configurations, sparse grid techniques, chirping instabilities and runaway electron modes. Author-provided summaries of ten of the invited talks are included on pages 5 to 14 of this document.

There was a very strong showing by graduate students, postdocs, and young scientists at the meeting. More than 32 students attended the conference with an overwhelming majority presenting papers. A list of all participating students can be found on page 4.
Six “Student Poster Awards” were given to the following students for their exceptional presentations:

**Nicholas Roberds (Auburn University)** “Simulations of Sawtothing in CTH Using NIMROD”

**Silvia Espinosa-Gutiez (MIT)** “Efficient magnetic fields for supporting toroidal plasmas”

**Garth Whelan (University of Wisconsin-Madison)** ”Wavenumber-resolved energy transfer involving zonal flows in ITG turbulence”

**Ben Zhu (Dartmouth College)** “Global study of turbulent transport in the tokamak edge region”

**Florian Effenberg (University of Wisconsin-Madison)** “3-D Modeling of Edge Transport and Plasma Surface Interaction for Wendelstein 7-X Startup Plasmas”

** Jeff Lestz (PPPL)** “Hybrid MHD/particle simulation of sub-cyclotron Alfvén Eigenmodes in NSTX”

Student poster award winners. From left to right: Nicholas Roberds, Silvia Espinosa-Gutiez, Diego del Castillo-Negrete (Oak Ridge National Laboratory - Chair of the Sherwood Executive committee), Garth Whelan, Ben Zhu, Jeff Lestz (Not pictured: Florian Effenburg).
List of student attendees:

Eissa Al-Nasrallan (University of Wisconsin-Madison)
John Boguski (University of Wisconsin-Madison)
Tyler Cote (University of Wisconsin-Madison)
Jeffery Kollasch (University of Wisconsin-Madison)
Lucas Morton (University of Wisconsin-Madison)
Tonatiuh Sanchez-Vizuet (University of Delaware)
Jason Smoniewski (University of Wisconsin-Madison)
Andrea Becerra (University of Wisconsin-Madison)
Nicholas Roberds (Auburn University)
Kyle Bunkers (University of Wisconsin-Madison)
Meng Li (University of Texas-Austin)
Silvia Espinosa-Gutiez (MIT)
Garth Whelan (University of Wisconsin-Madison)
Torrin Bechtel (University of Wisconsin-Madison)
Zachary Wiliams (University of Wisconsin-Madison)
Adrian Fraser (University of Wisconsin-Madison)
Zz Riford (University of Wisconsin-Madison)

**Benjamin Faber (University of Wisconsin-Madison) [invited speaker]**

Jae Hoen Ahn (CEA – IRFM, France)
Ben Zhu (Dartmouth College)
Michael Halfmoon (University of Tulsa)
Calvin Lau (University of California, Irvine)
Sam Taimourzadeh (University of California, Irvine)
Christopher Flint (William & Mary)
Armen Oganesov (William & Mary)
Yao Zhou (PPPL)

Florian Effenberg (University of Wisconsin-Madison)

**Vinicius Duarte (PPPL) [invited speaker]**

**Chang Liu (PPPL) [invited speaker]**

Sean Miller (University of Washington)
Brian Cornille (University of Wisconsin-Madison)
Jeff Lestz (PPPL)

4 - 2016 International Sherwood Fusion Theory Conference Highlights
Physics of plasmoid-mediated reconnection and flux closure in simulations of Coaxial Helicity Injection

Fatima Ebrahimi\textsuperscript{1} and R. Raman\textsuperscript{2}

\textsuperscript{1} Department of astrophysical sciences, Princeton University, and PPPL, \textsuperscript{2} University of Washington

To produce plasmas that undergo fusion reactions indefinitely, non-inductive current drive techniques are essential. In a low-aspect-ratio Spherical Torus (ST), and in particular in an ST based fusion reactor, due to the restricted space for a central solenoid, elimination of the central solenoid, and thus non-inductive current-drive techniques, is necessary. Transient Coaxial Helicity Injection (CHI) is a leading candidate for plasma start-up and current formation in NSTX-U. Magnetic reconnection is essential for formation of closed flux surfaces and start-up plasma current in CHI. Here, we explore fundamental reconnection physics, in particular reconnecting plasmoids physics, in resistive MHD simulations of transient CHI. We report two major findings: 1) formation of an elongated Sweet-Parker (S-P) current sheet and a transition to plasmoid instability has for the first time been demonstrated by simulations of CHI experiments in a large-scale toroidal fusion plasma in the absence of any pre-existing instability\textsuperscript{1} and 2) a large-volume flux closure, and large fraction conversion of injected open flux to closed flux, in the NSTX-U geometry has also now been demonstrated for the first time\textsuperscript{2}. Simulations have been performed using the extended MHD NIMROD code in a realistic geometry with a toroidal guide field and using experimental NSTX poloidal coil currents.

It is found that as the helicity and plasma are injected into the device, the oppositely directed field lines in the injector region are (a) forced to re-connect through a local S-P type reconnection\textsuperscript{3,4} or (b) spontaneously reconnect when the elongated current sheet becomes MHD unstable (see Fig.1). Consistent with the theory, fundamental characteristics of the plasmoid instability, including fast reconnection rate, have been observed in these realistic simulations. Motivated by the simulations, experimental camera images have been revisited and suggest the existence of reconnecting plasmoids in NSTX (see Fig.2). As a result of the improved location of injector flux and shaping coils in NSTX-U, the simulations also show that the volume of flux closure is large and nearly all of the CHI-generated current is closed-flux current. It is found that the closed flux is over 70\% of the initial injector flux used to initiate the discharge. Work supported by DOE DE-SC0010565.

![Simulation vs Experiment](image1.png)

Figure 1: Top: Elongated current sheet formation during forced S-P reconnection. Bottom: Current sheet breaks up due to spontaneous reconnection at high S.

![Simulation vs Experiment](image2.png)

Figure 2: Left: Plasmoid formation in simulation of NSTX plasma during CHI/Right:Fast-camera image of NSTX plasma shows two discrete plasmoid–like bubble structures. First documentation of plasmoid formation in a laboratory (NSTX) predicted by realistic MHD simulations. [1]


Penetration and amplification of resonant perturbations in 3D ideal-MHD equilibria

S. R. Hudson, J. Loizu, S. Lazerson, A. Bhattacharjee and P. Helander.
Princeton Plasma Physics Laboratory

Understanding 3D ideal-MHD equilibria, as described by the ideal force-balance equation, $\nabla p = j \times B$, is fundamentally important for understanding both tokamaks & stellarators. Edge-localized modes are believed to be ideal, peeling-ballooning modes; and a ‘hot-topic’ of current research is to suppress these modes using resonant magnetic perturbations (RMPs). However, the nature of ideal-MHD equilibria in 3D geometry is profoundly affected by resonant surfaces, which beget a non-analytic dependence on the boundary. And, in order to preserve quasi-neutrality, non-physical infinite currents arise in equilibria with continuously-nested magnetic surfaces and smooth pressure & transform profiles. These difficulties are not fundamental flaws in ideal-MHD, which remains perhaps the most successful, relevant yet simplest model of plasma dynamics. It is just that, until recently, self-consistent tractable solutions to the ideal-MHD equilibrium equation for arbitrary 3D geometry had not been discovered.

Recently, for the first time, we computed the $1/x$ and delta-function current-densities, and we realized that self-consistent solutions demand locally-infinite shear at the resonant surfaces. We introduced a new class of solutions that admit additional delta-function current-densities that produce a discontinuity in the rotational-transform that removes the singularities. Our equilibrium solutions can be computed both perturbatively and using fully-nonlinear equilibrium calculations (with the SPEC code), and we present precise verification calculations. Most importantly, our solutions yield predictions that are in sharp contrast to previous predictions, with direct implications for understanding the penetration of RMPs: in ideal-MHD, a resonant perturbation penetrates past the rational surface and into the core of the plasma; and the perturbation is magnified by pressure inside the resonant surface, increasingly so as stability limits are approached. [Phys. Plasmas, 23, 055703 (2016)]
Nonlinear NIMROD modeling of DIII-D QH-mode discharges with broadband-MHD turbulence

J.R. KING¹, K. BARADA², K.H. BURRELL³, X. CHEN³, A.M. GAROFALO³, R.J. GROEBNER³, S.E. KRUGER¹, G.R. MCKEE³, M. ONO⁵, A.Y. PANKIN¹, T.L. RHODES², P.B. SNYDER³

¹Tech-X Corporation, Boulder, CO, USA; ²University of California, Los Angeles, CA, USA; ³General Atomics, San Diego, CA, USA; ⁴University of Wisconsin, Madison, WI, USA; ⁵Graduate University for Advanced Studies, Toki, Gifu, Japan

E-mail: jking@txcorp.com

It is desirable to have an ITER H-mode regime that is quiescent to edge-localized modes (ELMs). ELMs deposit large, localized and impulsive heat loads that can damage the divertor. A quiescent regime with edge harmonic oscillations (EHO) or broadband MHD activity is observed in some DIII-D, JET, JT-60U, and ASDEX-U discharge scenarios [Garofalo et al, PoP (2015); Burrell et al., PoP (2012); Garofalo et al, NF (2011) and refs. within]. These ELM-free discharges have the pedestal-plasma confinement necessary for burning-plasma operation on ITER. The mode activity associated with the EHO or broadband MHD is characterized by small toroidal-mode numbers (n≈1-5) and is thus suitable for simulation with global MHD codes. The particle transport is enhanced during QH-mode, leading to essentially steady-state profiles in the pedestal region. Relative to QH-mode operation with EHO, operation with broadband MHD tends to occur at higher densities and lower rotation and thus may be more relevant to ITER. Nonlinear NIMROD simulations initialized from a reconstruction of a DIII-D QH-mode discharge with broadband MHD saturate into a turbulent state.

Results from a nonlinear NIMROD simulation of DIII-D QH-mode shot 145098 at 4250ms with broadband MHD are presented. The measured toroidal and poloidal rotation profiles are included in the simulation as experimental observations indicate that the QH-mode operational regime is dependent on the rotation profile. The simulation develops into a saturated turbulent state and the n=1 and 2 modes become dominant through an inverse cascade. Each toroidal mode in the range of n=1-5 is dominant at a different time. The perturbations are advected and sheared apart in the counter-clockwise direction consistent with the direction of the poloidal flow inside the LCFS. Work towards validation through comparison to ECE, BES and Doppler reflectometry measurements is presented. Consistent with experimental observations during QH-mode, the simulated state leads to large particle transport relative to the thermal transport. A discussion of the transport assumptions built into our MHD modeling concludes that future QH-mode simulation studying the induced transport should run as a turbulence calculation where profiles are fixed and needs to include first-order FLR drift effects that stabilize high-n modes.
Spectrum of multi-region-relaxed magneto-hydrodynamic modes in slab geometry

or

Putting the D in MRMHD: a prescription for all that ails ideal MHD!

Robert L. Dewar,* L.H. (Alexis) Tuen, Matthew J. Hole
Plasma Theory & Modelling,
Centre for Plasmas and Fluids,
Australian National University

Bob Dewar of the Australian National University presented a dynamical extension of Taylor relaxation theory that shows promise of being both simpler and more physically applicable to hot plasmas, especially in 3-D geometries, than ideal MHD.
Gyrokinetic Delta-f Particle Simulation of Microtearing Turbulence  
J. Chowdhury, Y. Chen, W. Wan, S. E. Parker

We presented our linear and nonlinear results on microtearing mode (MTM) for parameters corresponding to experiments in the edge and core of NSTX using delta-f particle code GEM. We observe that the MTMs in the edge and core exhibit similar characteristics with respect to the dependence on toroidal mode number, electron beta and electron temperature gradient. However, they behave in a different manner with respect to dependence on collision frequency and effect of electrostatic potential. The edge MTM is independent of collisions and exists even when collision frequency is zero, the core MTM depends nonmonotonically on collision frequency and requires finite collisions for the mode to be unstable. The electrostatic potential is destabilizing in the edge and stabilizing in the core. Nonlinearly the MTM can produce substantial electron heat flux. The electromagnetic component carries bulk of the electron heat flux. Ion heat flux in these simulations is found to be very small.

The properties of the n=1, m=2 tearing mode is also studied, using a modified version of GEM for cylindrical geometry. The ions are fully gyrokinetic and electrons are drift kinetic. The linear properties of the tearing mode is compared with the earlier theories. We studied three cases, collisionless pure tearing mode, semicollisional pure tearing mode and collisionless drift tearing mode. For phi=0 we observe close agreement with earlier theories, as long as current layer is thin enough to satisfy constant psi approximation. The 1/3 dependence of the growth rate on collision frequency for semicollisional case is recovered for larger system size. Following are mode structures for a typical MTM (upper panel) and low n tearing mode (lower panel) observed in our simulations.

References:
A High Performance FRC (HPF) discharge in the C-2U experiment [1] may be studied as a logical sequence of events and dependencies. A suite of models has been used to help to configure the experiment and to understand the key physics of each of the processes.

First, two Compact Tori (CT) are formed by theta pinch, and translated towards each other to collide and merge into a single Field Reversed Configuration (FRC) in a central confinement vessel. This process depends on a complex interaction of external fields, pulsed power, eddy currents, neutral gas loading, ionization and shock heating, supersonic translation, and thermalization. Each of these features is captured by the extended MHD code LamyRidge, showing that the CTs translate at Mach ~ 2-3, and during the collision, a hot FRC is created by conversion of the ion kinetic energy to ion thermal energy.

Second, the FRC is stabilized to low order modes. The internal n=1 tilt mode can be effectively stabilized by kinetic ion effects, which is the motivation for the generation of a hot FRC. Experiment suggests that this is successful, and 3D hybrid PIC simulation with the HYM [2] and FPIC codes are in accord, indicating low growth rates and nonlinear saturation of the mode. The n=2 rotational mode is empirically stable/unstable with/without end-biasing of the SOL field lines. Simulations with the HYM code show that without biasing, the n=2 mode goes unstable after tilt saturation, but with negative radial electric field end biasing, the n=2 mode is stabilized. These observations agree with experiment.

The resulting macroscopically stable FRC has good confinement of energetic ion orbits. The third step then is to apply Neutral Beam (NB) heating. C-2U has 10MW of NB heating, at energy 15keV chosen to reduce the orbit size. Simulation with DEGAS2 [3] and fast ion Monte Carlo codes shows that the charge exchange trapping of the NB leaves behind a warm neutral cloud which acts as a charge exchange target for the fast ions. Further transport analyses with the hybrid thermal fluid/kinetic fast ion codes Q1D and Q2D show that the NB heating is nevertheless sufficient to sustain the FRC for times longer than the intrinsic transport timescales, and that fast ion pressure is ~50% of the total pressure. These observations agree with experiment. Strategic development of first-principles kinetic turbulent transport codes, effectively a port of the GTC code to FRC geometry [4], is also underway and shows early qualitative agreement with fluctuation measurements.

The above modeling codes and others, which have been benchmarked against analytic limits and tested extensively against C-2U experimental results, have been used in the planning of TAE’s coming experimental upgrade, C-2W

Sparse Grids for PIC Simulations

L.F. Ricketson\textsuperscript{1}, A.J. Cerfon\textsuperscript{1}

\textsuperscript{1}Courant Institute of Mathematical Sciences, New York, NY 10012

The particle-in-cell (PIC) method has been a standard tool for kinetic plasma simulation for over fifty years. While many improvements have been made in that time, simulations of the complex, three-dimensional systems that arise in modern fusion applications still require many hours on a massively parallel machine. A prominent reason for this is the interaction between grid- and particle-based errors. The figure of merit for statistical errors is $N_p/N_g$, where $N_p$ is the total particle number and $N_g$ the number of grid cells. For good grid resolution, $N_g$ must be large, especially in multiple dimensions. In order to keep $N_p/N_g$ correspondingly large, the total particle number becomes overwhelmingly enormous.

For the simplest PIC schemes - first order in time, second order in space - a simple calculation reveals that the computational effort $\kappa$ required to achieve an error of size $\varepsilon$ scales as

$$\kappa \sim \varepsilon^{-(3+d_x/2)},$$

where $d_x$ is the dimension of the underlying problem in position-space. Clearly, the problem is especially prevalent in multiple dimensions, where $N_g$ grows very rapidly with increasing grid resolution.

Sparse grids are a tool from the applied mathematics community for breaking the curse of dimensionality. For the commonly used piecewise-linear shape functions, a sparse grid with overall cell width $h$ achieves error $O(h^2 \log h^{d_x-1})$ with $N_g = O(h^{-1} \log h^{d_x-1})$. Compare this to a standard grid, where the error is $O(h^2)$, but $N_g = O(h^{-d_x})$.

In PIC, this has immediate and obvious advantages for the field solve. More importantly, though, the dramatic reduction in $N_g$ also means one can reduce the particle number $N_p$ without sacrificing statistical resolution. The end result is a new complexity scaling that is nearly independent of $d_x$.

We have implemented a 2-D (in position space), electrostatic PIC scheme with sparse grids to test their effectiveness. Though results are preliminary, we find good agreement with theory and standard PIC in linear Landau damping test cases - see fig. 1. We also observe considerable speedups compared to standard PIC when $N_g$ is large - see fig. 2. A 3-D implementation is forthcoming, where even more dramatic acceleration is expected. Incorporation of additional physics in more complex geometries is also planned.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Comparison of linear Landau damping for standard PIC (red), sparse PIC (blue), and analytically predicted decay rate (black).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Time snapshots of density from three simulations of the same nonlinear Landau damping problem. All have 1024 $\times$ 1024 effective resolution. Compared to (A), the sparse grid solution (B) has comparable statistical resolution but runs 30 times faster. Compared to (C), the sparse scheme runs in comparable time, but with considerably improved statistics.}
\end{figure}
Realistic characterization of chirping instabilities in tokamaks
Vinícius Duarte\textsuperscript{1,2}, Nikolai Gorelenkov\textsuperscript{2}, Herb Berk\textsuperscript{3}, Mario Podestà\textsuperscript{2}, Mike Van Zeeland\textsuperscript{4}, David Pace\textsuperscript{4} and Bill Heidbrink\textsuperscript{5}
\textsuperscript{1}Institute of Physics, University of São Paulo, São Paulo, SP, 05508-090, Brazil
\textsuperscript{2}Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ, 08543, USA
\textsuperscript{3}Institute for Fusion Studies, University of Texas, Austin, TX, 78712, USA
\textsuperscript{4}General Atomics, San Diego, CA, 92186, USA
\textsuperscript{5}University of California, Irvine, CA, 92697, USA

Frequency chirping instabilities are believed to be an important convective mechanism for losses of fast ions upon their interaction with Alfvénic modes. The possibility of chirping onset was studied using the theory of driven kinetic instabilities in the presence of dissipation. An extension was made to account for multiple resonance surfaces and realistic mode structures. In order to develop predictive capabilities, a criterion for chirping modes existence was derived and shown to be in accordance with observations in different tokamaks. The criterion involves fast particle velocity drag and pitch angle scattering. It was shown that Coulomb collisions alone are not sufficient for the consistency of this criterion with the experimentally observed modes. Microturbulence was brought to the model for the first time and was shown to be an important diffusive mechanism responsible for particle decorrelation from the coherent structures that support chirping modes. Figure 1 shows scans of the predicted criterion evaluated using the kinetic NOVA-K code for one mode in DIII-D that starts chirping after the plasma enters H mode. The chirping onset correlates with a substantial drop of fast ion particle diffusivity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Scans of the chirping criterion are shown for a DIII-D shot. If the criterion gives a positive (negative) number, it means that the mode is likely to be steady (chirping). The point representing the actual experimental condition crosses the boundary (black line) and transitions from the positive (left plot – before chirping was observed) to the negative region (right plot – during chirping observation) of the chirping criterion.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Scans of the chirping criterion are shown for a DIII-D shot. If the criterion gives a positive (negative) number, it means that the mode is likely to be steady (chirping). The point representing the actual experimental condition crosses the boundary (black line) and transitions from the positive (left plot – before chirping was observed) to the negative region (right plot – during chirping observation) of the chirping criterion.}
\end{figure}

It was shown that the application of the derived criterion is consistent with several observations on NSTX, DIII-D and TFTR. Further study is planned for the near future which seem to be important for tokamak operations on present day and future burning plasma experiments.
Adjoint method and runaway electron dynamics in momentum space

Chang Liu¹, Dylan P. Brennan¹, Eero Hirvijoki¹, Allen H. Boozer², Amitava Bhattacharjee¹
¹ Princeton Plasma Physics Laboratory, Princeton, NJ
² Columbia University, New York, NY

Runaway electron physics is an important aspect of the post-thermal collapse in disruptions, and is a critical area for current research. Theoretical and experimental studies have shown that various kinds of kinetic effects, including the collisional drag force, the pitch angle scattering, and the synchrotron and bremsstrahlung radiation can significantly affect the runaway electron dynamics. Also, each of these different forces dominates in different regions of momentum space.

In this study, we use a novel theoretical tool, the adjoint method, to study the runaway electron momentum space structure. The adjoint method can include all the aforementioned kinetic effects, overcome some of the limitations of other commonly used methods such as the test-particle and Monte-Carlo methods, and offer an alternative and insightful way to understand the physics of all methods of solution. Using the adjoint method, one can obtain results including the runaway probability function and the expected slowing-down time. These results are consistent with previous studies, including the increase of the critical electric field due to radiation effects and the runaway electron hysteresis. These methods can also be used to study the runaway electron avalanche and estimate the generation of seed runaway electrons in the thermal quench and the exponential growth rate in the current quench.

In addition, we use the adjoint method to study the role of large angle scattering in the runaway electron energy decay when the electric field is close to but less than the critical value (the marginal case). For this case, we develop a Boltzmann collision operator that includes both small and large angle scattering self-consistently. In contrast with the common belief that small angle scattering is much more important than large angle scattering in weakly coupled plasmas, we find that for the marginal case large-angle scattering plays an important role.

These results can help us better understand the runaway electron momentum space structure, and give insights into quiescent runaway electron (QRE) experiments and runaway electron mitigation in disruptions.

Fig 1: Runaway Probability Function ($P$) for $Z=7$. The probability has a localized smooth transition around a separatrix (dashed) at some momentum $p$, which is a function of pitch angle $\theta$.

Fig 2: Expected Slowing-down Time ($T$) for $Z=1$ and $E=1.5E_{\text{ci}}$ along $\theta=0$. The dependence of the forces on $p$ causes a transport barrier, above which the slowing down time rapidly increases. The solid lines include large angle collisions, indicating it is important in reducing the slowing down time.
Primary runaway electron generation and saturation in a tokamak

Zehua Guo, Chris Mcdevitt, Xianzhu Tang
Theoretical Division, Los Alamos National Lab, NM 87545

Understanding the physics of runaway electrons (RE) is essential for mitigating the most deleterious effect of tokamak disruptions. We have developed a parallel simulation code to study the dynamics of REs in tokamak geometries.

We first show that by taking into account both the energy and the pitch angle fluxes, a simple model is able to predict the presence of a local vortex structure and O point in the \((p,\xi \equiv p_\parallel/p)\) space. This is then confirmed by our simulation results. Formation of the global circulation and vortex structure is the result of competition among several effects: electric field acceleration, collisional drag, synchrotron radiation (SR) damping, pitch-angle diffusion and the squeezing in pitch-angle by electric field. As the REs reach higher energy, pitch-angle scatterings turn them away from \(\xi = -1\) so that the SR due to gyration becomes dominant eventually, leading to a phase-space return flow toward lower \(p\) and \(|\xi|\). The vortex structures and circulating flow in phase-space reach a steady state and produce a saturated primary RE population, which appears as a bump in the \((p,\xi)\) space as opposed to an attractor in energy space.

We also discuss the effects of toroidal geometry. Since trapped electrons can no longer experience parallel electric field acceleration, the phase-space flow is significantly modified. As the result, magnetic trapping can effectively reduce the primary RE population and the energy carried by them via changing the circulation pattern. A peculiar peaked runaway distribution near the trap-passing boundary inside the electric-deceleration region is understood by examining the phase space flow structure in the presence of a trap-region.

This study clearly motivates a reexamination of the Rosenbluth-Putvinski avalanche picture for tokamak runaway electrons, which predicts a saturation when the runaways take over the full plasma current. Our study shows that the complex phase space circulation, coupled with radial transport, can provide an additional means for secondary runaway saturation, and is an essential element in tokamak runaway physics.

Work supported by DOE OFES & LANL-LDRD.

![Figure 1](image1.png)

Figure 1: Left: local vortex structure for slab case; Right: \(f(p,\xi)\) for toroidal case.